

# 三角関数公式連関表

## 三角関数の定義, 相互関係

単位円  $x^2 + y^2 = 1$

$$\begin{cases} \sin \theta = (\text{P の } y \text{ 座標}) \\ \cos \theta = (\text{P の } x \text{ 座標}) \\ \tan \theta = \frac{\sin \theta}{\cos \theta} = (\text{OP の 傾き}) \end{cases}$$

$$\cos^2 \theta + \sin^2 \theta = 1 \quad \xrightarrow{\div \cos^2 \theta} \quad 1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$$

## 還元公式

$$\begin{cases} \sin(-\theta) = -\sin \theta \\ \cos(-\theta) = \cos \theta \\ \tan(-\theta) = -\tan \theta \end{cases} \quad \begin{cases} \sin(\theta + 2n\pi) = \sin \theta \\ \cos(\theta + 2n\pi) = \cos \theta \\ \tan(\theta + 2n\pi) = \tan \theta \end{cases} \quad \begin{cases} \sin(\theta + \pi) = -\sin \theta \\ \cos(\theta + \pi) = -\cos \theta \\ \tan(\theta + \pi) = \tan \theta \end{cases}$$

$$\begin{cases} \sin\left(\theta + \frac{\pi}{2}\right) = \cos \theta \\ \cos\left(\theta + \frac{\pi}{2}\right) = -\sin \theta \\ \tan\left(\theta + \frac{\pi}{2}\right) = -\frac{1}{\tan \theta} \end{cases} \quad \begin{cases} \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \\ \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \\ \tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan \theta} \end{cases} \quad \begin{cases} \sin(\pi - \theta) = \sin \theta \\ \cos(\pi - \theta) = -\cos \theta \\ \tan(\pi - \theta) = -\tan \theta \end{cases}$$

## 合成公式 (sin への合成)

$$a \sin \theta + b \cos \theta = \sqrt{a^2 + b^2} \left( \frac{a}{\sqrt{a^2 + b^2}} \sin \theta + \frac{b}{\sqrt{a^2 + b^2}} \cos \theta \right)$$

$$= \sqrt{a^2 + b^2} (\cos \alpha \sin \theta + \sin \alpha \cos \theta)$$

$$= \sqrt{a^2 + b^2} \sin(\theta + \alpha)$$

## 加法定理 (出発点!)

$$\begin{cases} \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{cases}$$

## 2倍角の公式

$$\begin{aligned} \sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 1 - 2 \sin^2 \alpha \quad (\text{sin 表示}) \\ &= 2 \cos^2 \alpha - 1 \quad (\text{cos 表示}) \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \end{aligned}$$

## 3倍角の公式

$$\begin{aligned} \sin 3\alpha &= \sin(\alpha + 2\alpha) \\ &= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\ &= \sin \alpha (1 - 2 \sin^2 \alpha) + \cos \alpha \cdot 2 \cos \alpha \sin \alpha \\ &= \sin \alpha - 2 \sin^3 \alpha + 2(1 - \sin^2 \alpha) \sin \alpha \\ &= 3 \sin \alpha - 4 \sin^3 \alpha \end{aligned}$$

$$\begin{aligned} \cos 3\alpha &= \cos(\alpha + 2\alpha) \\ &= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\ &= \cos \alpha (2 \cos^2 \alpha - 1) - \sin \alpha \cdot 2 \sin \alpha \cos \alpha \\ &= 2 \cos^3 \alpha - \cos \alpha - 2(1 - \cos^2 \alpha) \cos \alpha \\ &= 4 \cos^3 \alpha - 3 \cos \alpha \end{aligned}$$

余弦定理

$$\begin{cases} \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \end{cases}$$

## 半角の公式

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} \iff \sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2} \iff \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\tan^2 \frac{\alpha}{2} = \frac{\sin^2 \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2}} = \frac{\frac{1 - \cos \alpha}{2}}{\frac{1 + \cos \alpha}{2}} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

## 和積 (積和) 変換公式 ( $\alpha + \beta = A, \alpha - \beta = B$ )

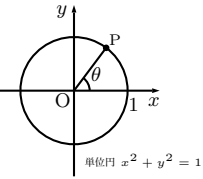
$$\begin{cases} \sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta \\ \sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos \alpha \sin \beta \\ \cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta \\ \cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \sin \alpha \sin \beta \end{cases} \quad \begin{cases} \sin \alpha \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \} \\ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos \alpha \sin \beta = \frac{1}{2} \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \} \\ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos \alpha \cos \beta = \frac{1}{2} \{ \cos(\alpha + \beta) + \cos(\alpha - \beta) \} \\ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin \alpha \sin \beta = -\frac{1}{2} \{ \cos(\alpha + \beta) - \cos(\alpha - \beta) \} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{cases}$$

## $\sin \alpha, \cos \alpha, \tan \alpha$ の $t \left( = \tan \frac{\alpha}{2} \right)$ による有理関数表示 (※数 III の三角関数の積分等で使用)

$$\begin{aligned} \sin \alpha &= 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & \cos \alpha &= 2 \cos^2 \frac{\alpha}{2} - 1 & \tan \alpha &= \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}} \\ &= 2 \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \cos^2 \frac{\alpha}{2} & &= 2 \frac{1}{1 + \tan^2 \frac{\alpha}{2}} - \frac{1 + \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} & &= \frac{2t}{1 - t^2} \\ &= 2 \tan \frac{\alpha}{2} \frac{1}{1 + \tan^2 \frac{\alpha}{2}} & &= \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} & &= \frac{1 - t^2}{1 + t^2} \\ &= \frac{2t}{1 + t^2} & &= \frac{1 - t^2}{1 + t^2} & & \end{aligned}$$

# 三角関数公式連関表 (問題 20分)

三角関数の定義, 相互関係



$$\begin{cases} \sin \theta = \\ \cos \theta = \\ \tan \theta = \end{cases}$$

還元公式

$$\begin{cases} \sin(-\theta) = \\ \cos(-\theta) = \\ \tan(-\theta) = \end{cases} \quad \begin{cases} \sin(\theta + 2n\pi) = \\ \cos(\theta + 2n\pi) = \\ \tan(\theta + 2n\pi) = \end{cases} \quad \begin{cases} \sin(\theta + \pi) = \\ \cos(\theta + \pi) = \\ \tan(\theta + \pi) = \end{cases}$$

$$\begin{cases} \sin\left(\theta + \frac{\pi}{2}\right) = \\ \cos\left(\theta + \frac{\pi}{2}\right) = \\ \tan\left(\theta + \frac{\pi}{2}\right) = \end{cases} \quad \begin{cases} \sin\left(\frac{\pi}{2} - \theta\right) = \\ \cos\left(\frac{\pi}{2} - \theta\right) = \\ \tan\left(\frac{\pi}{2} - \theta\right) = \end{cases} \quad \begin{cases} \sin(\pi - \theta) = \\ \cos(\pi - \theta) = \\ \tan(\pi - \theta) = \end{cases}$$

合成公式

$$a \sin \theta + b \cos \theta$$

加法定理 (出発点!)

$$\begin{cases} \sin(\alpha + \beta) = \\ \sin(\alpha - \beta) = \\ \cos(\alpha + \beta) = \\ \cos(\alpha - \beta) = \end{cases}$$

余弦定理

2倍角の公式

$$\begin{cases} \sin 2\alpha = \\ \cos 2\alpha = \\ = \\ = \\ \tan 2\alpha = \end{cases}$$

(sin 表示)

(cos 表示)

3倍角の公式

$$\begin{cases} \sin 3\alpha = \\ \cos 3\alpha = \end{cases}$$

$$\begin{cases} \tan(\alpha + \beta) = \\ \tan(\alpha - \beta) = \end{cases}$$

半角の公式

$$\begin{cases} \sin^2 \alpha = & \iff & \sin^2 \frac{\alpha}{2} = \\ \cos^2 \alpha = & \iff & \cos^2 \frac{\alpha}{2} = \end{cases} \quad \tan^2 \frac{\alpha}{2} =$$

和積 (積和) 変換公式  $(\alpha + \beta = A, \alpha - \beta = B)$

$$\begin{cases} \sin(\alpha + \beta) + \sin(\alpha - \beta) = & \begin{cases} \sin \alpha \cos \beta = \\ \sin A + \sin B = \end{cases} \\ \sin(\alpha + \beta) - \sin(\alpha - \beta) = & \begin{cases} \cos \alpha \sin \beta = \\ \sin A - \sin B = \end{cases} \\ \cos(\alpha + \beta) + \cos(\alpha - \beta) = & \begin{cases} \cos \alpha \cos \beta = \\ \cos A + \cos B = \end{cases} \\ \cos(\alpha + \beta) - \cos(\alpha - \beta) = & \begin{cases} \sin \alpha \sin \beta = \\ \cos A - \cos B = \end{cases} \end{cases}$$

$\sin \alpha, \cos \alpha, \tan \alpha$  の  $t \left( = \tan \frac{\alpha}{2} \right)$  による有理関数表示 (※数 III の三角関数の積分等で使用)

$$\begin{cases} \sin \alpha = \\ \cos \alpha = \\ \tan \alpha = \end{cases}$$