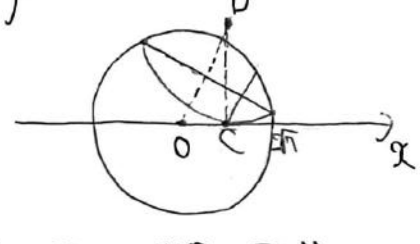
2021 昭和大(医)

1

(1)



C_1とC_2の半径は同じ.

$D(\sqrt{7}, 2\sqrt{7})$ //

$C_2 : (x-\sqrt{7})^2+(y-2\sqrt{7})^2=28$ //

(2) GはOとDの中点より

$G\left(\frac{\sqrt{7}}{2}, \sqrt{7}\right)$ //

$OG=\frac{\sqrt{7}}{2}\times\sqrt{5}=\frac{\sqrt{35}}{2}$

$EG=\sqrt{(2\sqrt{7})^2-\left(\frac{\sqrt{35}}{2}\right)^2}$

$=\sqrt{\frac{112-35}{4}}$

$=\frac{\sqrt{77}}{2}$

$C_3 : \left(x-\frac{\sqrt{7}}{2}\right)^2+(y-\sqrt{7})^2=\frac{77}{4}$ //

(3) C_3と$x=0$を連立.

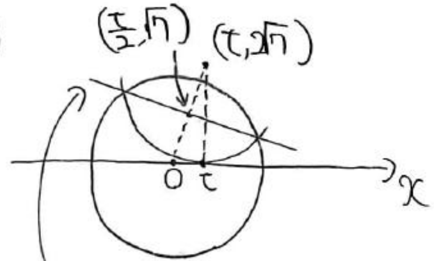
$(y-\sqrt{7})^2=\frac{70}{4}$

$\Leftrightarrow y-\sqrt{7}=\pm\frac{\sqrt{70}}{2}$

$\Leftrightarrow y=\sqrt{7}\pm\frac{\sqrt{70}}{2}$

この2点の距離は$\sqrt{70}$ //

(4)



$t\left(x-\frac{t}{2}\right)+2\sqrt{7}(y-\sqrt{7})=0$

$\Leftrightarrow y=-\frac{t}{2\sqrt{7}}x+\frac{t^2}{4\sqrt{7}}+\sqrt{7}$

$\Leftrightarrow y=\frac{1}{4\sqrt{7}}\left(t^2-2xt\right)+\sqrt{7}$

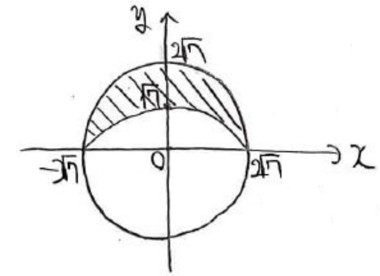
$\Leftrightarrow y=\frac{1}{4\sqrt{7}}(t-x)^2-\frac{x^2}{4\sqrt{7}}+\sqrt{7}$

$(-2\sqrt{7}\leqq t\leqq 2\sqrt{7})$

$-2\sqrt{7}\leqq x\leqq 2\sqrt{7}$ より

$y\geqq -\frac{x^2}{4\sqrt{7}}+\sqrt{7}$

弦PQはC_1の周および内部に存在するので求める範囲は



図の斜線部分. 境界線含む.

2

(1) x $(0\leqq x\leqq 9)$

$1\boxed{x}2$が8の倍数であればいい.

↓

$10x+102\equiv 2x+6\equiv 0 \pmod 8$

$\therefore x=1,5,9$ //

(2) 両辺自然対数とると

$\frac{n^2+5n-14}{2}\log(n^2-9n+19)=0$

↓

$(n+7)(n-2)=0$

または

$n^2-9n+19=1$

$\therefore n=2,3,6$ //

(3)

$m+(m+2)+(m+4)+\cdots+(m+2^n)$

$=m(n+1)+\frac{2-2^n\cdot 2}{1-2}$

$=m(n+1)+2^{n+1}-2=1000$

$\Leftrightarrow m(n+1)+2^{n+1}=1002$

$n=1 \to 2m=998$ ○

$n=2 \to 3m=994$

$n=3 \to 4m=986$

$n=4 \to 5m=970$ ○

$n=5 \to 6m=938$

$n=6 \to 7m=874$

$n=7 \to 8m=746$

$n=8 \to 9m=490$

$(n,m)=(1,499),(4,194)$

(*)は

$499, 501$ //

または

$194, 196, 198, 202, 210$ //

3

(1)

(1-1)

P(少なくとも1コ赤)

$= 1 - P(\text{オール白})$

$= 1 - \frac{7}{10}\cdot\frac{6}{9}\cdot\frac{5}{8}$

$= \frac{17}{24}$

(1-2)

P(3回目白で終了する)

$= P(\text{白}\times 2, \text{赤}\times 1 \text{ かつ 3回目白})$

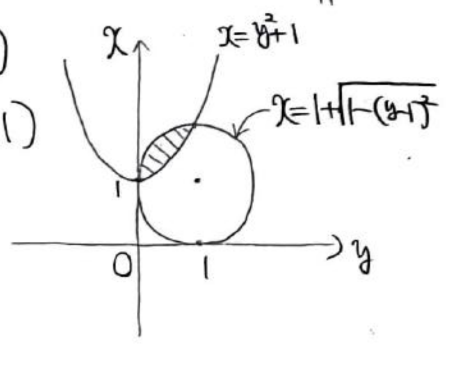
$= \left(\frac{7}{10}\cdot\frac{3}{9} + \frac{3}{10}\cdot\frac{7}{10}\right)\frac{6}{9}$

$= \frac{21}{10}\cdot\frac{2}{3}\left(\frac{1}{9}+\frac{1}{10}\right)$

$= \frac{7}{5}\cdot\frac{19}{90} = \frac{133}{450}$

(2)

(2-1)



S

$= \quad - \left(\int_0^1 (y^2+1)\,dy - 1\right)$

$= \frac{\pi}{4} + 1 - \left[\frac{1}{3}y^3 + y\right]_0^1$

$= \frac{\pi}{4} - \frac{1}{3}$

(2-2)

バウムクーヘン積分使います.

V

$= \int_0^1 2y\pi\left(\sqrt{1-(y-1)^2} - y^2\right)dy$

$= 2\pi\int_0^1 y\sqrt{1-(y-1)^2}\,dy$ $y-1=t$

$-2\pi\left[\frac{1}{4}y^4\right]_0^1$

$= 2\pi\int_{-1}^0 (t+1)\sqrt{1-t^2}\,dt$

$-\frac{1}{2}\pi$

$= \pi\int_{-1}^0 2t\sqrt{1-t^2} + 2\pi\int_{-1}^0 \sqrt{1-t^2}\,dt$

$-\frac{\pi}{2}$

$= \pi\left[-\frac{2}{3}(1-t^2)^{\frac{3}{2}}\right]_{-1}^0 + 2\pi\cdot\frac{\pi}{4}$

$-\frac{\pi}{2}$

$= \frac{\pi^2}{2} - \frac{7}{6}\pi$

4

(1) (1-1) $S = ab\pi$

(1-2)

$z = t$ のとき $(t \geqq -2)$.

$px^2 + qy^2 = t+2$

$\Leftrightarrow \frac{x^2}{\frac{t+2}{p}} + \frac{y^2}{\frac{t+2}{q}} = 1$

このときの楕円内の面積を

$S(t)$とすると

$S(t) = \sqrt{\frac{t+2}{p}}\sqrt{\frac{t+2}{q}}\pi = \frac{\pi}{\sqrt{pq}}(t+2)$

V

$= \int_{-2}^0 S(t)\,dt$

$= \left[\frac{\pi}{\sqrt{pq}}\left(\frac{t^2}{2}+2t\right)\right]_{-2}^0 = \frac{2\pi}{\sqrt{pq}}$

(2)

(2-1)

P(Bが勝つ)

$= P(\text{Bが}H\times 3, \text{Aが}H\times 2\text{以下})$

$+ P(\text{Bが}H\times 2, \text{Aが}H\times 1\text{以下})$

$+ P(\text{Bが}H\times 1, \text{Aが}H\times 0)$

$= \frac{1}{8}\left({}_4C_2\frac{1}{16} + {}_4C_1\frac{1}{16} + \frac{1}{16}\right)$

$+ {}_3C_2\frac{1}{8}\left({}_4C_1\frac{1}{16} + \frac{1}{16}\right)$

$+ {}_3C_1\frac{1}{8}\cdot\frac{1}{16}$

$= \frac{1}{128}(6+4+1+12+3+3) = \frac{29}{128}$

P(引き分け)

$= \frac{{}_4C_3 + {}_3C_2\cdot{}_4C_2 + {}_3C_1\cdot{}_4C_1 + 1}{128}$

$= \frac{35}{128}$

$P(\text{Aが勝つ}) = 1 - \frac{29+35}{128} = \frac{1}{2}$

(2-2) 1枚の額を

金貨…kC 銀貨…C

とする.

$\frac{29}{128}\times 4kC > \frac{1}{2}\times 3C$

$\Leftrightarrow k > \frac{3}{2}\cdot\frac{128}{4\cdot 29} = \frac{48}{29}$

となればいいので, 金貨は銀貨の

少なくとも2倍あればいい.