2021 順天堂大（医）

$\boxed{\text{I}}$

(1) $|z_1|$

(a) $= \sqrt{\frac{9}{4}+4}$

$= \frac{5}{2}$

$z_1 = \frac{5}{2}\left(\frac{3}{5}+\frac{4}{5}i\right)$

$\therefore \sin\alpha = \underline{\frac{4}{5}}$

(b)

z_3

$= \left(\frac{3}{5}+\frac{9}{5}i\right)\{\cos(-\alpha)+i\sin(-\alpha)\}$

$= \left(\frac{3}{5}+\frac{9}{5}i\right)\left(\frac{3}{5}-\frac{4}{5}i\right)$

$= \frac{9}{25}+\frac{15}{25}i+\frac{36}{25}$

$= \underline{\frac{9}{5}+\frac{3}{5}i}$

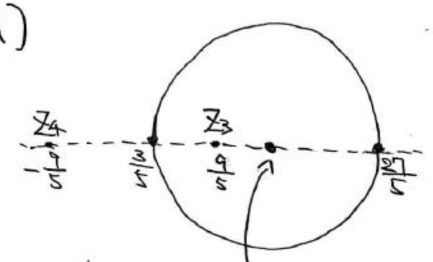
(c)

z_4

$= \left(\frac{3}{5}+\frac{9}{5}i\right)i$

$= \underline{\frac{-9}{5}+\frac{3}{5}i}$

(d)



中心：$\underline{3+\frac{3}{5}i}$

半径 $\underline{\frac{12}{5}}$

(2)

(a) $\int_0^a (x^3-a^2x)$

$= \left[\frac{1}{4}x^4-\frac{a^2}{2}x^2\right]_0^a$

$= -\frac{1}{4}a^4 \quad \underline{p=\frac{-1}{4} \quad q=4}$

$\left|\int_0^a (x^3-a^2x)dx\right|$

$= \left|-\frac{1}{4}a^4\right| = 4 \quad \therefore \underline{a=2}$

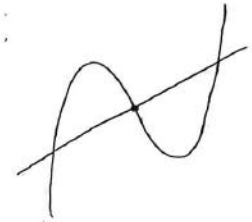
(b)

$f(x) = x^3-6x^2+10x+1$

$f'(x) = 3x^2-12x+10$

$f''(x) = 6x-12$

変曲点 $\underline{(2, 5)}$



3つの交点のx座標を

$2-\alpha, 2, 2+\alpha$ とする.

$\int_2^{2+\alpha}\{-(x-(2-\alpha))(x-2)(x-(2+\alpha))\}dx$

$= -\int_2^{2+\alpha}(x-2+\alpha)(x-2)(x-2-\alpha)dx$

$= -\int_2^{2+\alpha}\{(x-2)^2(x-2-\alpha)$

$\qquad +\alpha(x-2)(x-2-\alpha)\}dx$

$= -\int_2^{2+\alpha}(x-2)^2(x-2-\alpha)dx$

$-\alpha\int_2^{2+\alpha}(x-2)(x-2-\alpha)dx$

$= -\left\{\left[\frac{(x-2)^3}{3}(x-2-\alpha)\right]_2^{2+\alpha}\right.$

$\qquad \left.-\int_2^{2+\alpha}\frac{(x-2)^3}{3}dx\right\}$

$-\alpha\left\{-\frac{1}{6}\alpha^3\right\}$

$= \frac{\alpha^4}{12}+\frac{\alpha^4}{6} = \frac{\alpha^4}{4} = 4 \quad \therefore \alpha=2$

x座標が最も大きいものは $\underline{4}$

求める直線は

$y = \frac{9-5}{2}(x-2)+5$

$= \underline{2x+1}$

(3) (a)

$\bar{x}=0$

s_x^2

$= \frac{1}{2n+1}2\{1^2+2^2+\cdots+n^2\}$

$= \frac{2}{2n+1}\cdot\frac{1}{6}n(n+1)(2n+1)$

$= \frac{1}{3}n(n+1) = \frac{56}{3} \quad \therefore \underline{n=7}$

(b)

$a_1+a_2+a_3=3$

$\frac{1}{3}(a_1^2+a_2^2+a_3^2)-1^2 = \frac{26}{3}$

$\Leftrightarrow a_1^2+a_2^2+a_3^2=29$

$\Downarrow$

$-5 \leqq a_1$ かつ $a_3 \leqq 5$

a_1^2, a_2^2, a_3^2 の組が $4, 9, 16$

$a_1+a_2+a_3=3$ より

$a_1=-3, \ a_2=2, \ a_3=\underline{4}$

組が $0, 4, 25$, $a_1+a_2+a_3=3$ より

$a_1=-2, \ a_2=0, \ a_3=\underline{5}$

(c)

(a)のデータから $-2, 0, 5$ を除く

$(\text{平均}) = \frac{1}{12}(0-3) = \frac{-1}{4}$ //

(分散)

$= \frac{1}{12}(280-29) - \left(\frac{-1}{4}\right)^2$

$= \frac{251}{12} - \frac{1}{16} = \frac{1001}{48}$ //

(4) (a)

$2x^2+y^2+3x-2y+1$

$= 2\left(x+\frac{3}{4}\right)^2 + (y-1)^2 - \frac{9}{8}$

$x=2,\ y=-2$ のとき

最大値 23 //

$x=-\frac{3}{4},\ y=1$ のとき

最小値 $-\frac{9}{8} = \frac{-9}{8}$ //

(b)

$x^2-4xy+4y^2+2x+y+1$

$= x^2+(2-4y)x+4y^2+y+1$

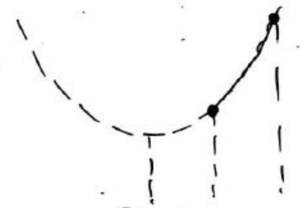
$= (x+1-2y)^2+5y = A$

$x=-2,\ y=2$ のとき

$A=35$

$x=2,\ y=-2$ のとき

$A=37$ ← 最大値 //



$2y-1 \quad -2 \quad 2$

$2y-1<-2 \Leftrightarrow y<-\frac{1}{2}$ のとき

$\min A = A|_{x=-2}$

$= 4y^2+9y+5$

$= 4\left(y+\frac{9}{8}\right)^2 - \frac{65}{16}$

$-2<2y-1<2 \Leftrightarrow -\frac{1}{2}<y<\frac{3}{2}$ のとき

$\min A = A|_{2y-1}$

$= 5y = -\frac{5}{2}$

$2<2y-1 \Leftrightarrow \frac{3}{2}<y$ のとき

$\min A = A|_{x=2}$

$= 4y^2-7y+9$

$= 4\left(y-\frac{7}{8}\right)^2+\frac{95}{16}$

以上より $\min A = \frac{-65}{16}$ //

$\boxed{\text{II}}$

(a) $AC = \frac{1+\sqrt{5}}{2}$

$\cos\angle BAC = \frac{\frac{1+\sqrt{5}}{4}}{1}$

$\cos^2\angle BAC = \frac{6+2\sqrt{5}}{16}$

$= \frac{3+\sqrt{5}}{8}$

$\sin^2\angle BAC = 1-\frac{3+\sqrt{5}}{8}$

$= \frac{5-\sqrt{5}}{8}$

正

$\frac{BC}{\sin\angle BAC} = 2r$

$\therefore r^2 = \frac{1}{4\sin^2\angle BAC}$

$= \frac{2}{5-\sqrt{5}} \times \frac{5+\sqrt{5}}{5+\sqrt{5}}$

$= \frac{5+\sqrt{5}}{10}$ //

S

$= \frac{1}{2}\cdot 1\cdot \phi \sin\angle BAC$

$+ \frac{1}{2}\phi\cdot\phi\sin\angle BAC$

$+ \frac{1}{2}\cdot 1\cdot\phi\sin\angle BAC$

$= \frac{5+\sqrt{5}}{4}\cdot\frac{1+\sqrt{5}}{2}\cdot\sin\angle BAC$

$= \frac{\sqrt{5}}{8}(1+\sqrt{5})^3\sin\angle BAC$

S^2

$= \frac{5}{64}(6+2\sqrt{5})^2\frac{5-\sqrt{5}}{8}$

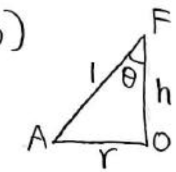
$= \frac{5\sqrt{5}(56+24\sqrt{5})(\sqrt{5}-1)}{64\cdot 8}$

$= \frac{5\sqrt{5}(7+3\sqrt{5})(\sqrt{5}-1)}{64}$

$= \frac{5\sqrt{5}(8+4\sqrt{5})}{64}$

$= \frac{25+10\sqrt{5}}{16}$ //

(b)



$h^2 = 1-r^2 = \frac{5-\sqrt{5}}{10}$ //

$\sin 2\theta = 2\sin\theta\cos\theta$

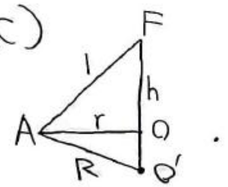
$= 2rh$

$= 2\sqrt{\frac{5+\sqrt{5}}{10}\cdot\frac{5-\sqrt{5}}{10}}$

$= \frac{2}{10}\sqrt{20}$

$= \frac{2\sqrt{5}}{5}$ //

(c)



余

$R^2 = 1 + R^2 - 2R\cos\theta$

$\Leftrightarrow 2R\cos\theta = 1$

$\Leftrightarrow R = \dfrac{1}{2\cos\theta} = \dfrac{1}{2h}$

$\therefore R^2 = \dfrac{1}{4h^2} = \dfrac{1}{4} \times \dfrac{10}{5-\sqrt{5}} = \underline{\dfrac{5+\sqrt{5}}{8}}$

$\boxed{\text{III}}$

(1)

$9\,)\,567$
$9\,)\,63$
7

$n_3(945-378)$

$= n_3(567)$

$= n_3(3^4 \cdot 7) = 4$

$d_3(945, 378)$

$= 3^{-4} = \underline{\dfrac{1}{81}}$

(2) $x = 0$ のとき

$n_p(mx) = n_p(x)$

$x \geqq 1$ のとき

$n_p(mx) \geqq n_p(x)$

以上より $n_p(mx) \geqq n_p(x)$

$\therefore p^{-n_p(mx)} \leqq p^{-n_p(x)}$

$\therefore d_p(mx, 0) \leqq d_p(x, 0)$

(3)

(i) $x = y$ のとき

$d_p(x, z) \leqq 0 + d_p(x, z)$

より与式をみたす.

(ii) $y = z$ のとき

$d_p(x, z) \leqq d_p(x, z) + 0$

より与式をみたす.

(iii) $z = x$ のとき

$0 \leqq d_p(x, y) + d_p(y, x)$

より与式をみたす.

(iv) x, y, z がすべて異なるとき

$x - y = \alpha p^a$

$y - z = \beta \cdot p^b$ (α, β, γ は p で割れない)

$x - z = \gamma \cdot p^c$

とおくと

$x - z = x - y + y - z$

$\gamma \cdot p^c = \alpha p^a + \beta \cdot p^b$

(i) $a > b$ のとき

$\gamma \cdot p^c = p^a(\alpha + \beta \cdot p^{b-a})$

$\alpha + \beta \cdot p^{b-a}$ は p で割れないので

$c = a$. これより

$p^{-c} = p^{-a} \leqq p^{-a} + p^{-b}$

$\therefore d_p(x, z) \leqq d_p(x, y) + d_p(y, z)$

(ii) $a < b$ のとき

(i) と同様に成り立つ.

(iii) $a = b$ のとき

$\gamma \cdot p^c = p^a(\alpha + \beta)$

$\alpha + \beta$ は p で割れる可能性があるので $c \geqq a$

$p^{-c} \leqq p^{-a} \leqq p^{-a} + p^{-b}$

$\therefore d_p(x, z) \leqq d_p(x, y) + d_p(y, z)$