

第1問

[1]

$$(1) \quad \begin{aligned} a^2b^2+c^2 &= (a+b+c)^2 \\ &\quad -2(ab+bc+ca) \end{aligned}$$

$$\therefore ab+bc+ca = -6$$

$$(a-b)^2+(b-c)^2+(c-a)^2$$

$$= 2(a^2+b^2+c^2) - 2(ab+bc+ca) = 38$$

(2)

$$\begin{aligned} x+y &= b-c+c-a \\ &= b-a = -2\sqrt{5} \end{aligned}$$

$$x^2+y^2$$

$$\begin{aligned} &= (b-c)^2+(c-a)^2 \\ &= 38-(a-b)^2 = 18 \end{aligned}$$

$$x^2+y^2 = (x+y)^2 - 2xy$$

$$\therefore xy = 1$$

$$(a-b)(b-c)(c-a) = 2\sqrt{5}$$

[2]

$$\tan 6^\circ = 0.2867$$

$$\square 1 \quad AC = \textcircled{1} \quad BC = \textcircled{0.2867}$$

$$\text{実際 } AC = \square 4 \quad BC = \square 0.2867$$

$$\tan \angle BAC = \frac{0.2867}{4}$$

$$= 0.071675 \approx 0.072$$

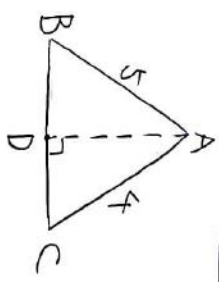
$\angle BAC$ は $4^\circ < 5^\circ$ の間 $\textcircled{2}$

[3]

(1) \textcircled{E}

$$\frac{4}{\sin \angle ABC} = 2:3$$

$$\therefore \sin \angle ABC = \frac{2}{3}$$



$$AD = 5 \sin \angle ABC = \frac{10}{3}$$

(2)

$$\textcircled{E} \quad \frac{AB}{\sin \angle ACD} = 6$$

$$\therefore AB = 6 \sin \angle ACD$$

同様に

$$AC = 6 \sin \angle ABC$$

$$\begin{aligned} \therefore 0 \leq AB \leq 6 \dots \textcircled{1} \\ 0 \leq AC \leq 6 \dots \textcircled{2} \end{aligned}$$

故に

$$AB = \frac{1}{2}(4-AC)$$

$$\textcircled{2} \# \quad 4 \leq AB \leq 7$$

$\textcircled{1}$ と対比して

$$4 \leq AB \leq 6$$

$\textcircled{E} \#$

$$\sin \angle ABC = \frac{AC}{6}$$

$$\begin{aligned} \therefore AD &= AB \sin \angle ABC \\ &= AB \cdot \frac{4-2AB}{6} \end{aligned}$$

$$= -\frac{1}{3}AB^2 + \frac{1}{3}AB$$

$$= -\frac{1}{3}(AB - \frac{1}{2})^2 + \frac{1}{12}$$

AB=4 のとき 最大値 $\frac{1}{12}$

第2問

[1]

(1)

$$\begin{aligned} x^2+4x-4 &= 0 \\ x^2-4x+4 &= 0 \end{aligned}$$

$$x = 3$$

$$x^2+x-2 = 0$$

$$x^2-2x+1 = 0$$

$$x = 2$$

$$x^2-6x+9 = 0 \dots \#$$

$$x^2+x-6 = 0$$

※ 重解のとき $x=9$

重解をばいさ

$$x^2-6x+9 = 0$$

$$\rightarrow x^2+9x-6 = 0$$

$$-(6+9)x + 9+6 = 0$$

$$\Leftrightarrow (x+6)(1-x) = 0$$

$x = -6$ のとき不適

$$0 = 10 \text{ のとき } x = 5$$

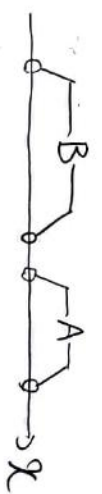
$$\# \# \# \quad x = 5, 9$$

(3)

③の移動...⑥
④ : ...①

(4)

$$A = \{x \mid 3 - \sqrt{9-x} < x < 3 + \sqrt{9-x}\}$$
$$B = \{x \mid -\frac{x - \sqrt{x^2+25}}{2} < x < -\frac{x + \sqrt{x^2+25}}{2}\}$$



$x \in A$ は $x \in B$ である③
 $x \in B \cap x \in A =$ ①

(3)

$$k_{ST} = \frac{S_T}{\sum_{S,T} S_T}$$
$$= \frac{735.3}{39.3 \times 29.9}$$
$$= 0.6257 \dots$$
$$\approx 0.63$$

(4) ヅ...③

第3問

(1)

① $n(n-1) = 1$
 $P(\text{1回目終了}) = \frac{1}{2}$

(i)

$$n(n \text{ 回目終了}) = \frac{n}{3}$$
$$\frac{ABC}{BCA} = \frac{CAB}{ACB}$$

$$P(\dots) = \frac{2}{3!} = \frac{1}{3}$$

(ii)

①は横軸の最大が6
②は n の総和が6

$$P(\text{4回以下終了})$$
$$= \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^2 \frac{1}{3} + \left(\frac{2}{3}\right)^3 \frac{1}{3}$$
$$= \frac{27+18+12+8}{81} = \frac{65}{81}$$

(2)

$n(n)$ が自分のとき
 $= 4 \cdot 2$
← 1人のときも選べる
 $= 8$
 $n(n)$ のみ

$n(2)$ が自分のとき

$$= 4 \cdot 3 \cdot 1$$
$$= 6$$

$n(4)$ が自分のとき

$$= 1$$

$n(1)$ が自分のとき

$$= 8 + 6 + 1 = 15$$

$P(\text{1回終了})$

$$= 1 - \frac{15}{4!}$$

$$= 1 - \frac{15}{24} = \frac{9}{4!}$$

$$= \frac{3}{8} = \frac{9}{4!}$$

(3)

$n(A)$ が自分のとき

$$= 5 \cdot 9$$
$$= 45$$

$n(2)$ が自分のとき

$$= 5 \cdot 3 \cdot 2$$
$$= 20$$

$n(3)$ が自分のとき

$$= 5 \cdot 3 \cdot 1$$
$$= 10$$

$n(5)$ が自分のとき

$$= 1$$

$P(\text{1回終了})$

$$= 1 - \frac{45+20+10+1}{5!}$$
$$= \frac{44}{120} = \frac{11}{30}$$

(4)

$\frac{1}{3}$ 残

$$= \frac{n(\text{1回終了})}{n(A, B, C, D \text{ が自分のとき})}$$

$$= \frac{44}{44}$$

$$= \frac{44 + n(\text{Eだけが自分のとき})}{44 + 9 \cdot 1}$$

$$= \frac{44}{53}$$

第4問

(1)

$$\sum_{k=0}^4 5^k \cdot 39 + 1$$

$$\Leftrightarrow 5^4 \cdot 1 - 2^4 \cdot 39 = 1$$

$$x=1, y=39 \leftarrow \text{最小}$$

$$\begin{cases} x = 2^k + 1 \\ y = 5^k + 39 \end{cases} \quad (k \in \mathbb{Z})$$

$k=0$ のとき $x=2^0+1$ の最小

$$x=17, y=664$$

(2)

$$625^2 = 5^8$$

$$m=39 \text{ とおす}$$

$$625^2 = (2^4 \cdot 39 + 1)^2$$

$$= (2^4 m + 1)^2$$

$$= 2^8 m^2 + 2^4 m + 1$$

(3)

$$5^5 x - 2^5 y = 1$$

$$\hookrightarrow 5^5 x - 625^2 = 5^5 y$$

$$\Leftrightarrow x - 5^3 = 2^5 z$$

$$\Leftrightarrow x = 125 + 32z \quad (z \in \mathbb{Z})$$

$$x = 125, y = 0 \text{ のとき}$$

$$5^5 - 2^5 y = 1$$

$$\Leftrightarrow 625^2 - 2^5 (2^3 m^2 + m) = 1$$

$$y = 8 \cdot 39^2 + 39$$

$$= 12207$$

(4)

$$11^4 = 2^4 x + 1$$

$$11(2^4 x + 1) x - 2^5 y = 1$$

$$\Leftrightarrow 2^4(118x - 2y) + 11x = 1$$

$$\rightarrow \frac{2^4(-2) + 11 \cdot 3 = 1}{2^4(118x - 2y + 2) + 11(3-x) = 0}$$

$$16(118x - 2y + 2) = 11(3-x)$$

$$3-x = 16s$$

$$\Leftrightarrow x = 3 - 16s \quad (s \in \mathbb{Z})$$

$$x = 19$$

$$119x - 2y + 2 = 11s$$

$$\Leftrightarrow 209x + 13 = 2y$$

$$11^4 = 14641$$

$$915 \leftarrow x$$

$$16 \overline{) 14641}$$

$$\underline{144}$$

$$\underline{24}$$

$$\underline{16}$$

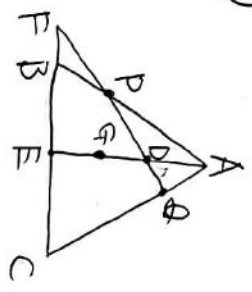
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$$2y = 209 \times 915 + 13$$

$$\therefore y = 95624$$

第5問

(1)



(1) $\frac{AD}{DE} = \frac{1}{2}$

(2)

$\frac{EF}{FB} \cdot \frac{BP}{PA} \cdot \frac{1}{2} = 1$

$\therefore \frac{BP}{AP} = 2 \times \frac{BF}{EF} \dots \textcircled{1}$

(3)

$\frac{CE}{FE} \cdot \frac{2}{1} \cdot \frac{AQ}{QC} = 1$

$\therefore \frac{AQ}{AQ} = 2 \times \frac{CE}{EF} \dots \textcircled{2}$

$\frac{BP}{AP} + \frac{AQ}{AQ}$

$= 2 \times \frac{BF+CF}{EF}$

$= 2 \times \frac{FB+FB+2BE}{FB+BE} = 4$

(2)

$\frac{2 \cdot AP}{AP} + \frac{6-AQ}{AQ} = 4$

$\Leftrightarrow \frac{9}{AP} + \frac{6}{AQ} = 6$

$\Leftrightarrow \frac{3}{AP} + \frac{2}{AQ} = 2$

(3)

$AP \cdot AB = AQ \cdot AC$

$\therefore AQ = \frac{3}{2} AP$

$\frac{3}{AP} + \frac{4}{3AP} = 2$

$\Leftrightarrow \frac{B}{3AP} = 2$

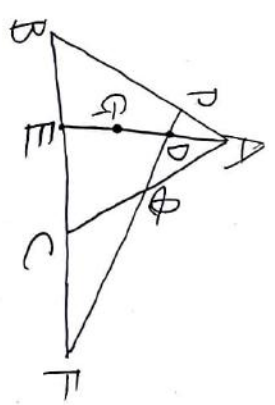
$\therefore AP = \frac{B}{6}, AQ = \frac{B}{4}$

(3)

$6 = 6 - \frac{B}{4}$

$\frac{1}{B} = 2 \times \frac{CF}{EF}$

また) FはCOの直線BC上にある。



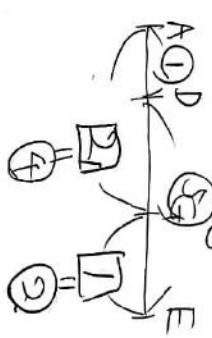
$\frac{1}{B} = 2 \times \frac{CF}{4+CF}$

$\Leftrightarrow 44 + 11CF = 26CF$

$\therefore CF = \frac{44}{15}$

(3)

$\frac{BP}{AP} + \frac{AQ}{AQ} = \frac{DE}{AD} \times 2$



$\frac{AD}{DQ} = \frac{1}{3}$