2021 杏林(医)

I.

(a)

$P(黒\times 2)$

$= \dfrac{{}_4C_2}{{}_9C_2} = \dfrac{1}{6}$ //

$P(赤\times 1, 黒\times 1)$

$= \dfrac{2\times 4}{{}_9C_2} = \dfrac{2}{9}$ //

$P(黒\times 1, 白\times 1)$

$= \dfrac{4\times 3}{{}_9C_2} = \dfrac{1}{3}$ //

(b)

$P(赤\times 1, 黒\times 1)$

$= \dfrac{xy}{{}_{16}C_2} = \dfrac{1}{5}$

$5xy = 120 \Leftrightarrow xy = 24$ //

$P(黒\times 1, 白\times 1)$

$= \dfrac{yz}{{}_{16}C_2} = \dfrac{1}{3}$

$yz = 40$ //

$yz = y(16-x-y) = 40$

$\Leftrightarrow 16y - 24 - y^2 = 40$

$\Leftrightarrow 0 = y^2 - 16y + 64 \quad y = 8$

$\therefore x = 3$ //, $y = 8$ //, $z = 5$ //

(c)

$P(赤\times 1, 黒\times 1, 白\times 1)$

$= \dfrac{3\cdot 5\cdot 7}{{}_{15}C_3} = \dfrac{3}{13}$ //

$P_{玉が2種類}(赤が2コある)$

$= \dfrac{赤赤黒 + 白赤赤}{赤赤黒 + 赤黒黒 + 黒黒白 + 黒白白 + 白白赤 + 白赤赤}$

$= \dfrac{{}_3C_2\cdot 5 + 7\cdot {}_3C_2}{{}_3C_2\cdot 5 + 3\cdot {}_5C_2 + {}_5C_2\cdot 7 + 5\cdot {}_7C_2 + {}_7C_2\cdot 3 + 7\cdot {}_3C_2}$

$= \dfrac{9}{76}$ //

II

中心(a, b)とおく

$a^2 + (b-5)^2 = (a-7)^2 + (b-4)^2$

$\Leftrightarrow 0 = -14a + 2b + 40$

$a^2 + (b-5)^2 = (a-6)^2 + (b+3)^2$

$\Leftrightarrow 0 = -12a + 16b + 20$

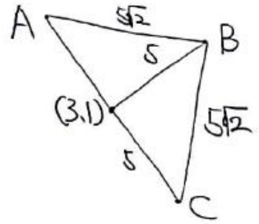
$0 = -28a + 4b + 80$

$-)\ 0 = -3a + 4b + 5$

$0 = -25a + 75$

$\therefore a = 3, b = 1$

半径 5 //　中心$(3, 1)$ //



短い$\overset{\frown}{BC} = 10\pi \times \dfrac{1}{4} = \dfrac{5}{2}\pi$ //

$\triangle ABC = 5\sqrt{2}\cdot 5\sqrt{2}\cdot \dfrac{1}{2}$

$= 25$ //

$25 = \dfrac{r}{2}(5\sqrt{2} + 5\sqrt{2} + 10)$

$\Leftrightarrow 5 = r(\sqrt{2} + 1)$

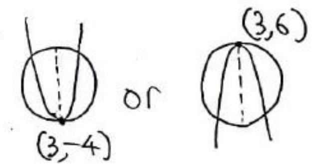
$\therefore r = 5(\sqrt{2} - 1)$ //

BとD$(-1, 4)$を通る放物線は

$y = a(x-7)(x+1) + 4$

$= a(x^2 - 6x - 7) + 4$

$= a(x-3)^2 - 16a + 4$



$-16a + 4 = -4, 6$

$-16a = -8, 2$

$\therefore a = \dfrac{1}{2}, -\dfrac{1}{8}$

$a = \dfrac{1}{2}$のとき

$y = \dfrac{1}{2}x^2 - 3x + \dfrac{1}{2}$ //

$a = -\dfrac{1}{8}$のとき

$y = -\dfrac{1}{8}x^2 + \dfrac{3}{4}x + \dfrac{39}{8}$ //

$\dfrac{1}{2}x^2 - 3x + \dfrac{1}{2} = -\dfrac{1}{8}x^2 + \dfrac{3}{4}x + \dfrac{39}{8}$

$4x^2 - 24x + 4 = -x^2 + 6x + 39$

$5x^2 - 30x - 35 = 0$

$x^2 - 6x - 7 = 0$

$x = -1, 7$

2つの放物線で囲まれた面積は

$\dfrac{\left|\frac{1}{2} - \left(-\frac{1}{8}\right)\right|}{6}(7+1)^3$

$= \dfrac{160}{3}$ //

Ⅲ $\frac{f(s)+f(t)+f(u)}{3}$

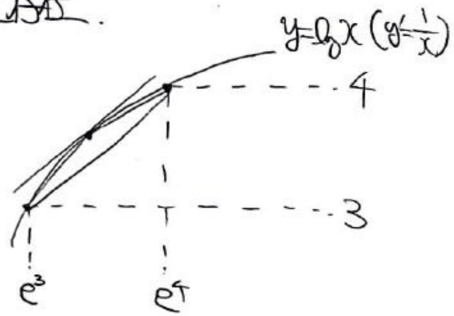
(a) ↓

$G\left(\frac{s+t+u}{3}, f\left((stu)^{\frac{1}{3}}\right)\right)$

② ⑦

$y=f(x)$ は上に凸で単調増加 ③

Gは $x>0$, $y<f(x)$ ② に存在.



$\frac{1}{e^4-e^3}=\frac{1}{t}$

$\Leftrightarrow t=e^3(-1+e)$ のとき

とき三角形の面積最大.

(b)

$f(a_n)=-3n+\alpha$

$a_n=e^{-3n+\alpha}$

公比 $\frac{1}{e^3}$ の等比数列 ⑥

$\log_b(a_2 \cdot a_{2021})-\log_b(a_1)^2$

$=\frac{\log a_2 \cdot a_{2021}}{\log \frac{a_{44}}{a_1}}-2\log\frac{a_1}{\frac{a_{44}}{a_1}}$

$=\frac{\log a_2+\log a_{2021}}{\log a_{44}-\log a_1}-2\frac{\log a_1}{\log a_{44}-\log a_1}$

$=\frac{\log a_2+\log a_{2021}-2\log a_1}{\log a_{44}-\log a_1}$

$=\frac{-6+\alpha-6063+\alpha-2(-3+\alpha)}{-132+3}$

$=\frac{-6063}{-129}$

$=47$

$\sum_{k=1}^{5} f(a_k)$

$=\sum_{k=1}^{5}(-3k+\alpha)$ $\curvearrowright \alpha=9$

$=-3\cdot\frac{1}{2}\cdot 5\cdot 6+5\alpha=0$

$f(a_n)=-3n+9$

$a_n=e^{-3n+9}$

$\prod_{k=1}^{5} a_k=e^{f(a_1)+f(a_2)+\cdots+f(a_5)}$

$=e^0$

$=1$

$\lim_{n\to\infty}\sum_{k=1}^{n} e^{-3k+9}$

$=\frac{e^6}{1-e^{-3}}$

$=\frac{e^9}{e^3-1}$

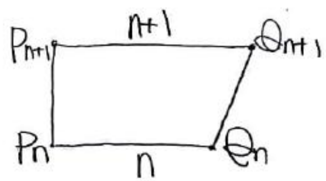
$n \geqq 5$ のとき

$f(|f(a_n)|)$

$=f(|-3n+9|)$

$=f(3n-9)$

$=\log(3n-9)$



S_n

$=(2n+1)\times\{\log(3n-6)-\log(3n-9)\}\times\frac{1}{2}$

$=\left(n+\frac{1}{2}\right)\log\frac{3n-6}{3n-9}$

$=\left(n+\frac{1}{2}\right)\log\left(1+\frac{1}{-3+n}\right)$

$\lim_{n\to\infty} S_n$

$=\lim_{n\to\infty}\frac{n+\frac{1}{2}}{n-3}\log\left(1+\frac{1}{n-3}\right)^{n-3}$

$=1$

(c) $T_n=\sum_{k=1}^{n}\log k$

$T_{100}-T_m-T_{100-m}$

$=\log(m+1)+\cdots+\log 100$

$-\{\log 1+\cdots+\log(100-m)\}$

$=\log\frac{100!}{m!(100-m)!}$

$m=0, 100$ のとき 最小値 0

$m=50$ のとき最大