2020 東邦大(医)

$\boxed{1}$

$$x^2+2x-2-y+k(-x^2+4x+10-y)=0$$

$\downarrow k=1$

$$6x+8-2y=0$$

$$\iff y=\underline{3x+4}$$

$x=y=0$

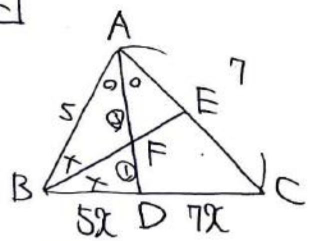
$$-2+10k=0 \quad \therefore k=\frac{1}{5}$$

$\downarrow$

$$\frac{4}{5}x^2+\frac{14}{5}x-\frac{6}{5}y=0$$

$$\therefore y=\underline{\frac{2}{3}x^2+\frac{7}{3}x}$$

$\boxed{2}$



メネラウス

$$\frac{12x}{5x}\cdot\frac{1}{3}\cdot\frac{AE}{EC}=1$$

$$\therefore \frac{AE}{EC}=\frac{5}{4}$$

$AE:EC$

$$=5:12x=5:4$$

$$\therefore x=\frac{1}{3} \quad BD=\underline{\frac{5}{3}}$$

メネラウス

$$\frac{9}{5}\cdot\frac{EF}{FB}\cdot\frac{5}{7}=1$$

$$\therefore \frac{EF}{FB}=\frac{7}{9} \quad \frac{BF}{EF}=\underline{\frac{9}{7}}$$

$\boxed{3}$

$$x^3+y^3+z^3=(x+y+z)(x^2+y^2+z^2)$$

$$-x(y^2+z^2)-y(x^2+z^2)-z(x^2+y^2)$$

$$=(x+y+z)(x^2+y^2+z^2)$$

$$-\{(x+y+z)(xy+yz+zx)-3xyz\}$$

$\downarrow$

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

$$\therefore 19=x^2+y^2+z^2-xy-yz-zx$$

$\downarrow$ 両辺 $+3(xy+yz+zx)$

$$19+3(xy+yz+zx)=(x+y+z)^2=1$$

$$\therefore xy+yz+zx=\underline{-6}$$

$$x^2+y^2+z^2=(x+y+z)^2-2(xy+yz+zx)$$

$$=13$$

$$x^4+y^4+z^4$$

$$=(x^2+y^2+z^2)^2-2(x^2y^2+y^2z^2+z^2x^2)$$

$$=169-2\{(xy+yz+zx)^2$$

$$-2(xy^2z+yz^2x+zx^2y)\}$$

$$=169-2\{36-2\underbrace{xyz}_{-2}\underbrace{(x+y+z)}_{1}\}$$

$$=\underline{89}$$

$\boxed{4}$

$$\begin{cases}\vec{a}\cdot\vec{b}=10 & \cdots ① \\ \vec{a}\cdot(\vec{b}-\vec{a})=-15 & \cdots ② \\ \vec{b}\cdot(\vec{b}-\vec{a})=-2 & \cdots ③\end{cases}$$

①−② $\quad$ ①+③

$$|\vec{a}|^2=25 \qquad |\vec{b}|^2=8$$

$$|\overrightarrow{AB}|^2=|\vec{b}|^2-2\vec{a}\cdot\vec{b}+|\vec{a}|^2$$

$$=13$$

$$\therefore |\overrightarrow{AB}|=\underline{\sqrt{13}}$$

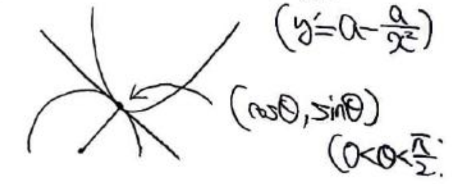
$$\underbrace{\vec{a}\cdot\vec{b}}_{10}=\underbrace{|\vec{a}|}_{5}\underbrace{|\vec{b}|}_{2\sqrt{2}}\cos\angle AOB$$

$$\therefore \cos\angle AOB=\frac{1}{\sqrt{2}}$$

正 $\quad 2R=\dfrac{|\overrightarrow{AB}|}{\sin\angle AOB}=\sqrt{26}$

$$\therefore R=\underline{\frac{\sqrt{26}}{2}}$$

$\boxed{5}$



解

$$\begin{cases}\sin\theta=a\left(\cos\theta+\dfrac{1}{\cos\theta}\right) \\ -\dfrac{1}{a-\frac{a}{\cos^2\theta}}=\tan\theta\end{cases}$$

$\downarrow$

$$\frac{-\cos^2\theta}{a\underbrace{(\cos^2\theta-1)}_{-\sin^2\theta}}=\frac{1}{a\tan^2\theta}=\tan\theta$$

$$\therefore a=\frac{1}{\tan^3\theta}$$

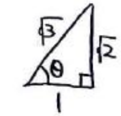
$\downarrow$

$$\sin\theta=\frac{1}{\tan^3\theta}\left(\cos\theta+\frac{1}{\cos\theta}\right)$$

$$\tan\theta=\frac{1}{\tan^3\theta}\left(1+\frac{1}{\cos^2\theta}\right)$$

$$=\frac{1}{\tan^3\theta}(2+\tan^2\theta)$$

$$\tan^4\theta-\tan^2\theta-2=0$$



$$\tan^2\theta=2$$

$$\therefore \tan\theta=\sqrt{2} \quad (\because 0<\theta<\tfrac{\pi}{2})$$

$a=\underline{\dfrac{\sqrt{2}}{4}}$ 接線 $\underline{\sqrt{2}x+2y=\sqrt{6}}$

$\boxed{6}$

$n+4=13k$
$n+13=4l$ （k, lは整数）

$\Downarrow$

$13k-4=4l-13$

$\Leftrightarrow 13k-4l=-9$

$\Downarrow$

$\begin{cases} k=4m-1 \\ l=13m-1 \end{cases}$ （mは整数）

$\therefore n=13(4m-1)-4$
$=52m-17$

(i) $m=2m'$のとき（m'は整数）

$n=104m'-17$

$n\equiv-17\equiv87 \pmod{104}$

(ii) $m=2m'+1$のとき（m'は整数）

$n=104m'+35$

$n\equiv35 \pmod{104}$

求める余りは小さい順に
$\underline{35, 87}$ //

$\boxed{7}$

$\overline{x_{30}}\geqq8$

$\Leftrightarrow 30\overline{x_{30}}\geqq240$

$\underbrace{\dfrac{27\overline{x_{27}}}{}}_{8}+x_{28}+x_{29}+x_{30}$

$\Leftrightarrow x_{28}+x_{29}+x_{30}\geqq24$

最小は$\underline{24}$ //

4	10	10		8	8	8
5	9	10			8	9
	10	10			8	10
6	8	10			9	9
	9	9			9	10
	9	10			10	10
	10	10		9	9	9
7	7	10			9	10
	8	9			10	10
	8	10		10	10	10
	9	9				
	9	10				
	10	10				

総数は

$3\times13+3!\times7+1\times3=\underline{84}$ //

$\boxed{8}$

$S_0=\sum_{k=0}^{32}{}_{32}C_k=(1+1)^{32}$

$\therefore \log_2 S_0=\underline{32}$ //

$S_1=\sum_{k=0}^{32}k\cdot{}_{32}C_k$

ここで

$(1+x)^n=\sum_{k=0}^{n}x^k\cdot{}_nC_k$

xで微分して

$n(1+x)^{n-1}=\sum_{k=1}^{n}kx^{k-1}{}_nC_k$

$\Downarrow x=1$

$n\cdot2^{n-1}=\sum_{k=1}^{n}k\cdot{}_nC_k \cdots ①$

これより

$\log_2 S_1$

$=\log_2\sum_{k=0}^{32}k\cdot{}_{32}C_k$

$=\log_2\sum_{k=1}^{32}k\cdot{}_{32}C_k$

$=\log_2 32\cdot2^{31}$ $\quad\therefore ①$

$=\log_2 2^{36}$

$=\underline{36}$ //

$\boxed{9}$

$\underset{1}{\tan\alpha}=\dfrac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$ $\quad$ $\underset{-1}{\tan\beta}=\dfrac{2\tan\frac{\beta}{2}}{1-\tan^2\frac{\beta}{2}}$

解くと $\tan\frac{\alpha}{2}=-1+\sqrt{2}$

$\tan\frac{\beta}{2}=1+\sqrt{2}$

$\therefore \tan\frac{\alpha}{2}+\tan\frac{\beta}{2}=\underline{2\sqrt{2}}$ //

$\int_0^1\dfrac{dx}{x^2-\sqrt{2}x+1}=\cdots=\underline{\dfrac{3\sqrt{2}}{8}\pi}$ //

$\int_0^1\dfrac{x^2+1}{x^4+1}dx$

$=\int_0^1\dfrac{x^2+1}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)}dx$

$=\int_0^1\frac{1}{2}\left(\dfrac{1}{x^2+\sqrt{2}x+1}+\dfrac{1}{x^2-\sqrt{2}x+1}\right)dx$

$=\cdots=\underline{\dfrac{\sqrt{2}}{4}\pi}$ //

$\boxed{10}$

$\dfrac{1}{4+x}\leqq a$ かつ $a\leqq\dfrac{1}{4+x}+bx$

これをみたすには $a=\frac{1}{4}$ であることが必要

$\dfrac{1}{4}-\dfrac{1}{4+x}\leqq bx$

$\Leftrightarrow 4+x-4\leqq bx\cdot4(4+x)$

$\Leftrightarrow x(4bx+16b-1)\geqq0$

これが常に成り立つには $b\geqq\underline{\frac{1}{16}}$ //

また $S_n=\sum_{k=1}^{n}\dfrac{\frac{k}{n^2}}{4+\frac{k}{n^2}}$

$\sum_{k=1}^{n}\left(\dfrac{k}{4n^2}-\dfrac{k^2}{16n^4}\right)\leqq\sum_{k=1}^{n}\dfrac{\frac{k}{n^2}}{4+\frac{k}{n^2}}\leqq\sum_{k=1}^{n}\dfrac{k}{4n^2}$

はさみうちで

$\lim_{n\to\infty}S_n=\underline{\dfrac{1}{8}}$ //