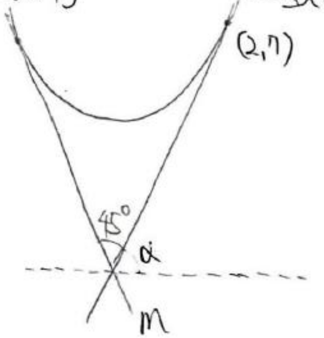
2019 帝京大(医) ②

[1]

(1) $y=\frac{1}{2}x^2+x+3$ $(y'=x+1)$

$y=3(x-2)+7$
$=3x+1$



$\tan(\alpha+45^\circ)$

$=\dfrac{\tan\alpha+1}{1-\tan\alpha}$

$=\dfrac{4}{-2}=\underline{-2}$ …ア

mの接点 $x+1=-2$ ∴$x=\underline{-3}$

(G, l, mで囲まれた面積)

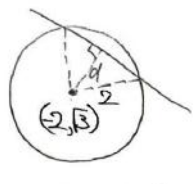
$=\dfrac{\frac{1}{2}}{12}\{2-(-3)\}^3$

$=\underline{\dfrac{125}{24}}$ $\boxed{\text{(2次関数の)}\frac{1}{12}\text{公式}}$

(2)

円 $(x+2)^2-4+(y-\sqrt{3})^2=0$

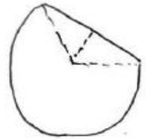
$\Leftrightarrow (x+2)^2+(y-\sqrt{3})^2=4$



$d=\dfrac{|-2\sqrt{3}-3\sqrt{3}+3\sqrt{3}|}{\sqrt{3+9}}$

$=\dfrac{2\sqrt{3}}{2\sqrt{3}}=1$

$AB=\underline{2\sqrt{3}}$



$=4\pi\times\frac{2}{3}+\sqrt{3}$

$=\underline{\frac{8}{3}\pi+\sqrt{3}}$

[2]

(1) $f(3\alpha+\frac{\pi}{2})$

$=2\sin\alpha=\frac{10}{13}$

$f(3\beta+2\pi)$

$=2\sin(\beta+\frac{\pi}{2})=2\cos\beta=\frac{6}{5}$

∴ $\begin{cases}\sin\alpha=\frac{5}{13}\\ \cos\beta=\frac{3}{5}\end{cases}$

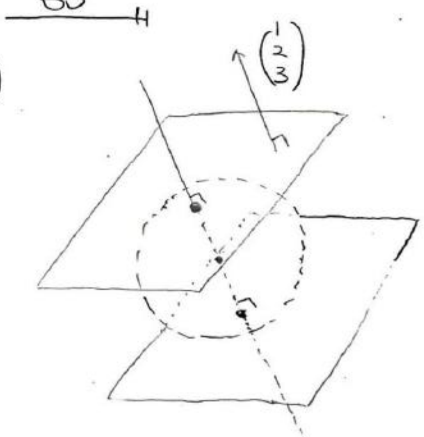
$\cos(\alpha+\beta)$

$=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\frac{12}{13}\cdot\frac{3}{5}-\frac{5}{13}\cdot\frac{4}{5}$

$=\underline{\frac{16}{65}}$

(2)



球と直線: $\begin{cases}x=t\\ y=2t\\ z=3t\end{cases}$ $(t\in\mathbb{R})$

の交点が $x+2y+3z$ がMまたはmをとるときの接点、連立すると

$t^2+4t^2+9t^2=1$

∴$t=\pm\frac{1}{\sqrt{14}}$

$M=\frac{1}{\sqrt{14}}+\frac{4}{\sqrt{14}}+\frac{9}{\sqrt{14}}=\sqrt{14}$

$m=-\sqrt{14}$ より $M-m=\underline{2\sqrt{14}}$

別 $\vec{p}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ $\vec{n}=\begin{pmatrix}1\\2\\3\end{pmatrix}$

とすると

$\vec{p}\cdot\vec{n}=\underbrace{|\vec{p}|}_{1}\underbrace{|\vec{n}|}_{\sqrt{14}}\cos\theta$

$\Leftrightarrow \cos\theta=\dfrac{x+2y+3z}{\sqrt{14}}$

↓

$-1\leqq\dfrac{x+2y+3z}{\sqrt{14}}\leqq 1$

$\Leftrightarrow \underbrace{-\sqrt{14}}_{m}\leqq x+2y+3z\leqq\underbrace{\sqrt{14}}_{M}$

(3)

余

$\left(a\dfrac{c^2+a^2-b^2}{2ca}+b\dfrac{b^2+c^2-a^2}{2bc}\right)\cos C=\dfrac{c}{2}$

$\Leftrightarrow \dfrac{2c^2}{2c}\cdot\cos C=\dfrac{c}{2}$

$\Leftrightarrow \cos C=\dfrac{1}{2}$

∴$C=\underline{\frac{1}{3}\pi}$

$\triangle ABC=\frac{1}{2}ab\sin C=\frac{3\sqrt{3}}{2}$

$\Leftrightarrow ab=6$

$\cos C=\dfrac{a^2+b^2-c^2}{2ab}=\dfrac{a^2+b^2-7}{12}=\dfrac{1}{2}$

$\therefore a^2+b^2=13$

$(a+b)^2=13+2ab=25$

$\therefore a+b=5$

$\therefore a+b+c=\underline{5+\sqrt{7}}$

(4) $|(p+qi)^2|=5$

$\therefore |p+qi|=\sqrt{5}$

$\therefore p^2+q^2=\underline{5}$

[3]

(1) $2^x=t$ とおくと

$f(x)=\frac{1}{4}t^2-2t+\frac{4}{t^2}-\frac{8}{t}+\log_2 a$

$\downarrow \frac{t}{2}+\frac{2}{t}=u$ とおく

$f(x)=u^2-2-4u+\log_2 a$

$=(u-2)^2+\log_2 a-6$

(相加平均)≧(相乗平均) で $u\geqq 2$

より $\min f(x)=\log_2 a-6\geqq 0$

$\Leftrightarrow a\geqq 2^6=64$

$\therefore \min a=\underline{64}$

(2)

$\log_3 9+\frac{\log_3(1-y)}{\log_3\sqrt{3}}<\log_3(3+3y^2)$

$\Leftrightarrow \log_3 9(1-y)^2<\log_3(3+3y^2)$

真数条件 $1-y>0 \Leftrightarrow y<1$ …①

$9(1-y)^2<3+3y^2$

$\Leftrightarrow 3-6y+3y^2<1+y^2$

$\Leftrightarrow y^2-3y+1<0$

$\Leftrightarrow \frac{3-\sqrt{5}}{2}<y<\frac{3+\sqrt{5}}{2}$ …②

①, ②より

$\frac{3-\sqrt{5}}{2}<y<1$

$\therefore \underline{3-\sqrt{5}}<x<\underline{2}$

(3)

$P(x)=x^5+4x^4+2x^3+px^2+qx+1$

$=(x^2+2x+3)Q_1(x)$ とおく

$px^2+qx+1=(x^2+2x+3)Q_2(x)+ax+b$ とおく

$\downarrow$

$x^5+4x^4+2x^3$

$=(x^2+2x+3)Q_1(x)$

$-(x^2+2x+3)Q_2(x)-ax-b$

$=(x^2+2x+3)\{Q_1(x)-Q_2(x)\}-ax-b$

$$
\begin{array}{rrrrrrr}
 & 1 & 2 & -5 & 4 & & \\
\hline
1\ 2\ 3\,) & 1 & 4 & 2 & 0 & 0 & 0 \\
 & 1 & 2 & 3 & & & \\
\hline
 & & 2 & -1 & 0 & & \\
 & & 2 & 4 & 6 & & \\
\hline
 & & & -5 & -6 & & \\
 & & & -5 & -10 & -15 & \\
\hline
 & & & & 4 & 15 & 0 \\
 & & & & 4 & 8 & 12 \\
\hline
 & & & & & 7 & -12
\end{array}
$$

$\therefore a=-7,\ b=12$

(求める余り) $=\underline{-7}x+\underline{12}$

[4]

(1) (与式)

$=\sum_{k=1}^{60}\frac{\sqrt{2k-1}-\sqrt{2k+1}}{-2}$

$=\frac{1-\sqrt{3}}{-2}+\frac{\sqrt{3}-\sqrt{5}}{-2}+\cdots+\frac{\sqrt{119}-\sqrt{121}}{-2}$

$=\frac{1-\sqrt{121}}{-2}=\underline{5}$

(2)

$\frac{a+2b+54}{6}=13$

$\Leftrightarrow a+2b=24$

$s^2=\frac{a^2+2b^2+324\cdot 3}{6}-169$

$=\frac{a^2+2b^2}{6}-7=29$

$\Leftrightarrow a^2+2b^2=216$

$\downarrow$

$(24-2b)^2+2b^2=216$

$\Leftrightarrow 3b^2-48b+180=0$

$\Leftrightarrow b^2-16b+60=0$

$a<b$ より $a=\underline{4}$, $b=\underline{10}$

(3)

エ

$=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$

$+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 3}+\frac{1}{1\cdot 4}+\frac{1}{2\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 4}$

$+\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{1\cdot 2\cdot 4}+\frac{1}{1\cdot 3\cdot 4}$

$+\frac{1}{1\cdot 2\cdot 3\cdot 4}$

$=(1+\frac{1}{1})(1+\frac{1}{2})(1+\frac{1}{3})(1+\frac{1}{4})-1$

$=\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}-1=\underline{4}$

オ

$=(1+\frac{1}{1})(1+\frac{1}{2})(1+\frac{1}{3})\cdots(1+\frac{1}{20})-1$

$=\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdots\frac{21}{20}-1$

$=\underline{20}$