2020 埼玉医科大（後期）

$\boxed{1}$

問1.

$200!! \leftarrow$ 2重階乗

$= 2 \cdot 4 \cdot 6 \cdots 198 \cdot 200$

$= 2^{100} \cdot 100!$

$= 2^{a} \cdot 3^{b} \cdot 5^{c} \cdot 7^{d} \cdots 97^{1}$

cを求める. 1〜100までで

5の倍数 … 20コ

25 〃 … 4コ

より $c = 20 + 4 = 24$.

200!!は0が $\underline{24}$ コ並ぶ

$199!!$

$= 1 \cdot 3 \cdot 5 \cdots 199$

偶数がないので末尾の数字は $\underline{5}$

問2.

$P(\text{陽性の病気}) = \frac{1}{5} \cdot \frac{9}{10} = \frac{18}{100}$

$P(\text{陽性の平気}) = \frac{4}{5} \cdot \frac{1}{20} = \frac{4}{100}$

$P_{\text{陽性}}(\text{平気})$

$= \dfrac{P(\text{陽性の平気})}{P(\text{陽性})}$

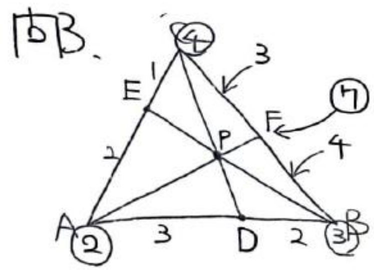
$= \dfrac{\frac{4}{100}}{\frac{18}{100} + \frac{4}{100}} = \frac{2}{11}$

$\fallingdotseq 0.18$ $\quad \underline{18\%}$

$11 \overline{)20}$ → 0.181

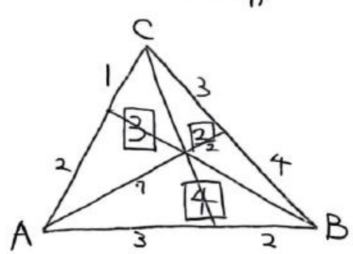
11, 90, 88, 2

問3.



$AP : PF = 7 : 2$

$\therefore \dfrac{AP}{AF} = \underline{\dfrac{7}{9}}$



$I : J : K = 4 : 2 : 3$

$= \underline{\frac{4}{3}} : \underline{\frac{2}{3}} : 1$

$\boxed{2}$

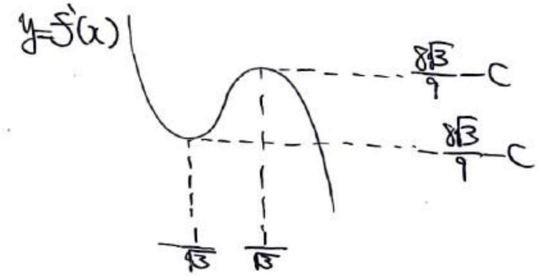
問1

$f(x) = -x^4 + 2x^2 - cx$

$f'(x) = -4x^3 + 4x - c$

$f''(x) = -12x^2 + 4$

$= -4(3x^2 - 1)$



$f'(x)$の符号が負から正に変化するには

$\frac{8\sqrt{3}}{9} - c > 0$ かつ $-\frac{8\sqrt{3}}{9} - c < 0$

$\therefore \underline{\frac{-8\sqrt{3}}{9}} < c < \underline{\frac{8\sqrt{3}}{9}}$

このとき極小値をとるxの範囲は

$\underline{\frac{-\sqrt{3}}{3}} < x < \underline{\frac{\sqrt{3}}{3}}$

問2

$x = \frac{1}{2}$で極小値をとるならば

$f'\left(\frac{1}{2}\right) = -\frac{1}{2} + 2 - c = 0$

$\therefore c = \frac{3}{2}$

$f'(x) = -4x^3 + 4x - \frac{3}{2}$

$= -\left(4x^3 - 4x + \frac{3}{2}\right)$

$= -\left(x - \frac{1}{2}\right)(4x^2 + 2x - 3)$

$4x^2 + 2x - 3 = 0$ の解を α, β

$(\alpha < \beta)$ とする.

$$
\begin{array}{r}
-\frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{4} \\
4\ 2\ -3\ \overline{)\,-1 \quad 0 \quad 2 \quad -\frac{3}{2} \quad 0} \\
-1 \quad -\frac{1}{2} \quad \frac{3}{4} \\
\hline
\frac{1}{2} \quad \frac{5}{4} \quad -\frac{3}{2} \\
\frac{1}{2} \quad \frac{1}{4} \quad -\frac{3}{8} \\
\hline
1 \quad -\frac{9}{8} \quad 0 \\
1 \quad \frac{1}{2} \quad -\frac{3}{4} \\
\hline
-\frac{13}{8} \quad \frac{3}{4}
\end{array}
$$

$f(x)$

$= (4x^2 + 2x - 3)\left(-\frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{4}\right)$

$\quad - \frac{13}{8}x + \frac{3}{4}$

$\max f(x)$

$= f(\alpha)$

$= -\frac{13}{8} \cdot \frac{-1-\sqrt{13}}{4} + \frac{3}{4}$

$= \underline{\frac{1}{32}(37 + 13\sqrt{13})}$

$\alpha = -\frac{1}{4}(1 + \sqrt{13})$

$\boxed{3}$

$$T_k = T_{k,3}$$
$$= \left[\frac{1}{k}t^k(1-t)^3\right]_0^1 + \frac{3}{k}\int_0^1 t^k(1-t)^2 dt$$
$$= \frac{3}{k}T_{k+1,2}$$
$$= \frac{3}{k}\cdot\frac{2}{k+1}\cdot T_{k+2,1}$$
$$= \frac{3}{k}\cdot\frac{2}{k+1}\cdot\frac{1}{k+2}\cdot T_{k+3,0}$$
$$= \frac{6}{k(k+1)(k+2)}\int_0^1 t^{k+2}dt$$
$$= \frac{6}{k(k+1)(k+2)(k+3)}$$

問1

$$T_3 = \frac{6}{3\cdot4\cdot5\cdot6} = \underline{\frac{1}{60}}$$

問2

$$\sum_{k=1}^{n} T_k$$
$$= \sum_{k=1}^{n} 2\left\{\frac{1}{k(k+1)(k+2)} - \frac{1}{(k+1)(k+2)(k+3)}\right\}$$
$$= 2\left\{\frac{1}{1\cdot2\cdot3} - \frac{1}{(n+1)(n+2)(n+3)}\right\}$$
$$= \underline{\frac{1}{3} - \frac{2}{(n+1)(n+2)(n+3)}}$$

問3

$$\sum_{k=1}^{\infty} T_k$$
$$= \underline{\frac{1}{3}}$$

※ベータ関数

$$B(m,n) = \int_0^1 t^m(1-t)^n dt$$
$$= \cdots$$
$$= \frac{m!\,n!}{(m+n+1)!}$$

を導いたり暗記していたりすると強いです.

$\boxed{4}$

問1.

$$f(x) = a^x$$
$$f(10) = a^{10} = \frac{1}{10} = 10^{-1}$$
$$\therefore a = 10^{-0.1}$$
$$\therefore f(x) = 10^{-0.1x}$$

$$f(x) = \frac{1}{2}$$
$$\Leftrightarrow -0.1x = \log_{10}\frac{1}{2}$$
$$\Leftrightarrow 0.1x = \log_{10}2$$
$$\Leftrightarrow x = 10\underbrace{\log_{10}2}_{0.30} = \underline{3.0}$$

問2.

$$f(x) = \frac{1}{e}$$
$$\Leftrightarrow -0.1x = \log_{10}\frac{1}{e}$$
$$\Leftrightarrow 0.1x = \log_{10}e$$
$$\Leftrightarrow x = \frac{10}{\log_e 10} \fallingdotseq \underline{4.3}$$

$$\begin{array}{r} 4.34 \\ 23\overline{)100} \\ 92 \\ \hline 80 \\ 69 \\ \hline 110 \end{array}$$

問3.

$$f(x) = 10^{-0.1x} = e^{?}$$
$$\Leftrightarrow -0.1x \times \log_e 10 = ?$$
$$\downarrow$$
$$f(x)$$
$$= e^{-0.1x \times \underbrace{\log_e 10}_{2.3}}$$
$$= e^{-0.23x}$$

$$f(1)$$
$$= e^{-0.23}$$
$$\fallingdotseq 1 - 0.23$$
$$= \underline{0.77}$$