2023 共通テスト数学IIB

第1問

[1]

(1) $x=\frac{\pi}{6}$ のとき

$\sin x < \sin 2x \cdots \underline{0}$

$x=\frac{2}{3}\pi$ のとき

$\sin x > \sin 2x \cdots \underline{2}$

(2)

$\sin 2x - \sin x$

$= \sin x(\underline{2}\cos x - \underline{1})$

$\sin x > 0$ かつ $\cos x > \frac{1}{2}$ なのは

$0 < x < \underline{\frac{\pi}{3}}$

$\sin x < 0$ かつ $\cos x < \frac{1}{2}$ なのは

$\pi < x < \underline{\frac{5}{3}\pi}$

(3) $\sin 4x - \sin 3x = 2\cos\frac{7}{2}x\sin\frac{x}{2}$

ク… $\underline{0}$　ケ… $\underline{7}$

$\cos\frac{7}{2}x > 0$ かつ $\sin\frac{x}{2} > 0$ なのは

$0 < x < \underline{\frac{\pi}{7}}$　$\underline{\frac{3}{7}\pi} < x < \underline{\frac{5}{7}\pi}$

ちなみに $\sin\frac{x}{2} < 0$ にならない.

$\sin 4x > \sin 2x$ なのは

$0 < 2x < \frac{\pi}{3}$, $\pi < 2x < \frac{5}{3}\pi$

$\Leftrightarrow 0 < x < \frac{\pi}{6}$, $\frac{\pi}{2} < x < \frac{5}{6}\pi$

$\sin 3x > \sin 4x > \sin 2x$ なのは

$\frac{\pi}{7} < x < \underline{\frac{\pi}{6}}$, $\frac{5}{7}\pi < x < \underline{\frac{5}{6}\pi}$

[2]

(1) $\log_a b = x$

$\Leftrightarrow a^x = b \cdots \underline{2}$

(2)

(i) $\log_5 25 = \underline{2}$

$\log_9 27 = \underline{\frac{3}{2}}$

(ii) $\log_2 3 = \frac{p}{q}$ と仮定

$\Leftrightarrow 2^{\frac{p}{q}} = 3$

$\therefore 2^p = 3^q \cdots \underline{5}$

(iii) 同様に

$a^p = b^q$

a と b の一方が偶数, もう一方が奇数

なら $\log_a b \neq$ ① $\cdots \underline{5}$

第2問

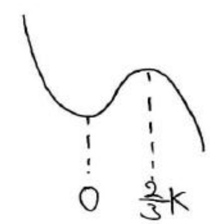
[1]

(1) x 軸との共有点

$(0,0)$ と $(k,0) \cdots \underline{4}$

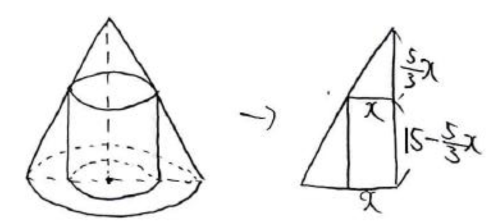
$f'(x) = \underline{-3}x^2 + \underline{2k}x$

$= x(2k-3x)$



$x=0 \cdots \underline{0}$ で極小値 $0 \cdots \underline{0}$

$x=\frac{2}{3}k \cdots \underline{3}$ で極大値 $\underline{\frac{4}{27}k^3} \cdots \underline{9}$



$V = x^2\pi \times (15 - \frac{5}{3}x)$

$= \underline{\frac{5}{3}}\pi x^2(\underline{9-x})$

$x = \frac{2}{3}k = 6$ で V は最大値

$\frac{5}{3}\pi \times \frac{4}{27}k^3 = \underline{180\pi}$

[2]

(1)

$\int_0^{30}(\frac{1}{5}x+3)dx$

$= [\frac{1}{10}x^2+3x]_0^{30}$

$= \underline{180}$

$\int(\frac{1}{100}x^2 - \frac{1}{6}x + 5)dx$

$= \underline{\frac{1}{300}x^3 - \frac{1}{12}x^2 + 5x + C}$

(2)

(i)

$S(t)$

$= \int_0^t(\frac{1}{5}x+3)$

$= \frac{1}{10}t^2 + 3t > 400$

$\Leftrightarrow t^2 + 30t - 4000 > 0$

$\Leftrightarrow (t+80)(t-50) > 0$

$t > 50 \cdots \underline{4}$

(ii)

$\int_{30}^{40} f(x)dx < \int_{40}^{50} f(x)dx$

$\cdots \underline{0}$

セ…④

第3問

(1)

(i) $P(X \geqq m)$

$= P\left(\frac{X-m}{\sigma} \geqq 0\right) = \frac{1}{2}$

(ii)

$E(\overline{X}) = m$ …④

$\sigma(\overline{X}) = \frac{\sigma}{\sqrt{n}}$ …②

$z_0 = 1.65$

$-1.65 \leqq Z = \frac{30-m}{\frac{3.6}{\sqrt{400}}} \leqq 1.65$

$\Leftrightarrow -1.65 \times 0.18 \leqq 30 - m \leqq \underbrace{1.65 \times 0.18}_{0.297}$

$\Leftrightarrow \quad \leqq m \leqq 30.297$

$\uparrow$ $30-0.297$

近いのは ④

(2)

$P(\text{Sサイズ}) = \frac{1}{2}$

P_0

$= {}_{50}C_{25}\left(\frac{1}{2}\right)^{25}\left(\frac{1}{2}\right)^{25}$

セ $= np = (50+k)\frac{1}{2} = \frac{50+k}{2}$ …②

ソ $= npq = \frac{50+k}{4}$ …⑦

$P_k = P(25 \leqq U_k \leqq 25+k)$

$= P\left(-\frac{k}{2} \leqq U_k - \frac{50+k}{2} \leqq \frac{k}{2}\right)$

$= P\left(-\frac{k}{\sqrt{50+k}} \leqq Y \leqq \frac{k}{\sqrt{50+k}}\right)$

タ…⓪

$\alpha^2 \geqq 4\beta^2$

$\Leftrightarrow k^2 \geqq 4(50+k)$

$\Leftrightarrow k^2 - 4k - 200 \geqq 0$

$k \geqq 2 + \sqrt{4+200}$

$\min k = k_0 = 17$

第4問

(1) $a_{n+1} = 1.01 a_n + p$

$a_3 = 1.01[1.01(10+p)+p]+p$ …②

イ…⓪ ウ…③

$a_{n+1} + 100p = 1.01(a_n + 100p)$

エ…④ オ…⓪

カ…② キ…③

$a_n = 10 \times 1.01^{n-1}$

$+ p\underbrace{\sum_{k=1}^{n} 1.01^{k-1}}$ …②

$\Downarrow$

$\frac{1-1.01^n}{1-1.01}$

$= 100(1.01^n - 1)$ …①

$1.01 \times a_{10} \geqq 30$ …③

$\Leftrightarrow$

$1.01 \times (10 \times 1.01^9 + p \cdot 100(1.01^9 - 1))$

$\geqq 30$

$\Leftrightarrow p \times 101(1.01^{10} - 1)$

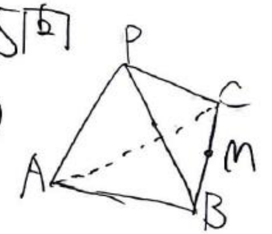
$\geqq 30 - 10 \times 1.01^{10}$

$\Leftrightarrow p \geqq \frac{30 - 10 \times 1.01^{10}}{101(1.01^{10} - 1)}$

(3) ソ…⑧

第5問

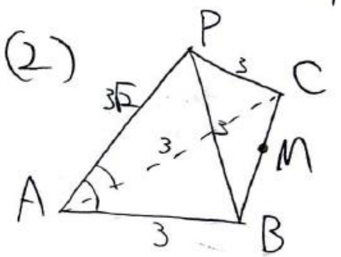
(1)



$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC}$

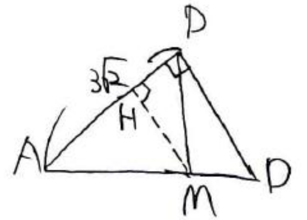
オ $= \cos\theta$ …①

(2)



$\overrightarrow{AP} \cdot \overrightarrow{AB} = \overrightarrow{AP} \cdot \overrightarrow{AC} = 3\sqrt{2} \cdot 3\cos 45^\circ$

$= 9$



$\overrightarrow{AP} \cdot \overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AP} \cdot \overrightarrow{AB} + \overrightarrow{AP} \cdot \overrightarrow{AC})$

$= 9$

$\therefore \underbrace{|\overrightarrow{AM}|\cos\angle PAD}_{AH} = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$

$= \frac{3\sqrt{2}}{2}$

$\therefore \overrightarrow{AD} = 2\overrightarrow{AM}$

$k = 1$のとき $\cdots 2$

(3)

(i) $\overrightarrow{PA} \cdot \overrightarrow{PQ} = 0$

$\Leftrightarrow \overrightarrow{PA} \cdot (\underbrace{\overrightarrow{AQ}}_{\overrightarrow{AB}+\overrightarrow{AC}} - \overrightarrow{AP}) = 0$

$\therefore \overrightarrow{AP} \cdot \overrightarrow{AB} + \overrightarrow{AP} \cdot \overrightarrow{AC} = \overrightarrow{AP} \cdot \overrightarrow{AP}$

$\cdots 0$

$(|\overrightarrow{AP}||\overrightarrow{AB}| + |\overrightarrow{AP}||\overrightarrow{AC}|)\cos\theta$
$= |\overrightarrow{AP}|^2$

$\therefore |\overrightarrow{AB}|\cos\theta + |\overrightarrow{AC}|\cos\theta = |\overrightarrow{AP}|$

$\cdots 3$

(ii) $k\overrightarrow{AP} \cdot \overrightarrow{AB} = \overrightarrow{AP} \cdot \overrightarrow{AC}$

$\hookrightarrow k|\overrightarrow{AB}| = |\overrightarrow{AC}| \cdots 0$

このとき

$AB' = |\overrightarrow{AB}|\cos\theta$

$AC' = |\overrightarrow{AC}|\cos\theta$

$= k|\overrightarrow{AB}|\cos\theta$

$\cdots 4$