2022 慶應理工

1.

(1) (i)

$\vec{x}\cdot\vec{a} = p|\vec{a}|^2 = 4p$

$|\vec{x}|^2 = |p\vec{a}+q\vec{b}+r\vec{c}|^2 = p^2|\vec{a}|^2+q^2|\vec{b}|^2+r^2|\vec{c}|^2 = 4p^2+q^2+r^2$

$|\vec{x}| = \sqrt{4p^2+q^2+r^2}$

(ii) $\vec{x} = \begin{pmatrix} s \\ 0 \\ z \end{pmatrix}$ $p=s,\ q=\cos\theta,\ r=\sin\theta$

とする. $\vec{x}\cdot\vec{a}=4p$ より

$5\sqrt{3}+z=4s$

$|\vec{x}|^2=4p^2+q^2+r^2$ より

$25+z^2=4s^2+1$

$\Leftrightarrow 96+4z^2=16s^2$

$96+4z^2=z^2+10\sqrt{3}z+75$

$\Leftrightarrow 3z^2-10\sqrt{3}z+21=0$

$\Leftrightarrow (z-\sqrt{3})(3z-7\sqrt{3})=0$

$\Leftrightarrow z=\sqrt{3},\ \dfrac{7\sqrt{3}}{3}$

(2) $k\in\mathbb{Z}$ とする.

$n=2k+1$ とする.

$\left[\dfrac{3n+2}{2}\right]=\left[3k+\dfrac{5}{2}\right]=3k+2$

$n\left[\dfrac{3n+2}{2}\right]=(2k+1)(3k+2)$

$2k+1$ は奇数であり, $3k+2$ は 3 の倍数でないので $\alpha,\beta\in\mathbb{Z}$ として

$$\begin{cases} 2k+1=3\alpha \\ 3k+2=2\beta \end{cases}$$

と表せる.

$6k=9\alpha-3=4\beta-4$

$\Leftrightarrow 9\alpha-4\beta=-1$

$-)\ 9(-1)-4(-2)=-1$

$9(\alpha+1)-4(\beta+2)=0$

$\Leftrightarrow 9\underbrace{(\alpha+1)}_{4l}=4\underbrace{(\beta+2)}_{9l}$

$\alpha=4l-1\quad(l\in\mathbb{Z})$

$\therefore n=2k+1=12l-3$

n は 12 で割ったときの余りが 9

2.

(1) C_1 と C_2 を連立

$(x-2)^2-\dfrac{x^2}{9}=r^2-1$

$\Leftrightarrow 8x^2-36x+36-9r^2+9=0$

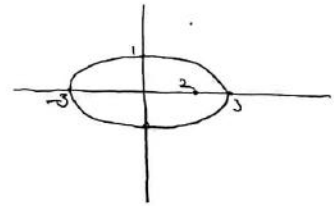
$\Leftrightarrow 8x^2-36x+45-9r^2=0 \cdots ☆$

$\dfrac{D}{4}=324-8(45-9r^2)$

$=4(81-90+18r^2)$

$=36(2r^2-1)\geqq 0$

$\therefore r\geqq\dfrac{1}{\sqrt{2}}$



図より $-3\leqq x\leqq 3$ で共有.

C_2

$\dfrac{1}{\sqrt{2}}\leqq r\leqq 5$

※解の配置で解いても可.

(2) $r=1$ のとき☆は

$8x^2-36x+36=0$

$\Leftrightarrow 2x^2-9x+9=0$

$\Leftrightarrow (2x-3)(x-3)=0$

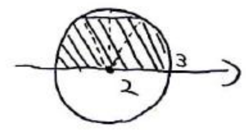
$\therefore x=\dfrac{3}{2},\ 3$

$x=\dfrac{3}{2}$ のとき

$\left(\dfrac{3}{2}-2\right)^2+y_0^2=1$

$\Leftrightarrow y_0^2=\dfrac{3}{4}$

$\therefore y_0=\dfrac{\sqrt{3}}{2}$



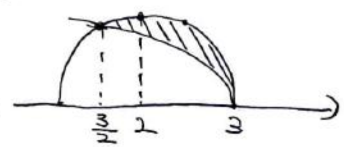
(ケ)

$=\dfrac{\pi}{2}-(\triangle-\triangle)$

$=\dfrac{\pi}{2}-\left(\dfrac{\pi}{6}-\dfrac{1}{2}\cdot1\cdot1\cdot\sin\dfrac{\pi}{3}\right)$

$=\dfrac{\pi}{3}+\dfrac{\sqrt{3}}{4}$

(3)



(コ)

$=\displaystyle\int_{\frac{3}{2}}^{3}2\pi x\left\{\sqrt{1-(x-2)^2}-\sqrt{1-\frac{x^2}{9}}\right\}dx$

$=\displaystyle\int_{\frac{3}{2}}^{3}2\pi x\sqrt{1-(x-2)^2}\,dx$

$-\displaystyle\int_{\frac{3}{2}}^{3}2\pi x\left(1-\frac{x^2}{9}\right)^{\frac{1}{2}}dx$

$= \int_{-\frac{1}{2}}^{1} 2\pi(t+2)\sqrt{1-t^2}\,dt$

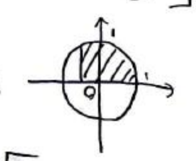
$+ \left[2\pi\left(1-\frac{2x^2}{9}\right)^{\frac{3}{2}} \cdot \frac{9}{2} \cdot \frac{2}{3} \right]_{\frac{3}{2}}^{3}$

$= \int_{-\frac{1}{2}}^{1} 2\pi t\sqrt{1-t^2}\,dt$

$+ 4\pi \int_{-\frac{1}{2}}^{1} \sqrt{1-t^2}\,dt$

$- 6\pi\left(\frac{3}{4}\right)^{\frac{3}{2}}$

$= \left[-\pi(1-t^2)^{\frac{3}{2}} \cdot \frac{2}{3} \right]_{-\frac{1}{2}}^{1}$

$+ 4\pi \times$ 

$- 6\pi \frac{3\sqrt{3}}{8}$

$= \pi\left(\frac{3}{4}\right)^{\frac{3}{2}} \cdot \frac{2}{3}$

$+ 4\pi \times \left(\frac{\pi}{3} + \frac{\sqrt{3}}{8} \right)$

$- 6\pi \cdot \frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}\pi}{8}\left(-\frac{16}{3}\right) + \frac{4\pi^2}{3} + \frac{\sqrt{3}\pi}{2}$

$= \frac{4\pi^2}{3} - 2\sqrt{3}\pi + \frac{\sqrt{3}\pi}{2}$

$= \frac{4\pi^2}{3} - \frac{3\sqrt{3}\pi}{2}$ //

※バウムクーヘンは立式は簡単ですが
積分が大変でした.

3.

(1)

P(2回で白2コ)

$= P(T \to H) + P(H \to T) \cdot \frac{1}{2}$

$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$ //

(2)

$P_{3回でH\times2,T}$(白2コ)について

(分子)

$= P(T \to H \to H)$

$+ P(H \to T \to H)$

$+ P(H \to H \to T)$

$= \left(\frac{1}{2}\right)^3$

$+ \frac{1}{2}\left(\frac{1}{2} \cdot \frac{1}{2}\right) \cdot \frac{1}{2}$

$+ \frac{1}{2} \cdot \frac{1}{2} \cdot \left(\frac{1}{2} \cdot \frac{1}{3}\right)$

$= \frac{11}{48}$

$\therefore P_{3回でH\times2,T}$(白2コ)

$= \frac{11}{48} \div {}_3C_2\left(\frac{1}{2}\right)^3 = \frac{11}{18}$ //

(3)

P(白玉のみ)

$= \left(\frac{1}{2}\right)^k = \frac{1}{2^k}$ //

(4)

$P_{1回目で赤}$(白がkコ)について

(分子)

$= \underbrace{\frac{1}{2}}_{H} \times \left(\underbrace{\frac{1}{2}}_{T} \cdot \frac{1}{2}\right) \times \left(\underbrace{\frac{1}{2}}_{T} \cdot \frac{2}{3}\right)$

$\times \left(\frac{1}{2} \cdot \frac{3}{4}\right) \times \cdots \times \left(\frac{1}{2} \cdot \frac{k-1}{k}\right)$

$= \left(\frac{1}{2}\right)^k \frac{1}{k}$

(分母) $= \frac{1}{2}$

$\therefore P_{1回目赤}$(白がkコ) $= \dfrac{\left(\frac{1}{2}\right)^k \frac{1}{k}}{\frac{1}{2}}$

$= \frac{1}{k \cdot 2^{k-1}}$ //

(5)

P(2回目で赤かつ白がkコ)

$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{1}{2} \cdot \frac{2}{3}\right) \times \left(\frac{1}{2} \cdot \frac{3}{4}\right)$

$\times \cdots \times \left(\frac{1}{2} \cdot \frac{k-1}{k}\right)$

$= \frac{2}{k \cdot 2^k}$

P(3回目で赤かつ白がkコ)

$= \frac{3}{k \cdot 2^k}$

同様に

P(i回目で赤かつ白がkコ)

$= \frac{i}{k \cdot 2^k}$

求める確率は

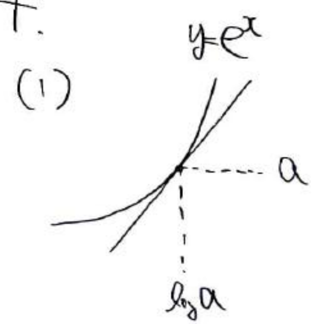
$\sum_{i=1}^{k} \frac{i}{k \cdot 2^k}$

$= \frac{1}{k \cdot 2^k} \cdot \frac{1}{2} k(k+1)$

$= \frac{k+1}{2^{k+1}}$

4.

(1)



$y=e^x$ の傾きが a である接線は

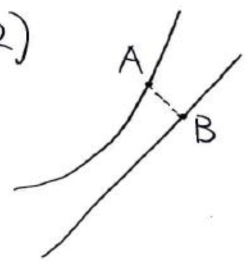
$y = a(x - \log a) + a$

$= ax + a(1 - \log a)$

$a=0$ のとき $b>0$

$a>0$ のとき $b \geqq a(1-\log a)$ //

(2)



直線AB: $y=-(x-t)+e^t$
$=-x+t+e^t$

$y=x$ と連立すると

$x=-x+t+e^t$

$\Leftrightarrow x=\dfrac{t+e^t}{2} \cdots$ タ //

円Dの半径は

$\left(\dfrac{t+e^t}{2}-t\right)\times\sqrt{2}$

$=\dfrac{e^t-1}{\sqrt{2}}$ //

また

$X(t)=\dfrac{2t+3\frac{t+e^t}{2}}{3+2}$

$=\dfrac{4t+3t+3e^t}{10}$

$=\dfrac{1}{10}(7t+3e^t)$

$Y(t)=\dfrac{2e^t+3\frac{t+e^t}{2}}{3+2}$

$=\dfrac{4e^t+3t+3e^t}{10}$

$=\dfrac{1}{10}(3t+7e^t)$

$\sqrt{\{X(t)\}^2+\{Y(t)\}^2}$

$=\dfrac{1}{10}\sqrt{(7t+3e^t)^2+(3t+7e^t)^2}$

$=\dfrac{1}{10}\sqrt{58t^2+84te^t+58e^{2t}}$

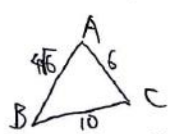
$\displaystyle\lim_{t\to\infty}\frac{Y(t)-kX(t)}{\sqrt{\{X(t)\}^2+\{Y(t)\}^2}}$

$=\displaystyle\lim_{t\to\infty}\frac{3t+7e^t-7kt-3ke^t}{\sqrt{58t^2+84te^t+58e^{2t}}}$

$=\displaystyle\lim_{t\to\infty}\frac{(3-7k)te^{-t}+(7-3k)}{\sqrt{58(te^{-t})^2+84te^{-t}+58}}=0$

こうなるには $7-3k=0$ $\therefore k=\dfrac{7}{3}$ //

5.



(1)

$\cos\angle ABC=\dfrac{96+100-36}{2\cdot4\sqrt{6}\cdot10}$

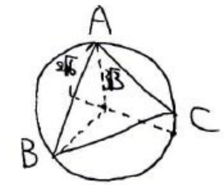
$=\dfrac{2}{\sqrt{6}}$

$\sin\angle ABC=\dfrac{\sqrt{2}}{\sqrt{6}}=\dfrac{1}{\sqrt{3}}$

(正)

$\dfrac{6}{\sin\angle ABC}=2R$

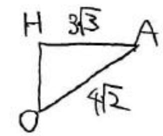
$\therefore R=3\times\sqrt{3}=3\sqrt{3} \cdots$ ト //



(ナ) $=4\sqrt{6}\times\left\{\sqrt{(3\sqrt{3})^2-(2\sqrt{6})^2}+3\sqrt{3}\right\}\times\dfrac{1}{2}$

$=4\sqrt{6}\times2\sqrt{3}$

$=24\sqrt{2}$ //



$OH=\sqrt{(4\sqrt{2})^2-(3\sqrt{3})^2}=\sqrt{5}$

(3)

$\triangle ABC=\dfrac{1}{2}\cdot4\sqrt{6}\cdot10\sin\angle ABC$

$=20\sqrt{6}\cdot\dfrac{1}{\sqrt{3}}$

$=20\sqrt{2}$

(ヌ)

$=20\sqrt{2}\times(\sqrt{5}+4\sqrt{2})\times\dfrac{1}{3}$

$=\dfrac{20(\sqrt{10}+8)}{3}$ //