2021 埼玉医科大(後期)

$\boxed{1}$

問1.

$x=\frac{1}{2}\pm i$

$\downarrow$

$x^2-1x+\frac{5}{4}=0$

問2.

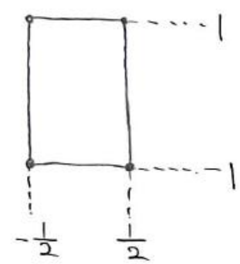
$$
\begin{array}{r|lllll}
 & & & 1 & 1 & p-\frac{1}{4} \\
\hline
1\ \ -1\ \ \frac{5}{4} & 1 & 0 & p & 0 & q \\
 & 1 & -1 & \frac{5}{4} & & \\
\hline
 & & 1 & p-\frac{5}{4} & 0 & \\
 & & 1 & -1 & \frac{5}{4} & \\
\hline
 & & & p-\frac{1}{4} & -\frac{5}{4} & q \\
 & & & p-\frac{1}{4} & -p+\frac{1}{4} & \frac{5}{4}p-\frac{5}{16} \\
\hline
 & & & & p-\frac{3}{2} & q-\frac{5}{4}p+\frac{5}{16}
\end{array}
$$

$\downarrow$

$$\begin{cases} p=\frac{3}{2} \\ q-\frac{15}{8}+\frac{5}{16}=0 \rightarrow q=\frac{25}{16} \end{cases}$$

$g(x)=(x^2-x+\frac{5}{4})(x^2+x+\frac{5}{4})$

$g(x)=0 \Leftrightarrow x=\frac{1}{2}\pm i,\ \frac{-1\pm 2i}{2}$



面積は 2

問3.

$\frac{4x^2-5}{g(x)}$

$=\frac{4x^2-5}{(x^2-x+\frac{5}{4})(x^2+x+\frac{5}{4})}$

$=\frac{ax-b}{x^2-x+\frac{5}{4}}-\frac{cx+d}{x^2+x+\frac{5}{4}}$

$\downarrow$

$4x^2-5$

$=(ax-b)(x^2+x+\frac{5}{4})$

$\quad -(cx+d)(x^2-x+\frac{5}{4})$

$=(a-c)x^3+(a-b+c-d)x^2$

$\quad +(\frac{5}{4}a-b-\frac{5}{4}c+d)x-\frac{5}{4}b-\frac{5}{4}d$

$a=c,\ 2a-b-d=4,$

$-b+d=0\quad -\frac{5}{2}b=-5$

$\therefore b=d=2,\ a=c=4$

$\therefore \frac{4x^2-5}{g(x)}=\frac{4x-2}{h_1(x)}-\frac{4x+2}{h_2(x)}$

問4.

$\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{4x^2-5}{g(x)}dx$

$=\int_{-\frac{1}{2}}^{\frac{1}{2}}\left\{\frac{4x-2}{h_1(x)}-\frac{4x+2}{h_2(x)}\right\}dx$

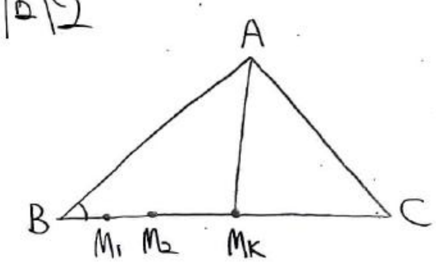
$=2\left[\log|h_1(x)|-\log|h_2(x)|\right]_{-\frac{1}{2}}^{\frac{1}{2}}$

$=2\{-\log 2-(\log 2-0)\}$

$=-4\log 2$

$\boxed{2}$

問2



$\triangle ABM_k$ に余

$AM_k^2=AB^2+BM_k^2-2AB\cdot M_k\cdot\cos\angle ABC$

$\quad =AB^2+\left(\frac{k}{2n+1}\right)^2-2AB^2\frac{k}{2n+1}$

$\sum_{k=1}^{2n}AM_k^2$

$=AB^2\cdot 2n+\frac{1}{(2n+1)^2}\frac{2n}{6}(2n+1)(4n+1)$

$\quad -\frac{2AB^2}{2n+1}\cdot\frac{2n}{2}(2n+1)$

$=AB^2\cdot 2n+\frac{n(4n+1)}{3(2n+1)}$

$\quad -AB^2\cdot 2n$

$=\frac{n(4n+1)}{3(2n+1)}=\frac{1}{3}\cdot\frac{n(4n+1)}{2n+1}$

問1.

$\sum_{k=1}^{4}AM_k^2=\frac{2\cdot 9}{3\cdot 5}=\frac{6}{5}$

問3.

$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{2n}AM_k^2$

$=\lim_{n\to\infty}\frac{1}{3}\cdot\frac{4n+1}{2n+1}=\frac{2}{3}$

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問1.

P(A→B→A→Bで負け)

$$= 1\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{25}{216}$$

問2.

P(Bがk回投げて負け)

$$= \underbrace{1\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdots\frac{5}{6}\cdot\frac{1}{6}}_{2k\text{回}}$$

$$= \frac{1}{6}\left(\frac{5}{6}\right)^{2k-2}$$

問3.

P(Bが負け)

$$= \sum_{k=1}^{\infty}\frac{1}{6}\left(\frac{5}{6}\right)^{2k-2}$$

$$= \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{6}{11}$$

問4.

$$\sum_{k=1}^{n}\frac{1}{6}\left(\frac{5}{6}\right)^{2k-2}$$

$$= \sum_{k=1}^{n}\frac{1}{6}\left(\frac{25}{36}\right)^{k-1}$$

$$= \frac{\frac{1}{6}-\frac{1}{6}\left(\frac{25}{36}\right)^{n}}{1-\frac{25}{36}}$$

$$= \frac{6}{11}-\frac{6}{11}\left(\frac{25}{36}\right)^{n} > \frac{1}{2}$$

$$\Leftrightarrow 12-12\left(\frac{25}{36}\right)^{n} > 11$$

$$\Leftrightarrow \left(\frac{25}{36}\right)^{n} < \frac{1}{12}$$

$$\Leftrightarrow 2n\log_{10}\frac{5}{6} < \log_{10}\frac{1}{12}$$

$$\Leftrightarrow 2n(\log_{10}5-\log_{10}6) < -\log_{10}12$$

$$\Leftrightarrow 2n > \frac{-2\log_{10}2-\log_{10}3}{\log_{10}5-\log_{10}2-\log_{10}3}$$

$$\Leftrightarrow 2n > \frac{-2\log_{10}2-\log_{10}3}{1-2\log_{10}2-\log_{10}3}$$

$$\Leftrightarrow n > \frac{-2\log_{10}2-\log_{10}3}{2-2\log_{10}4-2\log_{10}3}$$

$$= \frac{1.0790}{0.1582} \fallingdotseq 6.82\cdots$$

$\therefore n \geqq 7$