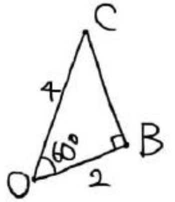
2020 昭和大(医) I期

1

求書けるべき

(1) $B(2\cos 15^\circ, 2\sin 15^\circ)$

$C(4\cos 75^\circ, 4\sin 75^\circ)$



$\overrightarrow{OB}$ と $\overrightarrow{OC}$ のなす角は 60° なので $\underline{\frac{\pi}{3}}$ //

$\triangle OBC = 2 \times 2\sqrt{3} \times \frac{1}{2} = \underline{2\sqrt{3}}$ //

(2)

$\beta' = 2\cos(-30^\circ) + i\,2\sin(-30^\circ)$

$= \underline{\sqrt{3} - i}$ //

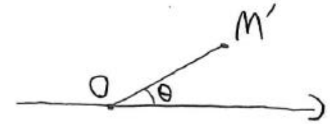
$\gamma' = 4\cos 30^\circ + i\,4\sin 30^\circ$

$= \underline{2\sqrt{3} + 2i}$ //

$\mu' = \frac{\beta' + \gamma'}{2}$

$= \underline{\frac{3\sqrt{3} + i}{2}}$ //

(3) $M'\left(\frac{3}{2}\sqrt{3}, \frac{1}{2}\right)$

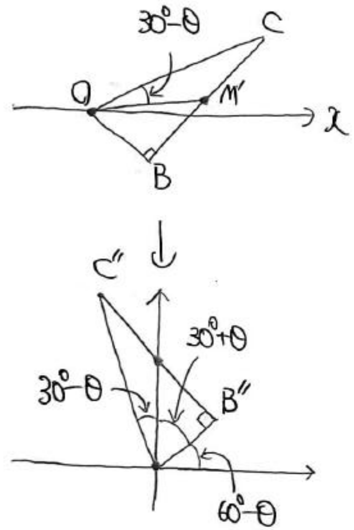


$OM' = \sqrt{\frac{27}{4} + \frac{1}{4}} = \sqrt{7}$

$\sin\theta = \frac{1}{2\sqrt{7}} = \underline{\frac{\sqrt{7}}{14}}$ //

$\cos\theta = \frac{3\sqrt{3}}{2\sqrt{7}} = \underline{\frac{3\sqrt{21}}{14}}$ //

(4) (3)の状況



$\cos(60^\circ - \theta)$

$= \cos 60^\circ \cos\theta + \sin 60^\circ \sin\theta$

$= \frac{3\sqrt{21}}{28} + \frac{\sqrt{21}}{28}$

$= \frac{\sqrt{21}}{7}$

$\sin(60^\circ - \theta)$

$= \sin 60^\circ \cos\theta - \cos 60^\circ \sin\theta$

$= \frac{9\sqrt{7}}{28} - \frac{\sqrt{7}}{28}$

$= \frac{2}{7}\sqrt{7}$

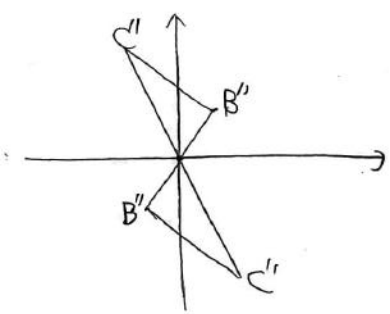
$\tan(60^\circ - \theta) = \frac{2\sqrt{7}}{\sqrt{21}} = \frac{2}{\sqrt{3}}$

$B''C''$ を通る直線は

$y = -\frac{\sqrt{3}}{2}\left(x - \frac{2\sqrt{21}}{7}\right) + \frac{4\sqrt{7}}{7}$

$= -\frac{\sqrt{3}}{2}x + \frac{3\sqrt{7}}{7} + \frac{4\sqrt{7}}{7}$

$= -\frac{\sqrt{3}}{2}x + \sqrt{7}$



2つあるので

$\underline{y = -\frac{\sqrt{3}}{2}x \pm \sqrt{7}}$ //

2

(1) a_2 がないので解けません.

全員正解扱いだそうです.

(2) 両辺 $2n-1$ をかけると

$\underbrace{(2n+1)(2n-1)a_n}_{b_n} = \underbrace{(2n-1)(2n-3)a_{n-1}}_{b_{n-1}}$

$\therefore b_n = 3 \cdot 1 \cdot a_1 = 1$

$\therefore \underline{a_n = \frac{1}{(2n-1)(2n+1)}}$ //

$\underline{\lim_{n\to\infty} a_n = 0}$ //

(3)

解

$8\alpha^2 - 4\alpha - 1 = 0$

$\Leftrightarrow \alpha = \frac{2 \pm \sqrt{12}}{8} = \frac{1 \pm \sqrt{3}}{4}$

$\alpha_1 = \frac{1+\sqrt{3}}{4}$ $\alpha_2 = \frac{1-\sqrt{3}}{4}$ とする.

$a_n = p\alpha_1^{n-1} + q\alpha_2^{n-1}$ とおける.

$a_1 = p + q = 1$

$a_2 = \frac{1+\sqrt{3}}{4}p + \frac{1-\sqrt{3}}{4}q = \frac{1}{2}$

↓

$\left(\frac{1+\sqrt{3}}{4} - \frac{1-\sqrt{3}}{4}\right)q = \frac{1+\sqrt{3}-2}{4}$

$\Leftrightarrow \frac{\sqrt{3}}{2}q = \frac{\sqrt{3}-1}{4}$

$\Leftrightarrow q = \frac{\sqrt{3}-1}{2\sqrt{3}} = \frac{3-\sqrt{3}}{6}$

$p = \frac{3+\sqrt{3}}{6}$

$$a_n=\frac{2\sqrt{3}}{3}\left\{\left(\frac{1+\sqrt{3}}{4}\right)^n-\left(\frac{1-\sqrt{3}}{4}\right)^n\right\}$$

$\therefore \lim a_n = \underline{0}$

$\boxed{3}$

(1)

(1-1) 焦点 $(0, \pm 5)$

(1-2)

漸近線 $y=\pm\frac{4}{3}x$

(2)

(2-1)

$P(A で赤, B で赤)+P(A で白, B で赤)$

$=\frac{4}{7}\cdot\frac{4}{8}+\frac{3}{7}\cdot\frac{3}{8}$

$=\underline{\frac{25}{56}}$

(2-2)

$P(A で赤, B で赤, A で赤, B で赤)$

$+P(\quad〃\quad A で白, 〃)$

$+P(A で白, B で赤, A で赤, B で赤)$

$+P(\quad〃\quad A で白, B で赤)$

$=\frac{4}{7}\cdot\frac{4}{8}\cdot\frac{3}{6}\cdot\frac{4}{8}$
$+\frac{4}{7}\cdot\frac{4}{8}\cdot\frac{3}{6}\cdot\frac{3}{8}$
$+\frac{3}{7}\cdot\frac{3}{8}\cdot\frac{4}{6}\cdot\frac{3}{8}$
$+\frac{3}{7}\cdot\frac{3}{8}\cdot\frac{2}{6}\cdot\frac{2}{8}$

$=\frac{\cancel{4}\cdot\cancel{4}\cdot\cancel{3}\cdot7+3\cdot\cancel{3}\cdot\cancel{4}\cdot\cancel{6}}{7\cdot\cancel{8}\cdot\cancel{6}\cdot\cancel{8}}$

$=\frac{10}{56}=\underline{\frac{5}{28}}$

(2-3)

$P(A で赤赤, B で赤赤)$

$+P(A で赤白, \quad 〃 \quad)$

$+P(A で白白, \quad 〃 \quad)$

$=\frac{{}_4C_2}{{}_7C_2}\cdot\frac{{}_5C_2}{{}_9C_2}+\frac{4\times3}{{}_7C_2}\cdot\frac{{}_4C_2}{{}_9C_2}+\frac{{}_3C_2}{{}_7C_2}\cdot\frac{{}_3C_2}{{}_9C_2}$

$=\frac{\overset{2}{\cancel{6}}\cdot10+12\cdot\overset{2}{\cancel{6}}+\cancel{3}\cdot3}{\underset{7}{\cancel{21}}\cdot36}$

$=\frac{20+24+3}{252}$

$=\underline{\frac{47}{252}}$

$\boxed{4}$

(1) $y=\log_3(5x-7)$

$=\frac{\log_e(5x-7)}{\log_e 3}$

$y'=\underline{\frac{5}{(5x-7)\log_e 3}}$

(2)

$\int_{1928}^{2020} f(x)\,dx$

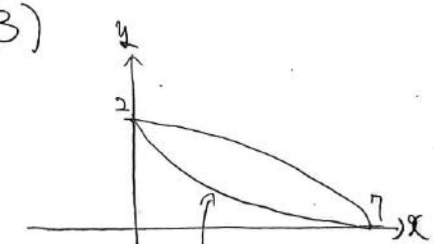
$=\int_{1928}^{1929} f(x)\,dx+\int_{1929}^{1930} f(x)\,dx+\cdots+\int_{1999}^{2020} f(x)\,dx$

$=1929+1930+\cdots+2020$

$=(1929+2020)\times72\times\frac{1}{2}$

$=\underline{181654}$

(3)



$\sqrt[3]{\frac{y}{2}}=1-\sqrt{\frac{x}{7}}$

$y=2\left(1-\sqrt{\frac{x}{7}}\right)^3$

求める体積を V とすると

V

$=\int_0^7\left\{\left(2\sqrt{1-\frac{x}{7}}\right)^2\pi-4\left(1-\sqrt{\frac{x}{7}}\right)^6\pi\right\}dx$

$=\pi\int_0^7\left\{4\left(1-\frac{x}{7}\right)-4\left(1-\sqrt{\frac{x}{7}}\right)^6\right\}dx$

$=4\pi\int_0^1\left\{1-t-(1-\sqrt{t})^6\right\}7\,dt$

$=28\pi\left[t-\frac{1}{2}t^2\right]_0^1$

$-28\pi\int_0^1(1-\sqrt{t})^6\,dt$

$\downarrow$ $1-\sqrt{t}=u$
$(1-u)^2=t$
$2(1-u)(-1)\,du=dt$

$=14\pi-28\pi\int_1^0 u^6(-2)(1-u)\,du$

$=14\pi-56\pi\int_0^1 u^6(1-u)\,du$

$=14\pi-56\pi\left\{\left[\frac{u^7}{7}(1-u)\right]_0^1-\int_0^1\frac{u^7}{7}(-1)\,du\right\}$

$=14\pi-56\pi\int_0^1\frac{u^7}{7}\,du$

$=14\pi-56\pi\cdot\frac{1}{56}$

$=\underline{13\pi}$

一応ベータ関数使えますが部分積分が速いです.