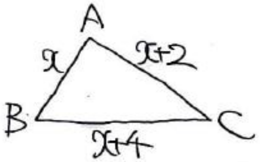
2022 国際医療福祉大（医）

第1問

(A)



$\cos A < 0$ 　余

$\iff x^2+(x+2)^2-(x+4)^2<0$

$\iff x^2-4x-12<0$

$\iff -2<x<6$

三角形の成立条件

$x+x+2>x+4$

$\iff x>2$

$\therefore \underline{2}<x<\underline{6}$ //

余

$\cos\theta = \cos A = \dfrac{-12}{2\cdot4\cdot6}$

$= -\dfrac{1}{4}$

$\tan\theta = \underline{-\sqrt{15}}$ //

(B)

解と係数の関係より

$$\begin{cases} \alpha+\beta+\gamma=2 \\ \alpha\beta+\beta\gamma+\gamma\alpha=-3 \\ \alpha\beta\gamma=-\dfrac{1}{3} \end{cases}$$

$\alpha^2+\beta^2+\gamma^2$

$=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=\underline{10}$ //

$\alpha^3+\beta^3+\gamma^3$

$=(\alpha^2+\beta^2+\gamma^2)(\alpha+\beta+\gamma)$

$\quad -\alpha^2(\beta+\gamma)-\beta^2(\gamma+\alpha)-\gamma^2(\alpha+\beta)$

$=20$

$\quad -\{(\alpha\beta+\beta\gamma+\gamma\alpha)(\alpha+\beta+\gamma)-3\alpha\beta\gamma\}$

$=\underline{25}$ //

$\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}+\dfrac{1}{\gamma^2}$

$=\dfrac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$

$=9\{(\alpha\beta+\beta\gamma+\gamma\alpha)^2-\alpha\beta\gamma(\alpha+\beta+\gamma)\times2\}$

$=9(9+\frac{1}{3}\cdot4)=\underline{93}$ //

(C)

$\log_{10}18^{50}$

$=50(\log_{10}2+2\log_{10}3)$

$=50\times1.2552=62.76$

$\log_{10}5=\log_{10}\dfrac{10}{2}$

$=1-0.3010$

$=0.6990$

$\log_{10}6=\log_{10}2+\log_{10}3$

$=0.7781$

↓

$\log_{10}5<0.76<\log_{10}6$

$\iff 62+\log_{10}5<\log_{10}18^{50}<62+\log_{10}6$

$\iff 5\cdot10^{62}<18^{50}<6\cdot10^{62}$

18^{50}は$\underline{63}$桁で最高位は$\underline{5}$ //

(D)

$E: 4(x-1)^2+9(y-2)^2=36$

$\iff \dfrac{(x-1)^2}{9}+\dfrac{(y-2)^2}{4}=1$

長軸 $\underline{6}$ // 　短軸 $\underline{4}$ //

焦点 $(\underline{1\pm\sqrt{5}}, \underline{2})$ //

E上の点を$(1+3\cos\theta, 2+2\sin\theta)$

とおくと，$y=\frac{1}{3}x+5$ つまり

$0=x-3y+15$ との距離は

$\dfrac{|1+3\cos\theta-3(2+2\sin\theta)+15|}{\sqrt{10}}$

$=\dfrac{1}{\sqrt{10}}|3\cos\theta-6\sin\theta+10|$

$=\dfrac{1}{\sqrt{10}}|3\sqrt{5}\cos(\theta+\alpha)+10|$

θに制限がないので最小値は

$\dfrac{1}{\sqrt{10}}(10-3\sqrt{5})$

$=\underline{\sqrt{10}-\dfrac{3\sqrt{2}}{2}}$ //

第2問

(1)

(i)

$$\begin{array}{r} 9\,\underline{)\,1800} \\ 8\,\underline{)\,200} \\ 25 \end{array}$$

1800

$=2^3\cdot3^2\cdot5^2$

(個数)$=4\cdot3\cdot3=\underline{36}$ //

$$\begin{array}{r} 9\,\underline{)\,1134} \\ 9\,\underline{)\,126} \\ 14 \end{array}$$

113400

$=2^3\cdot3^4\cdot5^2\cdot7$

(個数)$=4\cdot5\cdot3\cdot2$

$=\underline{120}$ //

$\gcd(a,b)$
$= 2^3\cdot3^2\cdot5^2 = \underline{1800}$

$N = 2^3\cdot3^2\cdot5^2\underbrace{(1+3^2\cdot7)}_{64=2^6}$
$= 2^9\cdot3^2\cdot5^2$

(個数) $= 10\cdot3\cdot3 = \underline{90}$

$\gcd(a,b,N) = 1800$
(1800の約数の個数)
$= 4\cdot3\cdot3 = \underline{36}$ …ス

$l = 2^x\cdot3^y\cdot5^z \begin{pmatrix} x=4\sim9 \\ y=0\sim2 \\ z=0\sim2 \end{pmatrix}$

lは全部で $6\cdot3\cdot3 = \underline{54}$

$\min l = 2^4 = \underline{16}$

(2)
$g\,\underline{)\,a \quad m}$
$\quad a' \quad m'$

↓
$g\underbrace{a'}_{a}m' = b$
$m' = 3^2\cdot7$

$a' = 2^s\cdot5^t \quad \begin{pmatrix} s=0\sim3 \\ t=0\sim2 \end{pmatrix}$

a'は $4\cdot3=12$通り, gも12通り
つまりmも$\underline{12}$通りある.

$a' = 2^3\cdot5^2$
$g = 3^2$
$m = gm' = 3^4\cdot7 = \underline{567}$

(3)
$a\,\underline{)\,b \quad n}$
$\quad 3^2\cdot7 \quad n'$

$n = an'$
$= 1800n' \leqq 240000$
$\Leftrightarrow n' \leqq \frac{400}{3} = 133.33\cdots$
133以下で3の倍数または7の倍数を取りのぞくと
$133 - (44+19-6)$
$= \underline{76}$

第3問

$a_1 = 3, \begin{cases} a_{2m} = 3a_{2m-1} \\ a_{2m+1} = a_{2m} + 3^{2m+1} \end{cases}$ $(m \in \mathbb{N})$

(1)
$a_3 = a_2 + 3^3$
$= 3a_1 + 3^3$
$= \underline{36}$

$a_4 = 3a_3 = \underline{108}$

(2)
$a_{2m+1} = 3a_{2m-1} + 3^{2m+1}$
↓
$c_{n+1} = \underline{3}c_n + \underline{3}^{2n+1}$ … $\underline{6}$

$\frac{c_{n+1}}{3^{n+1}} = \frac{c_n}{3^n} + 3^n$

$\frac{c_n}{3^n} = \frac{c_1}{3^1} + \sum_{k=1}^{n-1} 3^k$
$= 1 + \frac{3-3^n}{1-3}$
$= \frac{2}{2} + \frac{3^n-3}{2}$
$= \frac{3^n-1}{2}$

$\therefore c_n = \frac{1}{2}(3^{2n} - 3^n)$
$= \frac{1}{2}(\underline{9}^n - \underline{3}^n)$ ③ ③

$a_{2n} = 3a_{2n-1}$
$= \frac{3}{2}(9^n - 3^n)$

S_n
$= \sum_{k=1}^{2n} a_k$
$= \sum_{m=1}^{n} (a_{2m} + a_{2m-1})$
$= \sum_{m=1}^{n} 4a_{2m-1}$
$= \sum_{m=1}^{n} 2(9^m - 3^m)$
$= 2\left(\frac{9-9^{n+1}}{1-9} - \frac{3-3^{n+1}}{1-3}\right)$
$= \frac{9^{n+1}-9}{4} + 3 - 3^{n+1}$
$= \underline{\frac{1}{4}}\cdot\underline{9}^{n+1} - \underline{3}^{n+1} + \underline{\frac{3}{4}}$ ④

T_n
$= \sum_{k=1}^{n} k\cdot\frac{1}{2}(3^k - 1)$
$= \frac{3}{2}\sum_{k=1}^{n} k\cdot3^{k-1} - \frac{1}{4}n(n+1)$

ここで
$1 + x + x^2 + \cdots + x^n = \frac{1-x^{n+1}}{1-x}$
↓ xで微分
$1 + 2x + 3x^2 + \cdots + nx^{n-1}$
$= \frac{-(n+1)x^n(1-x) - (1-x^{n+1})(-1)}{(1-x)^2}$
$= \frac{nx^{n+1} - (n+1)x^n + 1}{(1-x)^2}$

$x=3$を代入して

$$\sum_{k=1}^{n} k\cdot 3^{k-1}$$

$$=\frac{n\cdot 3^{n+1}-(n+1)3^{n}+1}{4}$$

よって

T_n

$$=\frac{n\cdot 3^{n+2}-(n+1)3^{n+1}+3}{8}-\frac{1}{4}n(n+1)$$

$$=\frac{1}{8}\left\{(2n-1)3^{n+1}-2n(n+1)+3\right\}$$ //

第4問

$$\overrightarrow{PQ}=\begin{pmatrix}-(2-p)^2\\ p^2\\ 4\end{pmatrix}$$

直線PQ: $\begin{cases}x=(2-p)^2-(2-p)^2k\\ y=p^2k\\ z=4k\end{cases}$

$z=4k=t$のとき $k=\frac{t}{4}$

代入して

$x_r=\frac{1}{4}(4-t)(2-p)^2$

$y_r=\frac{1}{4}tp^2$ //

$z_r=t$

$-2\leqq p\leqq 2$のとき

$\underline{0}\leqq x_r\leqq \underline{4(4-t)}$ //

$$x_r=\frac{4-t}{4}(2-p)^2$$

$$\Leftrightarrow \frac{4}{4-t}x_r=(2-p)^2$$

$$\therefore \sqrt{\frac{4}{4-t}x_r}=2-p$$

$$\therefore p=2-2\sqrt{\frac{x_r}{4-t}}$$

$$\therefore y_r=t\left(1-\sqrt{\frac{x_r}{4-t}}\right)^2$$

$f(x)=t\left(1-\sqrt{\frac{x}{4-t}}\right)^2$のとき

$$\int_0^{4(4-t)} t\left(1-\sqrt{\frac{x}{4-t}}\right)^2dx$$

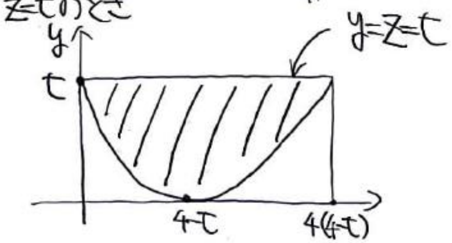
$$=t\int_0^{4(4-t)}\left(1+\frac{x}{4-t}-2\sqrt{\frac{x}{4-t}}\right)dx$$

$$=t\left[x+\frac{x^2}{8-2t}-\frac{4}{3}\sqrt{\frac{1}{4-t}}x^{\frac{3}{2}}\right]_0^{4(4-t)}$$

$$=t\left\{4(4-t)+8(4-t)-\frac{32}{3}(4-t)\right\}$$

$$=\frac{1}{3}(16t-4t^2)$$ //

$z=t$のとき



$S(t)$

$$=t\times 4(4-t)-\frac{1}{3}(16t-4t^2)$$

$$=\frac{1}{3}(32t-8t^2)$$ //

(Kの体積)

$$=\int_0^4 S(t)$$

$$=\left[\frac{16}{3}t^2-\frac{8}{9}t^3\right]_0^4$$

$$=\frac{256}{9}$$ //

$\frac{1}{6}$公式が使えると後で気が付く.