2022 昭和大(医)

1

$2\alpha\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=(1-t)\alpha+t$

$\Leftrightarrow \sqrt{3}\alpha i=(1-\alpha)t \cdots①$

$\downarrow \alpha=x+yi$

$\sqrt{3}xi-\sqrt{3}y=(1-x)t-yti$

$\downarrow$

$\begin{cases}-\sqrt{3}y=(1-x)t\\ \sqrt{3}x=-yt\end{cases}$

$\downarrow$

①で$\alpha=1$は不適. $\therefore x\neq 1$.

$\sqrt{3}x=-y\times\frac{-\sqrt{3}y}{1-x}$

$\Leftrightarrow x(1-x)=y^2$

$\Leftrightarrow 0=x^2-x+y^2$

$\Leftrightarrow \left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

中心$\underline{\frac{1}{2}}$, 半径$\underline{\frac{1}{2}}$の円

ただし$\underline{1}$は除く.

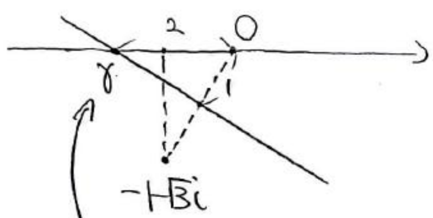
$\beta=\frac{z^6}{\alpha}\Leftrightarrow\alpha=\frac{z^6}{\beta}$

$\downarrow \left|\alpha-\frac{1}{2}\right|=\frac{1}{2}$

$\left|\frac{z^6}{\beta}-\frac{1}{2}\right|=\frac{1}{2}$ $\curvearrowleft \times 2|\beta|$

$\Leftrightarrow |\underbrace{2z^6}_{-1-\sqrt{3}i}-\beta|=|\beta|$

$\Leftrightarrow |\beta+1+\sqrt{3}i|=|\beta|$



$\underline{\gamma=-2}$

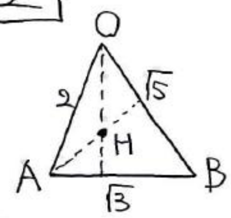
$\theta=\arg\left(1-\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{-2}\right)$

$=\arg\left(\frac{3}{4}+\frac{\sqrt{3}}{4}i\right)$

$=\arg\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

$=\underline{\frac{\pi}{6}}$

2



(1) $\underbrace{|\overrightarrow{AB}|^2}_{3}=|\vec{b}-\vec{a}|^2=\underbrace{|\vec{b}|^2}_{5}-2\vec{a}\cdot\vec{b}+\underbrace{|\vec{a}|^2}_{4}$

$\therefore \vec{a}\cdot\vec{b}=\underline{3}$

$\overrightarrow{OH}=s\vec{a}+t\vec{b}$ とおく.

$\overrightarrow{OH}\cdot\overrightarrow{AB}=(s\vec{a}+t\vec{b})\cdot(\vec{b}-\vec{a})$

$=3s-4s+5t-3t$

$=-s+2t=0 \Leftrightarrow s=2t$

$\overrightarrow{AH}\cdot\vec{b}=\{(s-1)\vec{a}+t\vec{b}\}\cdot\vec{b}$

$=3s-3+5t=0$

$\downarrow$

$11t=3$

$\therefore t=\frac{3}{11}, s=\frac{6}{11}$

$\underline{\overrightarrow{OH}=\frac{6}{11}\vec{a}+\frac{3}{11}\vec{b}}$

(3)

(a) $AD:DB=1:2$

$\therefore l_1=\frac{1}{3}\sqrt{3}=\underline{\frac{\sqrt{3}}{3}}$

(b)

$OD=\sqrt{2^2-\frac{1}{3}}$

$=\frac{\sqrt{11}}{\sqrt{3}}=\underline{\frac{\sqrt{33}}{3}}$

(4)

$\cos\angle AOB=\frac{3}{2\cdot\sqrt{5}}$

$\cos^2\angle AOB=\frac{9}{20}$

$\therefore \sin\angle AOB=\frac{\sqrt{11}}{2\sqrt{5}}$

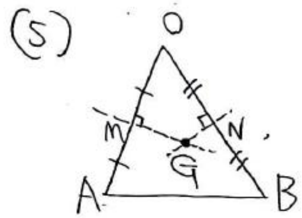
(正)

$\frac{\sqrt{3}}{\sin\angle AOB}=2R$

$\therefore R=\frac{\sqrt{3}}{\frac{\sqrt{11}}{\sqrt{5}}}$

$=\frac{\sqrt{15}}{\sqrt{11}}=\underline{\frac{\sqrt{165}}{11}}$

(5)



$\overrightarrow{OG}=p\vec{a}+q\vec{b}$ とおく.

$\overrightarrow{MG}\cdot\vec{a}$

$=\left\{\left(p-\frac{1}{2}\right)\vec{a}+q\vec{b}\right\}\cdot\vec{a}$

$=\left(p-\frac{1}{2}\right)4+3q$

$=4p+3q-2=0$

$\overrightarrow{NG}\cdot\vec{b}$

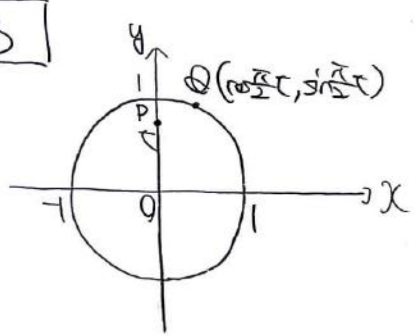
$=\left\{p\vec{a}+\left(q-\frac{1}{2}\right)\vec{b}\right\}\cdot\vec{b}$

$=3p+5q-\frac{5}{2}=0$

解くと $p=\frac{5}{22}$, $q=\frac{4}{11}$

$\therefore \underline{\overrightarrow{OG}=\frac{5}{22}\vec{a}+\frac{4}{11}\vec{b}}$

3



(1-1)

$Q\left(\cos\frac{\pi}{2}t, \sin\frac{\pi}{2}t\right)$

直線PQ

$$y=\frac{\sin\frac{\pi}{2}t-t}{\cos\frac{\pi}{2}t}x+t$$

Rのy座標は

$$\frac{\sin\frac{\pi}{2}t-t}{\cos\frac{\pi}{2}t}(-1)+t$$

$$=\frac{t\cos\frac{\pi}{2}t-\sin\frac{\pi}{2}t+t}{\cos\frac{\pi}{2}t}$$

$$\therefore R\left(-1, \frac{t\cos\frac{\pi}{2}t-\sin\frac{\pi}{2}t+t}{\cos\frac{\pi}{2}t}\right)$$ //

(1-2)

$$\lim_{t\to1-0}\frac{t\cos\frac{\pi}{2}t-\sin\frac{\pi}{2}t+t}{\cos\frac{\pi}{2}t}$$

$\downarrow$ $1-t=\theta \Leftrightarrow t=1-\theta$

$$=\lim_{\theta\to+0}\frac{(1-\theta)\cos\left(\frac{\pi}{2}-\frac{\pi}{2}\theta\right)-\sin\left(\frac{\pi}{2}-\frac{\pi}{2}\theta\right)+1-\theta}{\cos\left(\frac{\pi}{2}-\frac{\pi}{2}\theta\right)}$$

$$=\lim_{\theta\to+0}\frac{(1-\theta)\sin\frac{\pi}{2}\theta-\cos\frac{\pi}{2}\theta+1-\theta}{\sin\frac{\pi}{2}\theta}$$

$$=\lim_{\theta\to+0}\left(1-\theta+\frac{1-\cos\frac{\pi}{2}\theta}{\sin\frac{\pi}{2}\theta}-\frac{\frac{\pi}{2}\theta}{\sin\frac{\pi}{2}\theta}\times\frac{2}{\pi}\right)$$

$$=\lim_{\theta\to+0}\left(1-\theta+\frac{\sin\frac{\pi}{2}\theta}{1+\cos\frac{\pi}{2}\theta}-\frac{\frac{\pi}{2}\theta}{\sin\frac{\pi}{2}\theta}\times\frac{2}{\pi}\right)$$

$$=1-\frac{2}{\pi}$$ //

(2)

(2-1)

I_1

$$=\int_0^1\sqrt{1-x^2}\,dx$$

$$=\frac{\pi}{4}$$ //

(2-2)

I_{n+2}

$$=\int_0^1(1-x^2)^{\frac{n+2}{2}}dx \qquad x=\sin\theta,\ dx=\cos\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\cos^{n+2}\theta\cdot\cos\theta\,d\theta$$

$$=\left[\sin\theta\cdot\cos^{n+2}\theta\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\sin\theta\cdot(n+2)\cos^{n+1}\theta(-\sin\theta)\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\sin^2\theta\,(n+2)\cos^{n+1}\theta\,d\theta$$

$$=(n+2)\int_0^{\frac{\pi}{2}}\left(\cos^{n+1}\theta-\cos^{n+3}\theta\right)d\theta$$

$$=(n+2)(I_n-I_{n+2})$$

$$\Longleftrightarrow (n+3)I_{n+2}=(n+2)I_n$$

$$\therefore I_{n+2}=\frac{n+2}{n+3}I_n$$ //

(2-3)

$$I_5=\frac{5}{6}I_3=\frac{5}{6}\cdot\frac{3}{4}I_1=\frac{5}{32}\pi$$ //

4

(1) $5!=120$ //

(2)

(2-1) HとSとTが2文字

$$\frac{10!}{2!2!2!}=453600$$ //

(2-2)

$$\frac{8!}{2!}=20160$$ //

(2-3)

n(SHSHを含まない)

$=453600-$ n(SHSHを含む)

$$=453600-\frac{7!}{2!}$$

$$=451080$$ //

(2-4)

n(ST, TSの少なくとも一方を含む)

$=453600-$ n(ST, TS含まない)

(i) SS, TT

VHVPVHVOVWVAV $\leftarrow \frac{6!}{2!}$

$\frac{6!}{2!}\times{}_7C_2$

(ii) SS, T, T

VHVPVHVOVWVAV $\leftarrow \frac{6!}{2!}$

$\frac{6!}{2!}\times{}_7C_3\times3$

(iii) S, S, TT

VHVPVHVOVWVAV

(ii)と同じ

(iv) S, S, T, T

VHVPVHVOVWVAV

$\frac{6!}{2!}\times{}_7C_4\times\frac{4!}{2!2!}$

$$=453600-\frac{6!}{2!}\left({}_7C_2+{}_7C_3\times3\times2+{}_7C_4\times\frac{4!}{2!2!}\right)$$

$$=287280$$ //