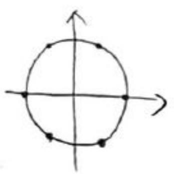
2020 金沢医科大(前期)

$\boxed{1}$

(1) $(a,b,c)=(3,6,6)$ のとき $T=7$ で最大

$(a,b,c)=(6,6,3)$ のとき $T=-6$ で最小

(2)



$P(T=0)$

$=P(\sin\frac{\pi a}{6}=1, b=1, c=3)$

$+P(\sin\frac{\pi a}{6}=1, b=2, c=2 \text{ or } 4)$

$+P(\sin\frac{\pi a}{6}=\frac{1}{2}, b=1, c=2 \text{ or } 4)$

$=\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{2}{6}+\frac{2}{6}\cdot\frac{1}{6}\cdot\frac{2}{6}$

$=\frac{7}{216}$

(3) $P(T$が正の偶数$)$

$=P(T=2)+P(T=4)+P(T=6)$

$=P(\sin\frac{\pi}{6}a=1, b=1, c=6)$

$+P(\sin\frac{\pi a}{6}=1, b=2, c=1 \text{ or } 5)$

$+P(\sin\frac{\pi a}{6}=\frac{1}{2}, b=3, c=1 \text{ or } 5)$

$+P(\sin\frac{\pi a}{6}=0, b=2, c=6)$

$+P(\sin\frac{\pi a}{6}=0, b=4, c=1 \text{ or } 5)$

$+P(\sin\frac{\pi a}{6}=1, b=3, c=6)$

$+P(\sin\frac{\pi a}{6}=1, b=6, c=1 \text{ or } 5)$

$+P(\sin\frac{\pi a}{6}=0, b=4, c=6)$

$+P(\sin\frac{\pi a}{6}=1, b=5, c=6)$

$+P(\sin\frac{\pi a}{6}=0, b=6, c=6)$

$=\frac{1}{216}+\frac{2}{216}+\frac{4}{216}+\frac{1}{216}$

$+\frac{2}{216}+\frac{1}{216}+\frac{2}{216}+\frac{1}{216}$

$+\frac{1}{216}+\frac{1}{216}$

$=\frac{16}{216}=\frac{2}{27}$

(4)

$P(T<3+\frac{\sqrt{3}}{2})$

$=1-P(T\geqq 3+\frac{\sqrt{3}}{2})$

$=1-\{P(T=3+\frac{\sqrt{3}}{2})+P(T\geqq 4)\}$

$=1-\{\frac{2}{6}\cdot\frac{3}{36}+\frac{4}{216}$

$+P(\sin\frac{\pi a}{6}\geqq\frac{1}{2}, b=4, c=6)$

$+P(b=5, c=6)+P(b=6, c=6)\}$

$=1-\frac{10}{216}-\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}-\frac{6}{216}-\frac{6}{216}$

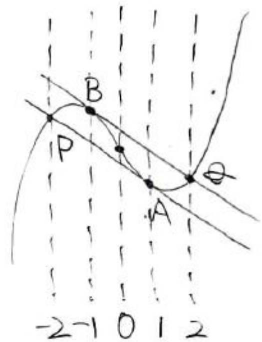
$=1-\frac{27}{216}$

$=\frac{7}{8}$

$\boxed{2}$

$y=x^3-9x$

$y'=3x^2-9 \quad y''=6x$



(1)

$l_1: y=-6(x-1)-8$

$=-6x-2$

$P(-2, 10)$

(2)

$l_2: y=-6(x+1)+8$

$=-6x+2$

$B(-1, 8)$

$Q(2, -10)$

(3) $\overrightarrow{AP}=\begin{pmatrix}-3\\18\end{pmatrix}$ $\overrightarrow{AQ}=\begin{pmatrix}1\\-2\end{pmatrix}$

$S_1=\sqrt{|\overrightarrow{AP}|^2|\overrightarrow{AQ}|^2-(\overrightarrow{AP}\cdot\overrightarrow{AQ})^2}$

$=\sqrt{333\cdot 5-(-39)^2}$

$=12$

(4) $S_2=S_3=\frac{1}{12}3^4$ $\boxed{\frac{1}{12}\text{公式}}$

$=\frac{27}{4}$

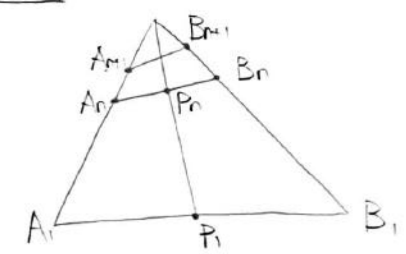
(5)

$\frac{S_1}{S_2+S_3}=\frac{12}{\frac{27}{2}}$

$=\frac{12}{27}\cdot 2$

$=\frac{8}{9}$

$\boxed{3}$



$\overrightarrow{OA_n} = \left(\frac{2}{3}\right)^{n-1}\overrightarrow{OA_1}$

$\overrightarrow{OB_n} = \left(\frac{1}{2}\right)^{n-1}\overrightarrow{OB_1}$

$\overrightarrow{OP_n} = (1-S_n)\overrightarrow{OA_n} + S_n\overrightarrow{OB_n}$

$= (1-S_n)\left(\frac{2}{3}\right)^{n-1}\overrightarrow{OA_1} + S_n\left(\frac{1}{2}\right)^{n-1}\overrightarrow{OB_1}$

$\overrightarrow{OP_n} = k\overrightarrow{OP_1}$

$= \frac{k}{2}\overrightarrow{OA_1} + \frac{k}{2}\overrightarrow{OB_1}$ より

$(1-S_n)\left(\frac{2}{3}\right)^{n-1} = S_n\left(\frac{1}{2}\right)^{n-1}$

$\Longleftrightarrow (1-S_n)2^{2n-2} = S_n \cdot 3^{n-1}$

$\Longleftrightarrow 2^{2n-2} = (3^{n-1} + 2^{2n-2})S_n$

$\therefore S_n = \underline{\dfrac{4^{n-1}}{3^{n-1}+4^{n-1}}}$

$\Downarrow n=2,3$

$S_2 = \underline{\dfrac{4}{7}}$ $S_3 = \underline{\dfrac{16}{25}}$

$\boxed{4}$

$C_1 : y = \frac{1}{2}x^2 + \frac{a}{2}$

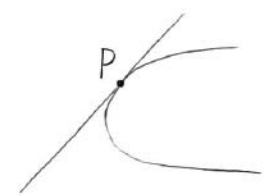
$l : y = x+3$

$\Downarrow$ 連立

$x^2 + a = 2x + 6$

$\Longleftrightarrow x^2 - 2x + a - 6 = 0$

接するので $a = \underline{7}$ $x=1$, $P\underline{(1,4)}$



C_2 が P を通るので

$b - 16 + 16 - c = 0$ $\therefore b = c$

C_2 の式を両辺 x で微分

$b - 2y\frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$y=4$ のとき

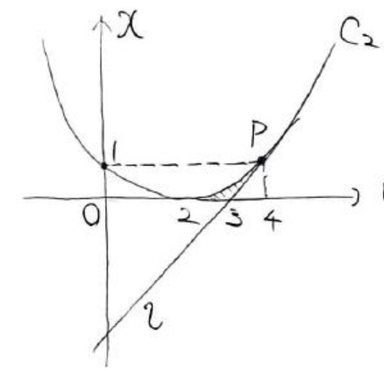
$b - 8\underbrace{\left.\frac{dy}{dx}\right|_{y=4}}_{1} + 4\underbrace{\left.\frac{dy}{dx}\right|_{y=4}}_{1} = 0$

$\therefore b = c = \underline{4}$

$C_2 : 4x - y^2 + 4y - 4 = 0$

$\Longleftrightarrow x = \frac{1}{4}y^2 - y + 1 = \frac{1}{4}(y-2)^2$

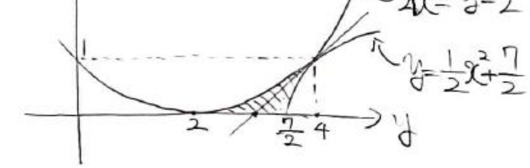
$l : x = y - 3$



$\frac{う}{え} = \int_2^4 \frac{1}{4}(y-2)^2 dy - \frac{1}{2}$

$= \left[\frac{1}{12}(y-2)^3\right]_2^4 - \frac{1}{2}$

$= \frac{8}{12} - \frac{6}{12} = \underline{\frac{1}{6}}$



$\frac{おか}{きく}\pi = \int_0^1 \left\{\left(\frac{1}{2}x^2 + \frac{7}{2}\right)^2 - (2\sqrt{x}+2)^2\right\}\pi dx$

$= \pi\int_0^1 \left(\frac{1}{4}x^4 + \frac{7}{2}x^2 - 4x - 8\sqrt{x} + \frac{33}{4}\right)dx$

$= \pi\left[\frac{1}{20}x^5 + \frac{7}{6}x^3 - 2x^2 - \frac{16}{3}x^{\frac{3}{2}} + \frac{33}{4}x\right]_0^1$

$= \pi\left(\frac{1}{20} + \frac{7}{6} - 2 - \frac{16}{3} + \frac{33}{4}\right)$

$= \underline{\frac{32}{15}\pi}$

$\frac{け}{こさ}\pi$

$= \int_2^4 \frac{1}{16}(y-2)^4 \pi dy$

$- \int_{\frac{7}{2}}^4 (2y-7)\pi dy$

$= \pi\left[\frac{1}{80}(y-2)^5\right]_2^4$

$- \pi\left[y^2 - 7y\right]_{\frac{7}{2}}^4$

$= \frac{2^5}{16 \cdot 5}\pi - \pi\left\{-12 - \left(\frac{49}{4} - \frac{49}{2}\right)\right\}$

$= \frac{2}{5}\pi + \pi\left(12 + \frac{49}{4} - \frac{98}{4}\right)$

$= \frac{8}{20}\pi + \pi\left(\frac{48+49-98}{4}\right)$

$= \frac{8}{20}\pi + \pi\left(-\frac{1}{4}\right)$

$= \underline{\frac{3}{20}\pi}$