2021 獨協医科大(医) 60分

1

(1) $P(S=7)$

$= P(1\text{が}5\text{回}, 2\text{が}1\text{回})$

$= {}_6C_5\left(\frac{3}{6}\right)^5\frac{2}{6}$

$= \frac{1}{16}$ //

$P(S=9)$

$= P(1\text{が}4\text{回}, 2\text{が}1\text{回}, 3\text{が}1\text{回})$

$\quad + P(1\text{が}3\text{回}, 2\text{が}3\text{回})$

$= \frac{6!}{4!}\left(\frac{1}{2}\right)^4\frac{1}{3}\cdot\frac{1}{6} + {}_6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{3}\right)^3$

$= \frac{85}{432}$ //

$P(1\text{が}4\text{回以上連続})$

$= P(4\text{回連続}) + P(5\text{回連続}) + P(6\text{回連続})$

$= \underbrace{\left(\frac{1}{2}\right)^5+\left(\frac{1}{2}\right)^6+\left(\frac{1}{2}\right)^5}_{\boxed{\begin{matrix}1111\sim ?\\ \sim 1111\sim\\ ?\sim 1111\end{matrix}}} + 2\left(\frac{1}{2}\right)^6$

$\quad + \left(\frac{1}{2}\right)^6$

$= \frac{2+1+2+2+1}{64} = \frac{1}{8}$ //

$P(1\text{が}4\text{回以上連続} \cap S\text{が偶数})$

$= \left\{\left(\frac{1}{2}\right)^4\left(\frac{1}{3}\right)^2 + \left(\frac{1}{2}\right)^5\frac{1}{6}\right\}\times 2$

$\quad + \left(\frac{1}{2}\right)^4\left(\frac{1}{3}\right)^2$

$\quad + \left(\frac{1}{2}\right)^4\left(\frac{1}{6}\right)^2$

111122
111131
211112
311113
221111
131111

$\quad + \left(\frac{1}{2}\right)^5\frac{1}{6}\times 2 + \left(\frac{1}{2}\right)^6$

111113
311111

$= \frac{1}{16}$

$P_{1\text{が}4\text{回以上連続}}(S\text{が偶数})$

$= \frac{\frac{1}{16}}{\frac{1}{8}} = \frac{1}{2}$ //

(2) $0.24_{(6)}$

$= 2\times\frac{1}{6} + 4\times\frac{1}{6^2}$

$= \frac{12+4}{36}$

$= \frac{4}{9}$ //

$0.2_{(a)} = \frac{2}{a}$

$0.12_{(b)} = \frac{1}{b} + \frac{2}{b^2} \quad (b \geqq 3)$

$\downarrow$

$\frac{2}{a} = \frac{b+2}{b^2}$ //

$\downarrow b+2=c$

$\frac{2}{a} = \frac{c}{(c-2)^2}$

$\Longleftrightarrow 2(c-2)^2 = ac$

$\Longleftrightarrow 8 = -2c^2 + 8c + ac$

$\Longleftrightarrow (-2c+8+a)c = 8$

$c=4$のとき $a=2$, $b=2$ …NG

$c=8$のとき

$-8+a=1$

$\therefore a=9,\ b=6$ //

2

(1) $\int \sin^2\theta\, d\theta$

$= \int \frac{1-\cos 2\theta}{2}\, d\theta$

$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C_1$

$\int \sin^2\theta\cos^2\theta\, d\theta$

$= \frac{1}{4}\int \sin^2 2\theta\, d\theta$

$= \frac{1}{4}\int \frac{1-\cos 4\theta}{2}\, d\theta$

$= \frac{1}{8}\theta - \frac{1}{32}\sin 4\theta + C_2$

(2)

$\frac{dx}{d\theta}$

$= 2(-\sin\theta)\cos\theta + 2(1+\cos\theta)(-\sin\theta)$

$= -2\sin\theta(1+2\cos\theta)$

$\frac{dy}{d\theta}$

$= 2(-\sin\theta)\sin\theta + 2(1+\cos\theta)\cos\theta$

$= 2\cos\theta + 2\cos 2\theta$

$= 2(2\cos^2\theta + \cos\theta - 1)$

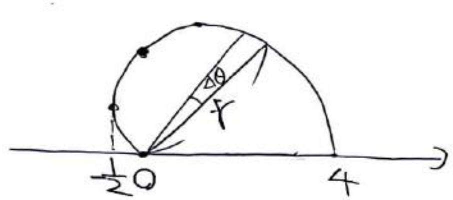
$= 2(2\cos\theta - 1)(\cos\theta + 1)$

θ	0	$\cdots$	$\frac{\pi}{3}$	$\cdots$	$\frac{2}{3}\pi$	$\cdots$	π
$\frac{dx}{d\theta}$		$-$	$-$	$-$	0	$+$	
x	4	$\searrow$	$\frac{3}{2}$	$\searrow$	$-\frac{1}{2}$	$\nearrow$	0
$\frac{dy}{d\theta}$		$+$	0	$-$	$-$	$-$	0
y	0	$\nearrow$	$\frac{3\sqrt{3}}{2}$	$\searrow$	$\frac{\sqrt{3}}{2}$	$\searrow$	0

xが最小なのは $\theta=\frac{2}{3}\pi$ で

点は $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

yが最大な点は $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$



$$\frac{S}{2}=\int_{-\frac{1}{2}}^{4} y_1\,dx-\int_{-\frac{1}{2}}^{0} y_2\,dx$$

Cを極方程式にすると

$$r=2(1+\cos\theta)$$

$\Delta\theta$が十分小さいとき図の扇形の面積は

$$r^2\times\pi\times\frac{\Delta\theta}{2\pi}$$

$$=\frac{1}{2}r^2\Delta\theta$$

これを $0\leqq\theta\leqq\pi$ で積分すると

$$\frac{S}{2}$$

$$=\int_0^{\pi}\frac{1}{2}\cdot4(1+\cos\theta)^2d\theta$$

$$=\int_0^{\pi}(2+4\cos\theta+2\cos^2\theta)d\theta$$

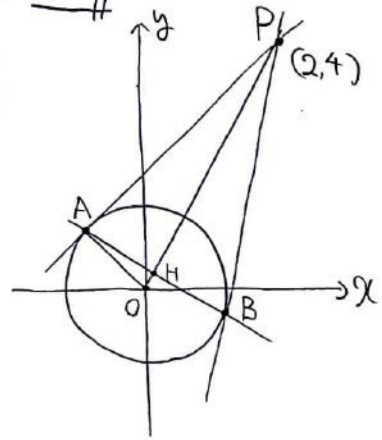
$$=\int_0^{\pi}(3+4\cos\theta+\cos2\theta)d\theta$$

$$=\left[3\theta+4\sin\theta+\frac{1}{2}\sin2\theta\right]_0^{\pi}$$

$$=3\pi$$

$\therefore S=6\pi$

$\boxed{3}$



$\triangle OAH\backsim\triangle OPA$ より

$OA:OP=OH:OA$

$\therefore OH\cdot OP=OA^2=1$

(1) Pが上のとき

$\triangle OPA\backsim\triangle APH$ より

$OP:AP=PA:PH$

$\therefore PH\cdot PO=AP^2=OP^2-OA^2=19$

P(2,4)を極にしたときの

直線AB(極線)は

$2x+4y=1\iff y=-\frac{1}{2}x+\frac{1}{4}$

(2)

(直線OPの傾き) $=\frac{1}{2}$

$P(2\alpha,\alpha)$ とおくと

(1)と同様にして

$PH\cdot PO=OP^2-OA^2=5\alpha^2-1$

ここで $PH=24OH$ より

$$PH=\frac{24}{25}OP$$

よって

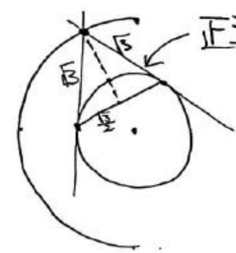
$\frac{24}{25}\cdot5\alpha^2=5\alpha^2-1$

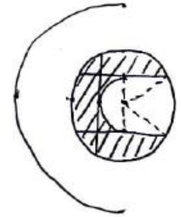
$\iff 24\alpha^2=25\alpha^2-5$

$\iff 5=\alpha^2$ $\therefore \alpha=\sqrt{5}$

$\therefore P(2\sqrt{5},\sqrt{5})$

(3)





斜線部分がS

$S=\pi-\frac{1}{2}\cdot\frac{1}{2}\pi\cdot\frac{1}{2}$

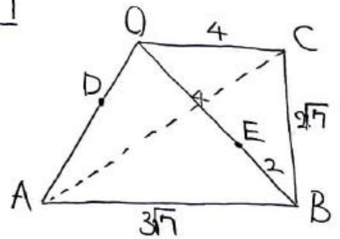
$-\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}\times2$

$-\pi\times\frac{1}{6}$

$=\frac{24-3-4}{24}\pi-\frac{\sqrt{3}}{4}$

$=\frac{17}{24}\pi-\frac{\sqrt{3}}{4}$

$\boxed{4}$



(1)

$|\overrightarrow{BC}|^2 = |\overrightarrow{OC} - \overrightarrow{OB}|^2$

$28 = 16 - 2\overrightarrow{OB}\cdot\overrightarrow{OC} + 36$

$\therefore \overrightarrow{OB}\cdot\overrightarrow{OC} = \underline{12}$

$S = \frac{1}{2}\sqrt{|\overrightarrow{OB}|^2|\overrightarrow{OC}|^2 - (\overrightarrow{OB}\cdot\overrightarrow{OC})^2}$

$= \frac{1}{2}\sqrt{36\cdot 16 - 144}$

$= \frac{1}{2}\sqrt{12\cdot 36} = \underline{6\sqrt{3}}$

OAは△OBCに垂直.

$OA = \sqrt{63-36} = 3\sqrt{3}$

$\therefore V = 6\sqrt{3} \times 3\sqrt{3} \times \frac{1}{3}$

$= \underline{18}$

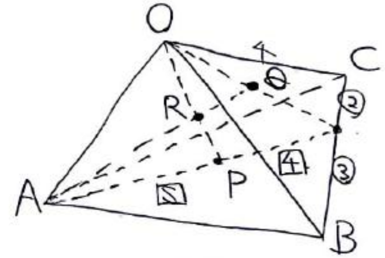
$\overrightarrow{OP} = \frac{4}{9}\overrightarrow{OA} + \frac{2}{9}\overrightarrow{OB} + \frac{1}{3}\overrightarrow{OC}$

$\Leftrightarrow \overrightarrow{AP} - \overrightarrow{AO} = -\frac{4}{9}\overrightarrow{AO} + \frac{2}{9}(\overrightarrow{AB} - \overrightarrow{AO}) + \frac{3}{9}(\overrightarrow{AC} - \overrightarrow{AO})$

$\therefore \overrightarrow{AP} = \underline{\frac{2}{9}}\overrightarrow{AB} + \underline{\frac{1}{3}}\overrightarrow{AC}$

(2)

$\overrightarrow{AP} = \frac{5}{9}\cdot\frac{2\overrightarrow{AB} + 3\overrightarrow{AC}}{5}$



$\overrightarrow{OQ} = k\cdot\frac{2\overrightarrow{OB} + 3\overrightarrow{OC}}{5}$

$= \frac{2k}{5}\overrightarrow{OB} + \frac{3k}{5}\overrightarrow{OC}$

$s : t = 2 : 3$ より $\frac{s}{t} = \underline{\frac{2}{3}}$

$|\overrightarrow{OQ}|^2$

$= \frac{4k^2}{25}|\overrightarrow{OB}|^2 + \frac{12k^2}{25}\overrightarrow{OB}\cdot\overrightarrow{OC} + \frac{9k^2}{25}|\overrightarrow{OC}|^2$

$= \frac{k^2}{25}(144 + 144 + 144)$

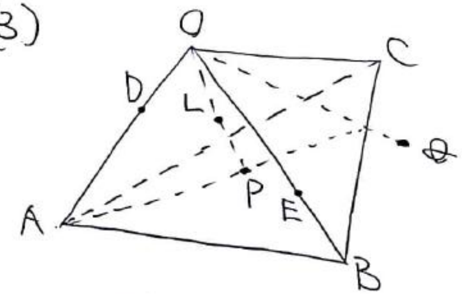
$= \frac{144}{25}3k^2$

$|\overrightarrow{OQ}| = \frac{12}{5}\sqrt{3}k = 3\sqrt{3}$

$\therefore k = \frac{5}{4}$

$\therefore s = \underline{\frac{1}{2}},\ t = \underline{\frac{3}{4}}$

(3)



$\overrightarrow{OL} = \alpha\overrightarrow{OP}$

$= \frac{4\alpha}{9}\overrightarrow{OA} + \frac{2\alpha}{9}\overrightarrow{OB} + \frac{\alpha}{3}\overrightarrow{OC}$

$\overrightarrow{OL} = \beta\overrightarrow{OQ} + \gamma\overrightarrow{OD} + (1-\beta-\gamma)\overrightarrow{OE}$

$= \frac{\beta}{2}\overrightarrow{OB} + \frac{3\beta}{4}\overrightarrow{OC} + \frac{2\gamma}{5}\overrightarrow{OA}$

$+ \frac{2(1-\beta-\gamma)}{3}\overrightarrow{OB}$

$= \frac{2\gamma}{5}\overrightarrow{OA} + \frac{4-\beta-4\gamma}{6}\overrightarrow{OB} + \frac{3\beta}{4}\overrightarrow{OC}$

$\frac{4\alpha}{9} = \frac{2\gamma}{5} \Leftrightarrow \gamma = \frac{10}{9}\alpha$

$\frac{\alpha}{3} = \frac{3\beta}{4} \Leftrightarrow \beta = \frac{4}{9}\alpha$

$\frac{2\alpha}{9} = \frac{4-\beta-4\gamma}{6}$

$\Leftrightarrow 4\alpha = 12 - 3\beta - 12\gamma$

$\Leftrightarrow 12\alpha = 36 - 4\alpha - 40\alpha$

$\Leftrightarrow 56\alpha = 36$

$\Leftrightarrow \alpha = \frac{9}{14}$

$\therefore \overrightarrow{OL} = \underline{\frac{9}{14}\overrightarrow{OP}}$

(四面体OLAB)

$= \frac{9}{14}$(四面体OPAB)

$= \frac{9}{14}\cdot V \times \frac{3}{5} \times \frac{5}{9}$

$= \frac{3}{14}V$

$= \frac{3}{14}\cdot 18$

$= \underline{\frac{27}{7}}$