2020 センター数学IIB

第1問

[1]

(1)

$\sqrt{3}\cos\left(\theta-\frac{\pi}{3}\right)$

$=\underline{\frac{\sqrt{3}}{2}}\cos\theta+\underline{\frac{3}{2}}\sin\theta$

①

$\Leftrightarrow -\frac{1}{2}\sin\theta-\frac{\sqrt{3}}{2}\cos\theta>0$

$\Leftrightarrow \sin\theta\cdot\frac{1}{2}+\cos\theta\cdot\frac{\sqrt{3}}{2}<0$

$\Leftrightarrow \sin\left(\theta+\underline{\frac{\pi}{3}}\right)<0$

$\pi<\theta+\frac{\pi}{3}<2\pi$

$\therefore \underline{\frac{2}{3}}\pi<\theta<\underline{\frac{5}{3}}\pi$

(2)

$\sin\theta+\cos\theta=\frac{7}{5}$

$\sin\theta\cos\theta=\frac{k}{25}$

$\Downarrow$

$\frac{49}{25}=1+\frac{2k}{25}\quad \therefore k=\underline{12}$

$(\sin\theta-\cos\theta)^2=1-2\sin\theta\cos\theta$

$=\frac{1}{25}$

$\therefore \sin\theta-\cos\theta=\frac{1}{5}$

$\therefore \sin\theta=\underline{\frac{4}{5}},\ \cos\theta=\underline{\frac{3}{5}}$

ちなみに

$\sin\frac{5}{12}\pi=\frac{\sqrt{6}+\sqrt{2}}{4}\fallingdotseq\frac{3.8}{4}$

$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\fallingdotseq 0.86$

$\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}\fallingdotseq 0.7$

$\therefore \frac{\pi}{4}\leqq\theta<\frac{\pi}{3}\ \cdots\ \underline{③}$

[2]

$t^{\frac{2}{3}}+t^{-\frac{2}{3}}=(t^{\frac{1}{3}}-t^{-\frac{1}{3}})^2+2$

$=\underline{11}$

$(t^{\frac{1}{3}}+t^{-\frac{1}{3}})^2=11+2$

$\therefore t^{\frac{1}{3}}+t^{-\frac{1}{3}}=\underline{\sqrt{13}}$

$t-t^{-1}$

$=(t^{\frac{1}{3}}-t^{-\frac{1}{3}})(t^{\frac{2}{3}}+1+t^{-\frac{2}{3}})$

$=-3\cdot 12$

$=\underline{-36}$

(2)

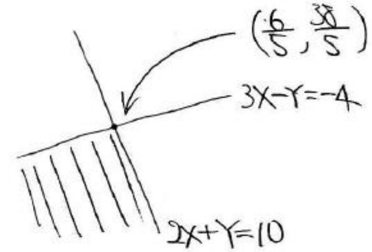
$X+\frac{1}{2}Y\leqq 5$

$\Leftrightarrow \underline{2}X+Y\leqq\underline{10}$

また

$\frac{\log_3\frac{y}{x^3}}{\log_3 81}=\frac{Y-3X}{4}\leqq 1$

$\Leftrightarrow \underline{3}X-Y\geqq\underline{-4}$



max $Y=\frac{38}{5}$

最大の整数 $Y=\underline{7}$

このとき

$X\leqq\frac{3}{2},\ X\geqq 1$

$\Leftrightarrow 1\leqq\log_3 x\leqq\frac{3}{2}$

$\Leftrightarrow 3\leqq x\leqq 3\sqrt{3}$

最大の整数 $x=\underline{5}$

第2問

(1)

$\ell: y=(2t+2)(x-t)+t^2+2t+1$

$=\underline{(2t+2)x-t^2+1}\ \cdots\ ①$

$\ell: y=\{2s-(4a-2)\}(x-s)$

$+s^2-(4a-2)s+4a^2+1$

$=(2s-4a+2)x$

$\underline{-s^2+4a^2+1}\ \cdots\ ②$

①=②

$t+1=s-2a+1$

$\Leftrightarrow t=s-2a$

また

$-t^2+1=-s^2+4a^2+1$

$\Leftrightarrow -(s-2a)^2=-s^2+4a^2$

$\Leftrightarrow 4as-4a^2=4a^2$

$\Leftrightarrow s=\underline{2a}\quad \therefore t=\underline{0}$

$\therefore \ell: \underline{y=2x+1}$

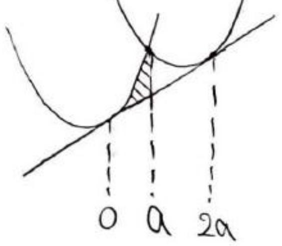
(2)

CとD連立

$\Downarrow$

$0=-4ax+4a^2$

$\therefore x=\underline{a}$



$S=\int_0^a x^2dx$

$=\left[\frac{1}{3}x^3\right]_0^a$

$=\frac{1}{3}a^3$

(3) $a>1$ のとき

$T=\int_0^1 x^2dx=\frac{1}{3}$

$\frac{1}{2}\leqq a<1$ のとき

$T=\int_0^a x^2dx+\int_a^1 (x-2a)^2dx$

$=\frac{a^3}{3}+\frac{1}{3}\{(1-2a)^3+a^3\}$

$=-2a^3+4a^2-2a+\frac{1}{3}$

(4)
U

$=2T-3S\,a^3$

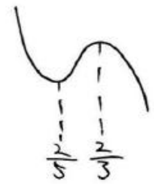
$=-5a^3+8a^2-4a+\frac{2}{3}$

U'

$=-15a^2+16a-4=0$

$\Leftrightarrow 15a^2-16a+4=0$

$\Leftrightarrow (3a-2)(5a-2)=0$



$a=\frac{2}{3}$ で最大値 $\frac{2}{27}$

第3問

(1)

$a_2=2(3^2-2\cdot3)=6$

(2)

$b_{n+1}=b_n+\frac{1}{(n+1)(n+2)}-\left(\frac{1}{3}\right)^{n+1}$

$\Leftrightarrow b_{n+1}-b_n=\left(\frac{1}{n+1}-\frac{1}{n+2}\right)-\left(\frac{1}{3}\right)^{n+1}$

$\sum_{k=1}^{n-1}\left(\frac{1}{k+1}-\frac{1}{k+2}\right)=\frac{1}{2}-\frac{1}{n+1}$

$=\frac{1}{2}\left(\frac{n-1}{n+1}\right)$

$\sum_{k=1}^{n-1}\left(\frac{1}{3}\right)^{k+1}=\frac{\frac{1}{9}-\left(\frac{1}{3}\right)^{n+1}}{1-\frac{1}{3}}=\frac{1-3\left(\frac{1}{3}\right)^n}{6}$

$=\frac{1}{6}-\frac{1}{2}\left(\frac{1}{3}\right)^n$

これを使うと

$b_n=b_1+\frac{1}{2}\cdot\frac{n-1}{n+1}-\frac{1}{6}+\frac{1}{2}\left(\frac{1}{3}\right)^n$

$=\frac{3n-3-n-1}{6(n+1)}$ 〃

$=\frac{n-2}{3(n+1)}+\frac{1}{2}\left(\frac{1}{3}\right)^n$

(3)
$c_n=3^{n-1}(n^2-4)+\frac{(n+1)(n+2)}{2}$

(4)

$a_{3k}\equiv 1$ (mod3)

$a_{3k+1}\equiv 0$ 〃

$a_{3k+2}\equiv 0$ 〃

$3\overline{)2020}$ 673
18
22
21
10
9
1

$\rightarrow 2020=673\cdot3+1$

$\sum_{k=1}^{2020}a_k$

$\equiv 674$

$\equiv 1$

第4問

(1)

$|\overrightarrow{OA}|=\sqrt{9+9+36}=3\sqrt{6}$

$|\overrightarrow{OB}|=\sqrt{32+16}=4\sqrt{3}$

$\overrightarrow{OA}\cdot\overrightarrow{OB}=36$

(2)

$\overrightarrow{OC}=s\vec{a}+t\vec{b}$

$\vec{a}\cdot\vec{c}=s|\vec{a}|^2+t\vec{a}\cdot\vec{b}$

$=54s+36t=0$

$\vec{b}\cdot\vec{c}=36s+48t=24$

$3s+2t=0$

$\therefore s=\frac{-2}{3}\quad t=1$

$|\vec{c}|^2=\frac{4}{9}54-\frac{4}{3}\vec{a}\cdot\vec{b}+48$

$=24-48+48=24$

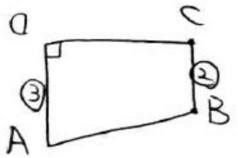
$\therefore |\overrightarrow{OC}|=2\sqrt{6}$

(3)

$\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}$

$=-5\vec{a}$

$=\frac{2}{3}\begin{pmatrix}3\\3\\-6\end{pmatrix}=\underline{\begin{pmatrix}2\\2\\-4\end{pmatrix}}$ //



台形…③ //

$(\text{面積})=(3\sqrt{6}+2\sqrt{6})2\sqrt{6}\cdot\frac{1}{2}$

$=\underline{30}$ //

(4)

$\overrightarrow{OC}=\overrightarrow{OB}-\overrightarrow{CB}=\begin{pmatrix}2\sqrt{3}\\-2\sqrt{3}\\0\end{pmatrix}$

$P(x,y,1)$とおくと

$\begin{cases}3x+3y-6=0\\2\sqrt{3}x-2\sqrt{3}y=2\sqrt{6}\end{cases}$

$\Leftrightarrow\begin{cases}x+y=2\\x-y=\sqrt{2}\end{cases}\quad\therefore y=\frac{2-\sqrt{2}}{2}$

$\therefore\underline{D\left(1+\frac{\sqrt{2}}{2},1-\frac{\sqrt{2}}{2},1\right)}$ //

$|\overrightarrow{OD}|=\sqrt{1+\frac{1}{2}+1+\frac{1}{2}+1}=2$

$\cos\angle COD=\frac{1}{2}\quad\therefore\angle COD=\underline{60^\circ}$ //

$(\text{高さ})=\underline{\sqrt{3}}$ //

$(PABC)=2\sqrt{6}\times2\sqrt{6}\times\frac{1}{2}\times\sqrt{3}\times\frac{1}{3}$

$=\underline{4\sqrt{3}}$ //

第5問

(1)

$E(X)=1\cdot\frac{54}{720}+2\cdot\frac{36}{720}+3\cdot\frac{18}{720}$

$=\frac{180}{720}=\underline{\frac{1}{4}}$ //

$E(X^2)=\frac{54+4\cdot36+9\cdot18}{720}$

$=\frac{360}{720}=\underline{\frac{1}{2}}$ //

$\sigma(X)=\sqrt{E(X^2)-(E(X))^2}$

$=\sqrt{\frac{8}{16}-\frac{1}{16}}$

$=\underline{\frac{\sqrt{7}}{4}}$ //

(2)

$E(Y)=np=600=\underline{240}$ //

$\sigma(Y)=\sqrt{np(1-p)}$

$=\sqrt{144}=\underline{12}$ //

$Z=\frac{Y-240}{12}$

$P(Y\leqq215)$

$=P\left(Z\leqq-\frac{25}{12}\right)$

$\fallingdotseq2.08$

$12\overline{)25}$ = 2.08, 24, 100, 96, 4

$=0.5-0.4812\fallingdotseq\underline{0.02}$ //

$p=0.2$のとき

$E(Y)=0.2\times600=120$

キタイの$\underline{\frac{1}{2}}$ // 倍

$\sigma(Y)=\sqrt{120\times0.8}$

$=\sqrt{96}=4\sqrt{6}$

ひょうの $\frac{4\sqrt{6}}{12}=\underline{\frac{\sqrt{6}}{3}}$ // 倍

(3) $E(U_1)=E(W)-60$

$=\underline{m-60}$ //

$\sigma(U_1)=\sigma(W_1-60)$

$=\underline{30}$ //

$\left|\frac{U-\bar{U}}{\frac{\sigma}{\sqrt{n}}}\right|\leqq1.96$

$\Leftrightarrow-1.96\leqq\frac{t-50}{3}\leqq1.96$

$\Leftrightarrow50-1.96\cdot3\leqq t\leqq50+1.96\cdot3$

$\Leftrightarrow44.12\leqq t\leqq55.88$

$\therefore\underline{44.1}\leqq t\leqq\underline{55.9}$ //