2022 共通テスト 数学IIB

第1問

[1]

$(x-2)^2+(y-5)^2 \leqq 25$

(1)

中心 $(2,5)$，半径 5

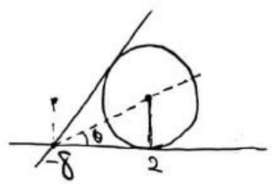
の周および内部 … ③

(2)

(i) $y=0$ はすぐわかる接線

(ii) カ … ⓪

(iii)



$\tan\theta = \frac{1}{2}$

(接線の傾き) $= \tan 2\theta$ … ①

(iv)

$\tan 2\theta = \frac{1}{1-\left(\frac{1}{2}\right)^2}$

$= \frac{4}{3}$

ℓ と D が共有点をもつのは ⑤

[2]

(1) $\log_3 9 = 2$

セ $= \left(\frac{1}{4}\right)^{-\frac{3}{2}}$

$= 4^{\frac{3}{2}}$

$= 8$

(2)

ソ : $b = a^t$ … ①

タ : $a = b^{\frac{1}{t}}$ … ①

(3)

$-1 < \log_a b < 0$，$1 < \log_a b$

$a > 1$ のとき チ … ③

$0 < a < 1$ 〃 ツ … ⓪

(4)

テ … ②

第2問

(1) $a=0$ のとき ①

$a<0$ 〃 ⓪

(2)

$f(x) = x^3 - 6ax + 16$

$f'(x) = 3x^2 - 6a$

$= 3(x^2 - 2a)$

$f(-\sqrt{2a}) = -2\sqrt{2}a^{\frac{3}{2}} + 6\sqrt{2}a^{\frac{3}{2}} + 16$

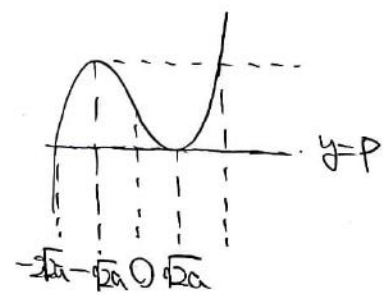
$= 4\sqrt{2}a^{\frac{3}{2}} + 16$

$f(\sqrt{2a}) = -4\sqrt{2}a^{\frac{3}{2}} + 16$

共有点 3 コ

$-4\sqrt{2}a^{\frac{3}{2}} + 16 < p < 4\sqrt{2}a^{\frac{3}{2}} + 16$

③ ②



$q = -2\sqrt{2}a^{\frac{1}{2}}$ $r = \sqrt{2}a^{\frac{1}{2}}$

(3) ①, ④

[2]

$g(x) = h(x)$

$\Leftrightarrow -3bx + 3b^2 = -x^2 + b^2$

$\Leftrightarrow x^2 - 3bx + 2b^2 = 0$

$\therefore \alpha = b$，$\beta = 2b$

$\alpha \leqq x \leqq \beta$ で $g(x) - h(x) < 0$

セ … ②

ソ … ①

$S - T$

$= \int_\alpha^\beta (h-g)\,dx - \int_\beta^t (g-h)\,dx$

$= \int_\alpha^t \{h(x) - g(x)\}\,dx$

タ … ②

$= \int_\alpha^t (-x^2 + 3bx - 2b^2)\,dx$

$= \left[-\frac{1}{3}x^3 + \frac{3b}{2}x^2 - 2b^2x\right]_\alpha^t$

$= -\frac{1}{3}t^3 + \frac{3b}{2}t^2 - 2b^2t$

$+ \frac{1}{3}b^3 - \frac{3b^3}{2} + 2b^3$

$= \frac{-1}{6}(2t^3 - 9bt^2 + 12b^2t - 5b^3)$

$s=t$ なのは

$$\begin{array}{rrrrl} 2 & -9 & 12 & -5 & \\ & 2 & -7 & & \lfloor 1 \\ \hline 2 & -7 & 5 & & \\ & 2 & -5 & & \lfloor 1 \\ \hline 2 & -5 & & & \end{array}$$

$t=\frac{5}{2}b$ のとき

第3問

(1)

Z は $B(400, 0.25)$

$E(Z)=400\times 0.25$

$=100$

(2)

$\sigma(R)=\sqrt{V(R)}$

$=\sqrt{V(\frac{Z}{400})}$

$=\frac{1}{400}\sqrt{V(Z)}$

$=\frac{1}{400}\sqrt{400\times\frac{1}{4}\times\frac{3}{4}}$

$=\frac{\sqrt{3}}{80}\cdots ②$

$P(R\geqq x)=0.0465$

$=0.5-0.4535$

$P(R\geqq x)$

$=P\left(\frac{R-0.25}{\frac{\sqrt{3}}{80}}\geqq\frac{x-0.25}{\frac{\sqrt{3}}{80}}\right)$

$\frac{x-0.25}{\frac{\sqrt{3}}{80}}=1.68$

$\Leftrightarrow x-0.25=\frac{\sqrt{3}}{80}\times 1.68$

$\Leftrightarrow x=\frac{\sqrt{3}}{10}\times 0.21+0.25$

$=0.03633+0.25$

$=0.28633$

$\fallingdotseq 0.286\cdots ②$

(3)

$P(100\leqq X\leqq 300)=1$

$\int_{100}^{300}(ax+b)dx$

$=\left[\frac{a}{2}x^2+bx\right]_{100}^{300}$

$=4\cdot 10^4a+2\cdot 10^2b=1$

$\Leftrightarrow 8\cdot 10^6a+4\cdot 10^4b=2\cdot 10^2$

$-\big)\ \frac{26}{3}10^6a+4\cdot 10^4b=180$

$-\frac{2}{3}\cdot 10^6a=20$

$\Leftrightarrow -\frac{1}{3}10^5a=1\quad\therefore a=-3\cdot 10^{-5}$

$-12\cdot 10^{-1}+200b=1$

$\Leftrightarrow 200b=2.2$

$\Leftrightarrow 1000b=11\quad\therefore b=11\cdot 10^{-3}$

$\therefore f(x)=-3\cdot 10^{-5}x+11\cdot 10^{-3}$

$P(200\leqq x\leqq 300)$

$=\int_{200}^{300}(ax+b)dx$

$=\left[\frac{a}{2}x^2+bx\right]_{200}^{300}$

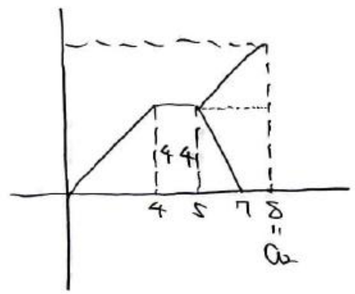
$=25000a+100b$

$=-\frac{75000}{100000}+\frac{1100}{1000}$

$=-\frac{3}{4}+1.1=0.35\cdots ②$

第4問 (1)

自転車が最初に歩行者に追いつくとき $(4,4)$



$a_2=8$ $b_2=7$

n 回目に出発した自転車が次に歩行者に追いつくとき

$(a_n+b_n,\ 2b_n)$

③ ④

図より

$a_{n+1}=a_n+2b_n+2$

$b_{n+1}=2b_n+b_n+1$

$=3b_n+1$

$b_{n+1}+\frac{1}{2}=3\left(b_n+\frac{1}{2}\right)$

解くと

$b_n=\left(b_1+\frac{1}{2}\right)3^{n-1}-\frac{1}{2}$

$=\frac{5}{2}3^{n-1}-\frac{1}{2}\cdots ⑦$

$a_{n+1}=a_n+5\cdot 3^{n-1}+1$

$a_n=\underset{2}{a_1}+\sum_{k=1}^{n-1}(5\cdot 3^{k-1}+1)$

$=2+5\frac{1-3^{n-1}}{1-3}+n-1$

$=n+1+5\cdot\frac{3^{n-1}-1}{2}$

$=\frac{5}{2}3^{n-1}+n-\frac{3}{2}\cdots ⑨$

(2)

$2b_n \leqq 300$

$\Leftrightarrow 5 \cdot 3^{n-1} - 1 \leqq 300$

$\Leftrightarrow 5 \cdot 3^{n-1} \leqq 301$

$\therefore n \leqq 4$　4回

$a_4 + b_4$

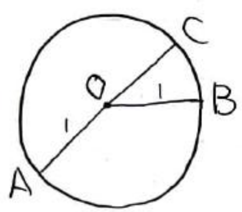
$= \frac{5}{2} \cdot 27 + 4 - \frac{3}{2}$

$+ \frac{5}{2} \cdot 27 - \frac{1}{2}$

$= 135 + 2 = 137$

第5問

(1)



$1 \cdot 1 \cdot \cos\angle AOB = \overrightarrow{OA} \cdot \overrightarrow{OB}$

$= \frac{-2}{3}$

$\overrightarrow{OP} = (1-t)\overrightarrow{OA} + t\overrightarrow{OB}$

$\overrightarrow{OQ} = (k - kt)\overrightarrow{OA} + tk\overrightarrow{OB}$

①　⓪

$\overrightarrow{CQ} = \overrightarrow{OQ} - \overrightarrow{OC}$

$= \overrightarrow{OQ} + \overrightarrow{OA}$

$= (k - kt + 1)\overrightarrow{OA} + tk\overrightarrow{OB}$

④　⓪

$\overrightarrow{OA} \cdot \overrightarrow{OP} = 1 - t + t\left(\frac{-2}{3}\right)$

$= -\frac{5}{3}t + 1 = 0$

$\therefore t = \frac{3}{5}$

(2)

$\overrightarrow{OC} \cdot \overrightarrow{CQ}$

$= -k + kt - 1 + \frac{2}{3}tk = 0$

$\Leftrightarrow \left(\frac{5}{3}t - 1\right)k = 1$

$\therefore k = \frac{3}{5t-3}$

$0 < t < \frac{3}{5}$ のとき $k < -1$ より

QはD₂かつE₂に含まれる

③

$\frac{3}{5} < t < 1$ のとき $k > 0$ より

D₁かつE₁に含まれる

⓪

(3)

$t = \frac{1}{2}$ のとき $k = -6$

$\overrightarrow{OP} = \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB}$

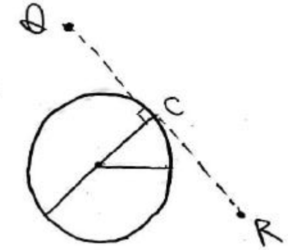
$|\overrightarrow{OP}|^2 = \frac{1}{4} + \frac{1}{4} + \frac{2}{4}\left(-\frac{2}{3}\right)$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$

$\therefore |\overrightarrow{OP}| = \frac{1}{\sqrt{6}}$

$|\overrightarrow{OQ}| = \frac{6}{\sqrt{6}} = \sqrt{6}$



$\overrightarrow{CR} = -\overrightarrow{CQ}$

$= (\underset{6}{-k} + \underset{-3}{kt} - 1)\overrightarrow{OA} - \underset{-3}{tk}\overrightarrow{OB}$

$= 2\overrightarrow{OA} + 3\overrightarrow{OB}$

$\overrightarrow{OR} = \overrightarrow{OC} + \overrightarrow{CR}$

$= \overrightarrow{OA} + 3\overrightarrow{OB}$

$= 4 \cdot \frac{\overrightarrow{OA} + 3\overrightarrow{OB}}{4}$

$t = \frac{3}{4}$ のとき $|\overrightarrow{OQ}| = \sqrt{6}$