2021 日本医科大(前期)

[I] 問1.

6回の後は

$P(\underbrace{a-2}_{0}, \underbrace{b}_{0}, \underbrace{c+2}_{0})$

$\therefore \underline{a=2,\ b=0,\ c=-2}$ //

問2.

$A(2, 0, -2)$

(1)

$z=-1$ になるには 5回中 Z で表が3回.

$_5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=\frac{10}{32}=\underline{\frac{5}{16}}$ //

(2)

$y=5$ になるのは 5回中 Y で表が5回.

$_5C_5\left(\frac{1}{2}\right)^5=\frac{1}{32}$

$y \leqq 4$ になるのは

$1-\frac{1}{32}=\frac{31}{32}$

$y \leqq 4$ かつ $z=-1$ になるのは

$\frac{5}{16}\cdot\frac{31}{32}=\underline{\frac{155}{512}}$ //

(3) $x>2$ なのは X で

$\underbrace{\left(\frac{1}{2}\right)^5}_{\text{表5回}}+\underbrace{{}_5C_4\left(\frac{1}{2}\right)^5}_{\text{表4回}}+\underbrace{{}_5C_3\left(\frac{1}{2}\right)^5}_{\text{表3回}}$

$=16\cdot\frac{1}{32}=\frac{1}{2}$

$y+z=2$ なのは

$\underbrace{\left(\frac{1}{2}\right)^5}_{\text{Y表5回}}\underbrace{{}_5C_2\left(\frac{1}{2}\right)^5}_{\text{Z表2回}}+\underbrace{{}_5C_4\left(\frac{1}{2}\right)^5}_{\text{Y表4回}}\underbrace{{}_5C_3\left(\frac{1}{2}\right)^5}_{\text{Z表3回}}$

$+\underbrace{{}_5C_3\left(\frac{1}{2}\right)^5}_{\text{Y表3回}}\underbrace{{}_5C_4\left(\frac{1}{2}\right)^5}_{\text{Z表4回}}+\underbrace{{}_5C_2\left(\frac{1}{2}\right)^5}_{\text{Y表2回}}\cdot\underbrace{\left(\frac{1}{2}\right)^5}_{\text{Z表5回}}$

$=(10+50)\left(\frac{1}{2}\right)^{10}\times 2$

$=15\cdot 4\cdot\left(\frac{1}{2}\right)^9$

$=\frac{15}{128}$

$x>2$ かつ $y+z=2$ になるのは

$\frac{1}{2}\cdot\frac{15}{128}=\underline{\frac{15}{216}}$ //

[II]

問1. $\underline{a=0}$ //

問2. $x>0$ のとき

$C: y=-x^2+7x \quad y'=-2x+7$

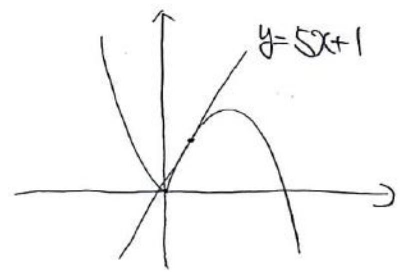
$(1, 6)$ での接線は

$y=5(x-1)+6$

$\therefore \underline{y=5x+1}$ //

問B.

$C: y=\begin{cases}-x^2+7x & (x \geqq 0)\\ x^2-5x & (x<0)\end{cases}$



$a>1$ のとき $N(a)=1$

$a=1$ 〃 $N(a)=2$

$0<a<1$ 〃 $N(a)=3$

$a=0$ 〃 $N(a)=2$

また

$-x^2+7x=5ax+a^4$

$\Leftrightarrow 0=x^2+(5a-7)x+a^4$

$D=(5a-7)^2-4a^4=0$

$\Leftrightarrow (5a-7+2a^2)(5a-7-2a^2)=0$

$\Leftrightarrow (2a+7)(a-1)\underbrace{(2a^2-5a+7)}_{>0}=0$

$\Leftrightarrow a=1, -\frac{7}{2}$

$-\frac{7}{2}<a<0$ のとき $N(a)=3$

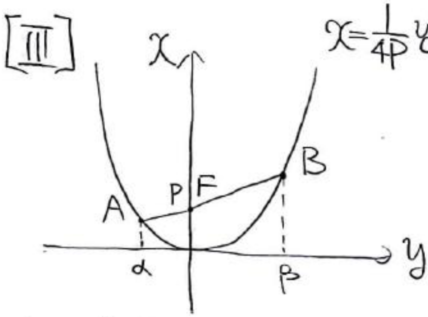
$a=-\frac{7}{2}$ 〃 $N(a)=2$

$a<-\frac{7}{2}$ 〃 $N(a)=1$

以上より

$N(a)=\begin{cases}1 & (a<-\frac{7}{2},\ 1<a)\\ 2 & (a=-\frac{7}{2}, 0, 1)\\ 3 & (-\frac{7}{2}<a<0,\ 0<a<1)\end{cases}$

[III]



上の座標平面で考える

$A\left(\alpha, \frac{\alpha^2}{4P}\right), B\left(\beta, \frac{\beta^2}{4P}\right)$

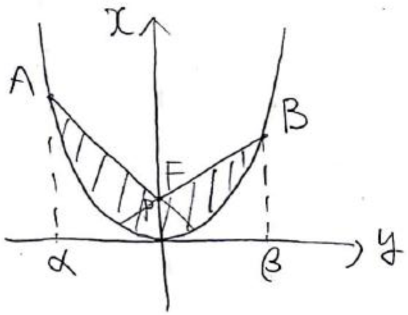
を通る直線は

$y=\frac{\beta+\alpha}{4P}(x-\beta)+\frac{\beta^2}{4P}$

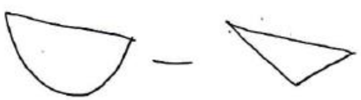
$=\frac{\alpha+\beta}{4P}x-\frac{\alpha\beta}{4P}$

切片 $\left(0, -\frac{\alpha\beta}{4P}\right)$ の y座標と P で比較する.

(i) $-\frac{\alpha\beta}{4p} \geq p$ のとき



S

$=$ 

$= \int_\alpha^\beta \left(\text{直線}AB - \frac{y^2}{4p}\right) dy$

$\quad - \left(-\frac{\alpha\beta}{4p} - p\right)(\beta-\alpha)\frac{1}{2}$

$= -\frac{1}{4p}\int_\alpha^\beta (y-\alpha)(y-\beta)\,dy$

$\quad + \frac{1}{2}\left(\frac{\alpha\beta}{4p}+p\right)(\beta-\alpha)$

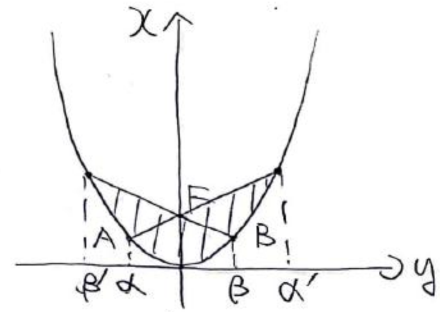
$= -\frac{1}{4p}\left\{-\frac{1}{6}(\beta-\alpha)^3\right\}$

$\quad + \frac{1}{8p}(\alpha\beta+4p^2)(\beta-\alpha)$

$= \frac{1}{24p}(\beta-\alpha)^3 + \frac{1}{8p}(\alpha\beta+4p^2)(\beta-\alpha)$

$= \frac{1}{24p}(\beta^3-\alpha^3) + \frac{p}{2}(\beta-\alpha)$ //

(ii) $-\frac{\alpha\beta}{4p} < p$ のとき



$\frac{y^2}{4p} = \frac{p - \frac{\alpha^2}{4p}}{-\alpha} y + p$

$y^2 - \frac{4p^2-\alpha^2}{-\alpha} y - 4p^2 = 0$

つまり $\alpha\alpha' = -4p^2$

これより $\alpha' = -\frac{4p^2}{\alpha}$ $\beta' = -\frac{4p^2}{\beta}$

(i)の結果使うと

S

$= \frac{1}{24p}(\beta'^3-\alpha'^3) + \frac{\beta}{2}(\beta'-\alpha')$

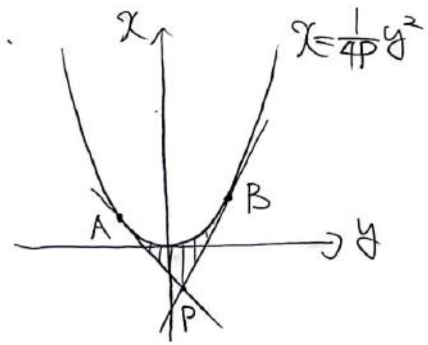
$= \frac{8}{3}p^5\frac{(\alpha-\beta)^3}{\alpha^3\beta^3} - \frac{2p^3}{\alpha^2\beta^2}(\beta-\alpha)(4p^2+\alpha\beta)$ //

※もしくと線分AF, BFとすると

(i)(ii)は共に(i)の値に

なります.

問2.



$\frac{1}{12}$公式(2次関数)を知っていると

楽です.

$T = \frac{\frac{1}{4p}}{12}(\beta-\alpha)^3$

$\quad = \frac{1}{48p}(\beta-\alpha)^3$ //

問3.

(ii)のとき $S>T$ なので

(i)のときを考える.

$\frac{1}{24p}(\beta^3-\alpha^3) + \frac{p}{2}(\beta-\alpha) = \frac{1}{48p}(\beta-\alpha)^3$

$\frac{1}{24p}(\beta^2+\beta\alpha+\alpha^2) + \frac{p}{2} = \frac{1}{48p}(\beta-\alpha)^2$

$\downarrow \div\alpha^2 \quad -\frac{\beta}{\alpha} = x$ とおく

$\frac{1}{24p}(x^2-x+1) + \frac{p}{2\alpha^2} = \frac{1}{48p}(-x-1)^2$

$2(x^2-x+1) + \frac{24p^2}{\alpha^2} = x^2+2x+1$

$\underbrace{\frac{24p^2}{\alpha^2}}_{>0} = -x^2+4x+1 > 0$

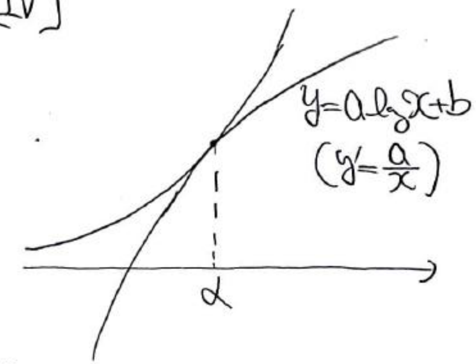
$x^2-4x-1<0$

$\therefore 2-\sqrt{3} < x < 2+\sqrt{3}$

$\therefore 2-\sqrt{3} < -\frac{\beta}{\alpha} < 2+\sqrt{3}$ //

[IV]

$y = e^{x^2}$ $(y' = 2xe^{x^2})$



問1.

$\begin{cases} a\log\alpha + b = e^{\alpha^2} \leftarrow y\text{一致} \\ 2\alpha e^{\alpha^2} = \frac{a}{\alpha} \leftarrow \text{傾き一致} \end{cases}$

$\therefore a = 2\alpha^2 e^{\alpha^2}$

$\quad b = e^{\alpha^2} - 2\alpha^2 e^{\alpha^2}\log\alpha$

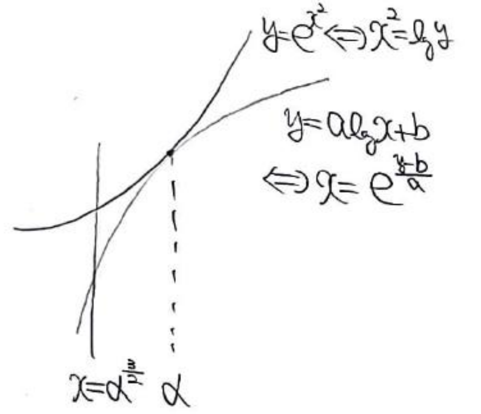
$\quad\;\; = e^{\alpha^2}(1-2\alpha^2\log\alpha)$ //

問2.

C_2とx軸の交点は

$a\log x + b = 0$

$\Leftrightarrow \log x = -\frac{b}{a}$



$V(\alpha)$ $\boxed{\text{バームクーヘン積分}}$

$$=\int_{\alpha^{\frac{3}{2}}}^{\alpha} 2\pi x\{e^{x^2}-a\log x-b\}dx$$

$$=2\pi\left[\frac{1}{2}e^{x^2}-\frac{ax^2}{2}\log x+\frac{ax^2}{4}-\frac{b}{2}x^2\right]_{\alpha^{\frac{3}{2}}}^{\alpha}$$

$$=2\pi\left\{\frac{1}{2}e^{\alpha^2}-\frac{a\alpha^2}{2}\log\alpha+\frac{a\alpha^2}{4}-\frac{b}{2}\alpha^2\right.$$
$$\left.-\left(\frac{1}{2}e^{\alpha^3}-\frac{a\alpha^3}{2}\log\alpha^{\frac{3}{2}}+\frac{a\alpha^3}{4}-\frac{b}{2}\alpha^3\right)\right\}$$

$$=\pi\left\{e^{\alpha^2}-e^{\alpha^3}-a\left(\alpha^2\log\alpha-\frac{3}{2}\alpha^3\log\alpha\right)\right.$$
$$\left.+\frac{a}{2}(\alpha^2-\alpha^3)-b(\alpha^2-\alpha^3)\right\}$$

$$=\pi e^{\alpha^2}\left\{1-e^{\alpha^3-\alpha^2}-2\alpha^4\left(\log\alpha-\frac{3}{2}\alpha\log\alpha\right)\right.$$
$$\left.+\alpha^4-\alpha^5-(1-2\alpha^2\log\alpha)(\alpha^2-\alpha^3)\right\}$$

$$=\pi e^{\alpha^2}\left(1-\alpha^2+\alpha^3+\alpha^4-\alpha^5\right.$$
$$\left.+\alpha^5\log\alpha-e^{\alpha^3-\alpha^2}\right)$$

問3.

$f(t)=e^{-t}-1+t-\frac{t^2}{2}$ とおく.

$f'(t)=-e^{-t}+1-t$

$f''(t)=e^{-t}-1\leqq 0$

$f'(t)$ は単調減少. $f'(0)=0$ より

$f'(t)\leqq 0$. これより $f(t)$ は

単調減少. $f(0)=0$ より

$f(t)\leqq 0$.

$g(t)=-\frac{t^3}{6}-f(t)$ とおく

$g'(t)=-\frac{t^2}{2}-f'(t)$

$g''(t)=-t-f''(t)$

$=-e^{-t}+1-t$

$=f'(t)\leqq 0$

$g'(t)$ は単調減少. $g'(0)=0$ より

$g'(t)\leqq 0$. これより $g(t)$ は

単調減少. $g(0)=0$ より

$g(t)\leqq 0$.

$\therefore -\frac{t^3}{6}\leqq e^{-t}-1+t-\frac{t^2}{2}\leqq 0$

問4.

問3より

$-\frac{t^3}{6}+\frac{t^2}{2}-t+1\leqq e^{-t}\leqq\frac{t^2}{2}-t+1$

$t=\alpha^2-\alpha^3$ とする

$-\frac{1}{6}(\alpha^2-\alpha^3)^3+\frac{1}{2}(\alpha^2-\alpha^3)^2-(\alpha^2-\alpha^3)+1$

$\leqq e^{\alpha^3-\alpha^2}\leqq\frac{1}{2}(\alpha^2-\alpha^3)^2-(\alpha^2-\alpha^3)+1$

これより

$\pi e^{\alpha^2}\left\{1-\alpha^2+\alpha^3+\alpha^4-\alpha^5+\alpha^5\log\alpha-\frac{1}{2}(\alpha^2-\alpha^3)^2+(\alpha^2-\alpha^3)-1\right\}$

$\leqq V(\alpha)\leqq\pi e^{\alpha^2}\left\{1-\alpha^2+\alpha^3+\alpha^4-\alpha^5+\alpha^5\log\alpha+\frac{1}{6}(\alpha^2-\alpha^3)^3-\frac{1}{2}(\alpha^2-\alpha^3)^2+(\alpha^2-\alpha^3)-\right.$

$\Leftrightarrow$

$\pi\frac{e^{\alpha^2}}{\alpha^c}\left(\frac{1}{2}\alpha^4-\frac{1}{2}\alpha^6+\alpha^5\log\alpha\right)\leqq\frac{V(\alpha)}{\alpha^c}\leqq\pi\frac{e^{\alpha^2}}{\alpha^c}\left(\frac{1}{2}\alpha^4-\frac{1}{3}\alpha^6-\frac{1}{2}\alpha^7+\frac{1}{2}\alpha^8-\frac{1}{6}\alpha^9\right.$
$\left.+\alpha^5\log\alpha\right)$

$\Leftrightarrow$

$\pi e^{\alpha^2}\frac{\frac{1}{2}-\frac{1}{2}\alpha^2+\alpha\log\alpha}{\alpha^{c-4}}\leqq\frac{V(\alpha)}{\alpha^c}\leqq\pi e^{\alpha^2}\frac{\frac{1}{2}-\frac{1}{3}\alpha^2-\frac{1}{2}\alpha^3+\frac{1}{2}\alpha^4-\frac{1}{6}\alpha^5+\alpha\log\alpha}{\alpha^{c-4}}$

$\lim_{\alpha\to+0}\alpha\log\alpha=0$ より最左辺と最右辺で $\alpha\to+0$ として有限確定

するには $\underline{c=4}$ である.

このとき $\lim_{\alpha\to+0}\frac{V(\alpha)}{\alpha^4}=\underline{\frac{\pi}{2}}$