2021 帝京大(医) ①

[1]

(1)

$f(x)$

$=3x^2\underbrace{\int_0^1 f(t)dt}_{a}+\int_0^1\left(f'(t)^2-2x\right)dt$

$=3ax^2+\int_0^1\left(f'(t)^2\right)dt-\left[2xt\right]_0^1$

$=3ax^2-2x+\int_0^1\left(f'(t)^2\right)dt$ とする

$f'(x)$

$=6ax-2$

$f(x)$

$=3ax^2-2x+\int_0^1(36a^2t^2-24at+4)dt$

$=3ax^2-2x+12a^2-12a+4$

a

$=\int_0^1(3at^2-2t+12a^2-12a+4)dt$

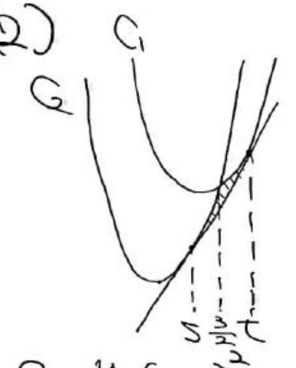
$=a-1+12a^2-12a+4$

$\Leftrightarrow 4a^2-4a+1=0$

$\therefore a=\frac{1}{2}$

$f(x)=\frac{3}{2}x^2-2x+1$ //

(2)



C_1: $y=(x-1)^2$

C_2: $y=x^2-2$

C_2 の $x=s$ での接線は

$y=2s(x-s)+s^2-2$

$=2sx-s^2-2$

C_1 と連立して

$x^2-2x+1=2sx-s^2-2$

$\Leftrightarrow x^2-2(1+s)x+s^2+3=0$

$\frac{D}{4}=(1+s)^2-s^2-3$

$=2s-2=0$

$\therefore s=1 \longrightarrow x^2-4x+4=0$

$\therefore t=2$

才 (傾き)$=2$ //

$=\int_1^{\frac{3}{2}}(x-1)^2dx+\int_{\frac{3}{2}}^2(x-2)^2dx$

$=\left[\frac{1}{3}(x-1)^3\right]_1^{\frac{3}{2}}+\left[\frac{1}{3}(x-2)^3\right]_{\frac{3}{2}}^2$

$=\frac{1}{3}\left(\frac{1}{2}\right)^3+\frac{1}{3}\left(\frac{1}{2}\right)^3=\frac{1}{12}$ //

[2]

(1) $P(\cos\theta,\sin\theta,0)$

$Q(-\cos\theta,-\sin\theta,0)$

$\overrightarrow{AP}=\begin{pmatrix}\cos\theta-2\\ \sin\theta-2\\ -1\end{pmatrix}$ $\overrightarrow{AQ}=\begin{pmatrix}-\cos\theta-2\\ -\sin\theta-2\\ -1\end{pmatrix}$

$\overrightarrow{AP}\cdot\overrightarrow{AQ}$

$=-(\cos^2\theta-4)-(\sin^2\theta-4)+1$

$=8$ //

$\triangle APQ=\frac{1}{2}\sqrt{|\overrightarrow{AP}|^2|\overrightarrow{AQ}|^2-(\overrightarrow{AP}\cdot\overrightarrow{AQ})^2}$

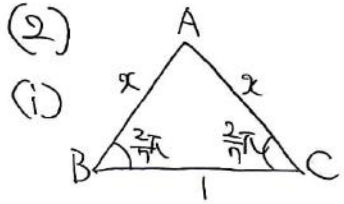
$\Leftrightarrow 3=\sqrt{|\overrightarrow{AP}|^2|\overrightarrow{AQ}|^2-64}$

$\therefore |\overrightarrow{AP}|^2|\overrightarrow{AQ}|^2=73$

$|\overrightarrow{AP}||\overrightarrow{AQ}|=\sqrt{73}$ //

(2)

(i)



正弦定理 $\frac{x}{\sin\frac{2}{7}\pi}=\frac{1}{\sin\frac{3}{7}\pi}$

$\Leftrightarrow \frac{1}{x}=\frac{\sin\frac{3}{7}\pi}{\sin\frac{2}{7}\pi}$

$a=3$ または 4. 最小なのは 3 //

(ii)

$-\sin\frac{8}{7}\pi=\sin\frac{6}{7}\pi$

$\Leftrightarrow -2\sin\frac{4}{7}\pi\cos\frac{4}{7}\pi=3\sin\frac{2}{7}\pi-4\sin^3\frac{2}{7}\pi$

$2\sin\frac{4}{7}\pi=2\sin\frac{2}{7}\pi\cos\frac{2}{7}\pi$

$\Leftrightarrow -4\cos\frac{2}{7}\pi\cos\frac{4}{7}\pi=3-4\sin^2\frac{2}{7}\pi$

$\Leftrightarrow -8\cos^3\frac{2}{7}\pi+4\cos\frac{2}{7}\pi$

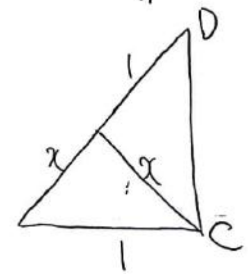
$=-1+4\cos^2\frac{2}{7}\pi$

$\downarrow \cos\frac{2}{7}\pi=\frac{1}{2x}$

$-\frac{1}{x^3}+\frac{2}{x}=-1+\frac{1}{x^2}$

$\Leftrightarrow -1+2x^2=-x^3+x$

$\therefore x^3+2x^2=x+1$ //



余弦定理より

$DC^2=1+x^2-2x\cos\frac{4}{7}\pi$

$$DC^2 = 1+x^2-2x\left(2\cos^2\frac{2}{7}\pi-1\right)$$
$$= 1+x^2-2x\left(\frac{1}{2x^2}-1\right)$$
$$= x^2+2x+1-\frac{1}{x}$$
$$= \frac{x^3+2x^2+x-1}{x}$$
$$= \frac{x+1+x-1}{x} = 2$$

$\therefore$ $DC = \underline{\sqrt{2}}$ //

[3]

(1) 真 $x>2$

$$\frac{\log_3(2x+3)}{\log_3\frac{1}{3}} - \log_3(x-2) > -2$$
$$\Leftrightarrow \log_3(2x+3)(x-2) < 2$$
$$\Leftrightarrow 2x^2-x-6 < 9$$
$$\Leftrightarrow 2x^2-x-15 < 0$$
$$\Leftrightarrow -\frac{5}{2} < x < 3$$

$\therefore$ $\underline{2<x<3}$ //

(2)

$f(x)$
$$= \left(\log_3 2+\log_3\frac{2}{x}\right)\frac{\log_3\frac{2}{x}}{\log_3\frac{1}{3}}$$
$$= -\left(\log_3\frac{2}{x}\right)^2-(\log_3 2)\log_3\frac{2}{x}$$
$$= -\left(\log_3\frac{2}{x}+\frac{1}{2}\log_3 2\right)^2 + \frac{1}{4}(\log_3 2)^2$$

$\frac{2}{x}=\frac{1}{\sqrt{2}} \Leftrightarrow \underline{x=2\sqrt{2}}$ // のとき最大値 $\underline{\frac{(\log_3 2)^2}{4}}$ //

[4]

(1)

P(2コとも赤)

$= P(赤\to赤\times2)+P(白\to赤\times2)$

$$= \frac{2}{6}\cdot\frac{{}_2C_2}{{}_6C_2}+\frac{4}{6}\cdot\frac{{}_2C_2}{{}_5C_2}$$
$$= \frac{1}{3}\cdot\frac{1}{15}+\frac{2}{3}\cdot\frac{1}{10}$$
$$= \frac{2+6}{90} = \underline{\frac{4}{45}}$$ //

P(2コとも白)

$= P(赤\to白\times2)+P(白\to白\times2)$

$$= \frac{2}{6}\cdot\frac{{}_4C_2}{{}_6C_2}+\frac{4}{6}\cdot\frac{{}_3C_2}{{}_5C_2}$$
$$= \underline{\frac{1}{3}}$$ //

(2) $m\in\mathbb{Z}$

$a_{4m+1} = 2m(4m+1)$ ← even

$a_{4m+2} = (4m+1)(2m+1)$ ← odd

$a_{4m+3} = (2m+1)(4m+3)$ ← odd

$a_{4m} = (4m-1)2m$ ← even

(i)
$$\sum_{k=1}^{8}(-1)^{a_k}$$
$$= \{1+(-1)+(-1)+1\}\times 2$$
$$= \underline{0}$$ //

(ii)
$$\sum_{k=1}^{12}(-1)^{a_k}k^2$$

$= 1+(-4)+(-9)+16$ ← 1列目

$+25+(-36)+(-49)+64$ ← 2列目

$+81+(-100)+(-121)+144$ ← 3列目

$= 4+4+4 = \underline{12}$ //

(iii) n列目の和は

$$(4n-3)^2-(4n-2)^2-(4n-1)^2+(4n)^2$$
$$= 4$$

$$\sum_{k=1}^{50}(-1)^{a_k}k^2$$
$$= 4\times12+49^2-50^2$$
$$= 48+(49+50)(49-50)$$
$$= 48-99$$
$$= \underline{-51}$$ //