2020 センター数学I・A

第1問

[1]

(1) $a^2-2a-8<0$

$\iff -2<a<4$

(2)

lが$(b,0)$を通る.

$0=(a^2-2a-8)b+a$

$\iff (a^2-2a-8)b=-a$

$a>0,\ b>0$のとき

$a^2-2a-8<0$

$\therefore 0<a<4$

$a\leqq 0,\ b>0$のとき

$a^2-2a-8>0$

$\therefore a<-2$

$a=\sqrt{3}$のとき

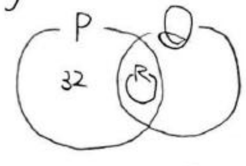
$(-5-2\sqrt{3})b=-\sqrt{3}$

$\iff b=\dfrac{\sqrt{3}}{5+2\sqrt{3}}\times\dfrac{5-2\sqrt{3}}{5-2\sqrt{3}}$

$\therefore b=\dfrac{5\sqrt{3}-6}{12}$

[2]

(1)



$32\in P\cap\overline{Q}$ …②

(2) $P\cap Q$に属する最小の自然数は

12

また $12\notin R$ …④

(3)

12は

「(pかつq)$\Rightarrow r$」…③

の反例である.

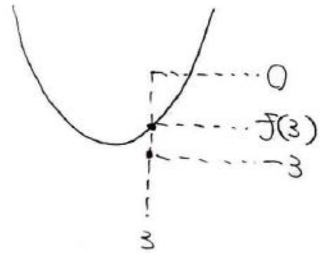
[3]

(1) $(c,0),\ (c+4,0)$を通る.

$y=(x-c)\{x-(c+4)\}$

$=x^2-2(c+2)x+c(c+4)$

$=f(x)$



求める条件は

$-3\leqq f(3)=c^2-2c-3\leqq 0$

$\Downarrow$

$\begin{cases} c^2-2c\geqq 0 \\ (c+1)(c-3)\leqq 0 \end{cases}$

$\iff \begin{cases} c\leqq 0,\ 2\leqq c \\ -1\leqq c\leqq 3 \end{cases}$

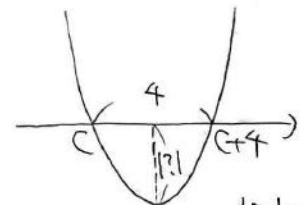
$\therefore -1\leqq c\leqq 0,\ 2\leqq c\leqq 3$

(2) Gが$(3,-1)$通る.

$-1=c^2-2c-3$

$\iff c^2-2c-2=0$

$\iff c=1\pm\sqrt{3}$ $\therefore c=1+\sqrt{3}$



$1:1=4$

Gは$y=x^2$をx軸方向に$3+\sqrt{3}$,

y軸方向に-4平行移動.

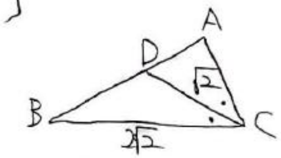
(y切片)$=c(c+4)$

$=(1+\sqrt{3})(5+\sqrt{3})$

$=8+6\sqrt{3}$

第2問

[1]



余

$BD^2=8+2-2\cdot 2\sqrt{2}\cdot\sqrt{2}\cdot\dfrac{3}{4}$

$=4$

$\therefore BD=2$

余

$\cos\angle BDC=\dfrac{4+2-8}{2\cdot 2\cdot\sqrt{2}}$

$=-\dfrac{1}{2\sqrt{2}}$

$\therefore \sin\angle BDC=\sin\angle ADC=\dfrac{\sqrt{7}}{2\sqrt{2}}=\dfrac{\sqrt{14}}{4}$

また $2\sqrt{2}:AC=2:AD$

$\iff 2AC=2\sqrt{2}AD$

$\therefore \dfrac{AC}{AD}=\sqrt{2}$ であるから

AC=$\sqrt{2}x$, AD=x として

余

$2x^2 = x^2+2-2\cdot x\cdot\sqrt{2}\cdot\frac{1}{2\sqrt{2}}$

$= x^2+2-x$

$\Leftrightarrow x^2+x-2=0$

$\therefore x = AD = 1$

余

$\cos\angle ACB = \dfrac{8+2-9}{2\cdot 2\sqrt{2}\cdot\sqrt{2}}$

$= \dfrac{1}{8}$

$\sin\angle ACB = \dfrac{3\sqrt{7}}{8}$

正

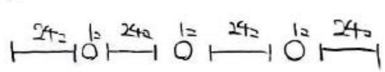
$\dfrac{3}{\sin\angle ACB} = 2R$

$\therefore R = \dfrac{3}{2}\cdot\dfrac{8}{3\sqrt{7}}$

$= \dfrac{4}{\sqrt{7}} = \dfrac{4\sqrt{7}}{7}$

[2]

(1)



3, 5

(2) 6

(3) 4

(4) 3

第3回

[1]

P(5回で少なくとも1回表)

$= 1-\left(\frac{1}{2}\right)^5$

$= \frac{31}{32} > 0.95 \cdots\cdots 0$

P(5回中3回赤)

$= {}_5C_3 P^3(1-P)^2$

$\frac{3}{5}$と言い切れない.

P(文字異なる)

$= \dfrac{1\cdot 2+2\cdot 2+2\cdot 1}{{}_5C_2} = \dfrac{4}{5} \cdots 2$

[2]

(1) $P(X=-2) = \frac{1}{4}$

$P(X=1) = {}_2C_1\cdot\frac{1}{2}\cdot\frac{1}{2}$

$= \frac{1}{2}$

(2)

再び 0点になるのは3回投げ終わったとき.

P(3回でX=0)

$= {}_3C_1\left(\frac{1}{2}\right)^3 = \frac{3}{8}$

(3)

$P(X=4)$

$= P(H\times 3, T\times 2)$

$- P(HTTHH)$

$= {}_5C_3\left(\frac{1}{2}\right)^5 - \left(\frac{1}{2}\right)^5\times 3$

$= \frac{7}{32}$

(4)

$P_{X=4}$(2回で1点)

$= \dfrac{P(HT\cdot HHT)}{P(X=4)}$

$= \dfrac{2\cdot 2\cdot\left(\frac{1}{2}\right)^5}{\frac{7}{32}}$

$= \frac{4}{7}$

第4回.

(1)

$100x = 236.\dot{3}\dot{6}$

$-)\quad x = 2.\dot{3}\dot{6}$

$99x = 234$

$\therefore x = \frac{26}{11}$

(2)

$y = 2.\dot{a}\dot{b}_{(7)}$

↓

$49y = 2ab.\dot{a}\dot{b}_{(7)}$

$-)\quad y = 2.\dot{a}\dot{b}_{(7)}$

$48y = 2ab-2_{(7)}$

$= 98+7a+b-2$

$\therefore y = \dfrac{96+7\times a+b}{48}$

↓ 分母が4

$96+7a+b=$(12の倍数)

$7a+b=12k$

$a=1, b=5$のとき $\frac{9}{4}$

$a=5, b=1$ 〃 $\frac{11}{4}$

このとき $7a+b=36$

$$y-2=\frac{7a+b}{48}$$

↓

$7a+b$は48以外の48の約数

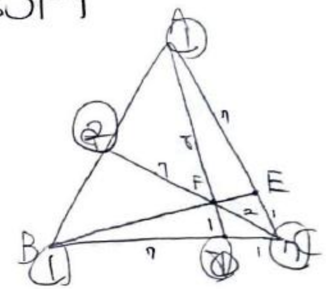
$\therefore 7a+b=1,2,3,4,6,\cancel{8},12$

$\cancel{24}$ ← $a=b=3$

↑ $a=b=1$

全部で $\underline{6コ}$

第5問

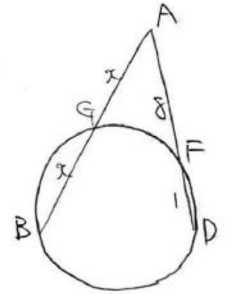


$$\frac{GB}{AG}=\underline{1} \qquad \frac{FD}{AF}=\underline{\frac{1}{8}}$$

$$\frac{FC}{GF}=\underline{\frac{2}{7}}$$

$$\frac{\triangle CDG}{\triangle BFG}=\frac{\frac{1}{2}\cdot\frac{1}{8}\cdot\triangle ABC}{\frac{1}{2}\cdot\frac{7}{9}\cdot\triangle ABC}$$

$$=\underline{\frac{9}{56}}$$



方べき

$x\cdot 2x=8\cdot 9$

$\therefore x^2=36 \quad \therefore x=6$

$\therefore AB=\underline{12}$

$AE=3\sqrt{7}$のとき

$AE\cdot AC=3\sqrt{7}\cdot 3\sqrt{7}\cdot\frac{8}{7}$

$=\underline{72}$

↓

$AE\cdot AC=AG\cdot GB$より

方べきの定理の逆から

四角形BGECは同一円周上.

$\therefore \angle AEG=180^\circ-\angle GEC$

$=\angle ABC \cdots \underline{②}$