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(理, 医, データサイエンス)

[I]

(1) $\alpha = 2^{\log_4 9}$

$\iff \log_2 \alpha = \log_4 9 = \dfrac{2\log_2 3}{\log_2 4}$

$\therefore \alpha = \underline{3}$ //

(2) $z^4 = z^2 - 1$

$\iff z^4 - z^2 + 1 = 0 \quad \times(z^2+1)$

$\iff z^6 + 1 = 0$

$\iff z^6 = -1$

$z^{40} + 2z^{10} + \dfrac{1}{z^{20}}$

$= (z^6)^6 z^4 + 2z^6 z^4 + \dfrac{z^4}{z^{24}}$

$= z^4 - 2z^4 + z^4$

$= \underline{0}$ //

(3) $f(x) = \dfrac{1}{2+\cos^2 x} + \dfrac{1}{1+\sin^2 x}$ ← $1-\cos^2 x$

$= \dfrac{4}{4-\cos^4 x} = \dfrac{64}{63}$

$\iff \dfrac{1}{4-\cos^4 x} = \dfrac{16}{63}$

$\iff 64 - 16\cos^4 x = 63$

$\iff 1 = 16\cos^4 x$

$\iff \cos^4 x = \dfrac{1}{16}$

$\therefore \cos x = \pm\dfrac{1}{2}$ より $x = 60^\circ, 120^\circ$

なので $\underline{2コ}$ //

(4)

P(5問中3問正解)

$= {}_5C_3 \left(\dfrac{1}{5}\right)^3 \left(\dfrac{4}{5}\right)^2 = \underline{\dfrac{32}{625}}$ //

[II]

(1)

P(和が3以上)

$= \dfrac{{}_cC_2 + c\cdot b}{{}_{a+b+c}C_2}$

$= \dfrac{\frac{c(c-1)}{2} + c\cdot b}{\frac{(a+b+c)(a+b+c-1)}{2}}$

$= \underline{\dfrac{c(c-1+2b)}{(a+b+c)(a+b+c-1)}}$ //

(2)

n回中k回表が出る確率は

${}_nC_k \left(\dfrac{1}{2}\right)^k \left(\dfrac{1}{2}\right)^{n-k} = \dfrac{{}_nC_k}{2^n}$

この後〇を引く確率は

$\dfrac{d+k}{d+e+n}$

求める確率は

$\displaystyle\sum_{k=0}^{n} \dfrac{{}_nC_k}{2^n} \cdot \dfrac{d+k}{d+e+n}$

$= \dfrac{1}{2^n(d+e+n)} \left(d\displaystyle\sum_{k=0}^{n} {}_nC_k + \sum_{k=0}^{n} k\cdot{}_nC_k\right)$

ここで

$(1+x)^n = \displaystyle\sum_{k=0}^{n} {}_nC_k x^k$ ↓ 微分

$n(1+x)^{n-1} = \displaystyle\sum_{k=1}^{n} k\cdot{}_nC_k x^{k-1}$

↓ $x=1$

$n\cdot 2^{n-1} = \displaystyle\sum_{k=0}^{n} k\cdot{}_nC_k$

$= \dfrac{1}{2^n(d+e+n)} \left(d\cdot 2^n + n\cdot 2^{n-1}\right)$

$= \underline{\dfrac{2d+n}{2(d+e+n)}}$ //

(3)

1 … m枚

無 … $x-m$;

求める確率は

$\underline{\dfrac{{}_mC_k \cdot {}_{x-m}C_{n-k}}{{}_xC_n}}$ //

[III]

$l: x-2 = \dfrac{y}{2} = z+1$

$m: x-3 = \dfrac{y-5}{-2} = \dfrac{z+5}{-2}$

それぞれの方向ベクトル $\vec{d_l}, \vec{d_m}$ は

$\vec{d_l} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \vec{d_m} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

$\vec{p} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ とおくと

$\vec{d_l}\cdot\vec{p} = a+2b+c = 0$

$\vec{d_m}\cdot\vec{p} = a-2b-2c = 0$

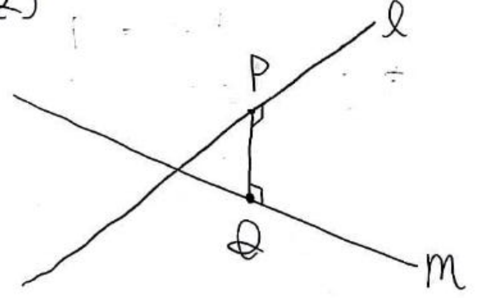
↓

$2a - c = 0 \qquad 3a + 2b = 0$

$\iff c = 2a \qquad \iff b = -\dfrac{3}{2}a$

$\therefore \vec{p} = \begin{pmatrix} a \\ -\frac{3}{2}a \\ 2a \end{pmatrix}$ (aは実数)

(2)



$\ell : \begin{cases} x = 2 + t_\ell \\ y = 2t_\ell \\ z = -1 + t_\ell \end{cases}$ $m : \begin{cases} x = 3 + t_m \\ y = 5 - 2t_m \\ z = -5 - 2t_m \end{cases}$

$P(2+p, 2p, -1+p)$

$Q(3+q, 5-2q, -5-2q)$

とおくと、

$\overrightarrow{PQ} = \begin{pmatrix} q-p+1 \\ -2q-2p+5 \\ -2q-p-4 \end{pmatrix} = \begin{pmatrix} k \\ -\frac{3}{2}k \\ 2k \end{pmatrix}$

解くと $k=-2,\ p=2,\ q=-1$

$\therefore \underline{P(4,4,1),\ Q(2,7,-3)}$ //

$n : \begin{cases} x = 4 + t \\ y = 4 - \frac{3}{2}t \\ z = 1 + 2t \end{cases}$ (tは実数)

//

(3) PQの中点は $\left(3, \frac{11}{2}, -1\right)$.

$(半径)^2 = (3-4)^2 + \left(\frac{11}{2} - 4\right)^2 + (-1-1)^2$

$= \frac{29}{4}$

$\therefore \underline{(x-3)^2 + \left(y - \frac{11}{2}\right)^2 + (z+1)^2 = \frac{29}{4}}$ //

[IV] $S(x) = \int_0^x \frac{1}{\sqrt{1-t^2}}\,dt$

(1)

$\lim_{x\to 0} \frac{S(x)}{x}$

$= \lim_{x\to 0} \frac{S(x) - S(0)}{x - 0}$

$= S'(0)$ $\left(S'(x) = \frac{1}{\sqrt{1-x^2}}\right)$

$= \underline{1}$ //

(2)

$S\left(\frac{1}{\sqrt{2}}\right)$ $\begin{array}{l} t = \sin\theta \\ dt = \cos\theta\, d\theta \end{array}$

$= \int_0^{\frac{\pi}{4}} \frac{\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$

$= \left[\theta\right]_0^{\frac{\pi}{4}}$

$= \underline{\frac{\pi}{4}}$ //

(3)

$\int \frac{t}{\sqrt{1-t^2}}\,dt$ $\begin{array}{l} \sqrt{1-t^2} = u \\ 1-t^2 = u^2 \\ -2t\,dt = 2u\,du \end{array}$

$= \int \frac{-u}{u}\,du$

$= -u + C$

$= \underline{-\sqrt{1-t^2} + C}$ (Cは積分定数) //

(4)

$\int_0^{\frac{1}{\sqrt{2}}} S(x)\,dx$

$= \left[xS(x)\right]_0^{\frac{1}{\sqrt{2}}} - \int_0^{\frac{1}{\sqrt{2}}} xS'(x)\,dx$

$= \frac{1}{\sqrt{2}} S\left(\frac{1}{\sqrt{2}}\right) - \int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{1-x^2}}\,dx$

$= \frac{1}{\sqrt{2}} \cdot \frac{\pi}{4} + \left[\sqrt{1-x^2}\right]_0^{\frac{1}{\sqrt{2}}}$

$= \underline{\frac{\sqrt{2}}{8}\pi + \frac{\sqrt{2}}{2} - 1}$ //

[V]

$E(X_i) = \sum_{k=1}^{a} k \cdot \frac{1}{a}$

$= \frac{1}{a} \cdot \frac{1}{2} \cdot a(a+1)$

$= \frac{1}{2}(a+1)$

$V(X_i) = E(X_i^2) - (E(X_i))^2$

$= \frac{1}{a} \cdot \frac{1}{6} a(a+1)(2a+1) - \frac{(a+1)^2}{4}$

$= \frac{a+1}{12}(4a+2-3a-3) = \frac{(a+1)(a-1)}{12}$

(1) $a=26$のとき

$E(X_i) = \underline{\frac{27}{2}}$ // $V(X_i) = \frac{\cancel{27}^{9} \cdot 25}{\cancel{12}_{4}} = \underline{\frac{225}{4}}$ //

(2)

$E(Y_i) = \sum_{k=1}^{13} (2k-1) \frac{1}{13}$

$= \frac{1}{13}(1+25) 13 \frac{1}{2}$

$= \underline{13}$ //

$V(Y_i) = E(Y_i^2) - 13^2$

$= \sum_{k=1}^{13} (2k-1)^2 \frac{1}{13} - 13^2$

$= \sum_{k=1}^{13} (4k^2 - 4k + 1) \frac{1}{13} - 13^2$

$= \frac{4}{13} \sum_{k=1}^{13} k(k-1) + 1 - \underbrace{13^2}_{169}$

$= \frac{4}{13} \sum_{k=1}^{12} k(k+1) - 168$

$= \frac{4}{\cancel{13}} \cdot \frac{1}{\cancel{3}} \cdot \cancel{12}^{4} \cdot \cancel{13} \cdot 14 - 168$

$= \underline{56}$ //

(3)

$$E(m)$$
$$=E\left(\frac{X_1+\cdots+X_n}{n}\right)$$
$$=\frac{1}{n}\cdot n\cdot\frac{27}{2}=\frac{27}{2}$$

$$\sigma(m)$$
$$=\sqrt{V(m)}$$
$$=\sqrt{V\left(\frac{X_1+\cdots+X_n}{n}\right)}$$
$$=\sqrt{\frac{1}{n^2}\cdot\frac{225}{4}\cdot n}=\frac{15}{2\sqrt{n}}$$

$$C=\frac{15}{2\sqrt{n}}\cdot\frac{2}{27}$$
$$=\frac{5}{9\sqrt{n}}<0.1$$
$$\Leftrightarrow \frac{50}{9}<\sqrt{n}$$
$$\therefore n>\frac{2500}{81}$$
$$\min n=\underline{31}_{//}$$

(4)

$$Z=\frac{12-m}{\frac{8}{\sqrt{100}}}$$
$$=\frac{5}{4}(12-m)$$

↓ 95%信頼区間は

$$-1.96\leqq\frac{5}{4}(12-m)\leqq 1.96$$
$$\Leftrightarrow -1.568\leqq 12-m\leqq 1.568$$
$$\Leftrightarrow 10.432\leqq m\leqq 13.568$$
$$\therefore \underline{10.43\leqq m\leqq 13.57}_{//}$$