2019 昭和大(医) I期

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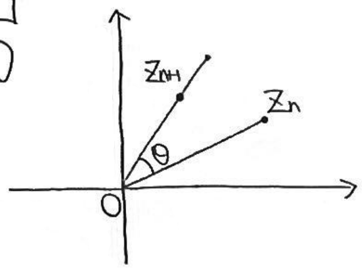
(1)



$$z_{n+1}=\frac{2}{3}(\cos\theta+i\sin\theta)z_n$$

↓

$$z_n= z_0\left\{\frac{2}{3}(\cos\theta+i\sin\theta)\right\}^n$$
$$=z_0\left(\frac{2}{3}\right)^n(\cos n\theta+i\sin n\theta)$$
$$=r\left(\frac{2}{3}\right)^n\{\cos(n\theta+\theta_0)$$
$$+i\sin(n\theta+\theta_0)\}_{/\!/}$$

(2)

$$z_{n+1}-z_n=\left\{\frac{2}{3}(\cos\theta+i\sin\theta)-1\right\}z_n$$
$$z_1-z_0=\left\{\frac{2}{3}(\cos\theta+i\sin\theta)-1\right\}z_0$$

↓

$$z_{n+1}-z_n=t(z_1-z_0)$$
$$\Leftrightarrow\ z_n=tz_0$$

つまり $n\theta=k\pi\ (k\in\mathbb{Z})$

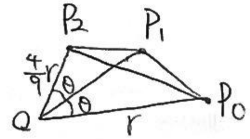
であれば t は実数となる.

$$\therefore\ n=\frac{k\pi}{\theta}\ (k\text{はnを自然数にする整数})_{/\!/}$$

(3)

$$\triangle P_nP_{n+1}P_{n+2}:\triangle P_{n+1}P_{n+2}P_{n+3}$$
$$=4:9\ \boxed{\text{相似比 }2:3}$$

$$\sum_{n=0}^{\infty}S_n$$
$$=\frac{\triangle P_0P_1P_2}{1-\frac{4}{9}}$$



$$=\frac{9}{5}(\triangle OP_0P_1+\triangle OP_1P_2-\triangle OP_0P_2)$$
$$=\frac{9}{5}\left(\frac{1}{2}\cdot\frac{2}{3}r^2\sin\theta+\frac{1}{2}\cdot\frac{8}{27}r^2\sin\theta\right.$$
$$\left.-\frac{1}{2}\cdot\frac{4}{9}r^2\sin2\theta\right)$$
$$=\frac{9r^2}{10}\left(\frac{2}{3}\sin\theta+\frac{8}{27}\sin\theta-\frac{4}{9}\sin2\theta\right)$$
$$=\frac{9r^2}{10}\left(\frac{26}{27}\sin\theta-\frac{8}{9}\sin\theta\cos\theta\right)$$
$$=r^2\sin\theta\left(\frac{13}{15}-\frac{4}{5}\cos\theta\right)$$
$$=\frac{1}{15}r^2\sin\theta(13-12\cos\theta)_{/\!/}$$

2

(1)

V…偶数または5の倍数

W…平方数でない)

$$\overline{A}\cap\overline{B}=W$$
$$\Leftrightarrow\ \overline{A\cup B}=W$$
$$\Leftrightarrow\ A\cup B=\overline{W}$$
$$B=(\overline{A}\cup B)\cap(A\cup B)$$
$$=V\cap\overline{W}$$
$$=\{4,16,25,36,64\}_{/\!/}$$

(2)

$$x={}_{99}C_{50}={}_{99}C_{49}$$
$$=\frac{99!}{49!\,50!}$$

1〜49で3の倍数…16コ

9の倍数…5コ

27の倍数…1コ

1〜99で3の倍数…33コ

9の倍数…11コ

27の倍数…3コ

81の倍数…1コ

$$x=\frac{2^{\alpha_1}\cdot3^{48}\cdot5^{\beta_1}\cdots97}{(2^{\alpha_2}\cdot3^{22}\cdot5^{\beta_2}\cdots47)(2^{\alpha_3}\cdot3^{22}\cdot5^{\beta_3}\cdots47)}$$
$$=2^{\alpha_4}\cdot3^4\cdot5^{\beta_4}\cdots$$

つまり x の約数で 3^a と表せる最大の数は $\underline{81}_{/\!/}$

(3)

$100=2^2\cdot5^2$　約数9コ

約数10コは $n=a\cdot b^4$

$(a,b)=(3,2),(5,2)$

約数12コは $n=a\cdot b^5$ または $a^2\cdot b^3$

または abc^2

$(a,b)=(3,2)$ または $(3,2)$

または $(a,b,c)=(5,2,3)$

$(5,3,2)$

$(7,3,2)$

以上より $n=\underline{7個}_{/\!/}$

(100の正の約数の和)

$=(1+2+4)(1+5+25)=217$

一方で

(48の正の約数の和)

$=(1+3)(1+2+4+8+16)=124$

(80の正の約数の和)

$= (1+5)(1+2+4+8+16) = 186$

(96の 〃)

$= (1+3)(1+2+4+8+16+32) = 252$

(72の 〃)

$= (1+3+9)(1+2+4+8) = 195$

(90の 〃)

$= (1+5)(1+2)(1+3+9) = 234$

(60の 〃)

$= (1+5)(1+3)(1+2+4) = 168$

(84の 〃)

$= (1+7)(1+3)(1+2+4) = 224$

この中で217より大きいのは

$\underline{n = 96,\ 90,\ 84}$ //

$\boxed{3}$

(1) $3x+7y=2019$

解くと

$$\begin{cases} x = 7k-4038 > 0 \\ y = -3k+2019 > 0 \end{cases}$$

↓

$$\frac{4038}{7} < k < \frac{2019}{3}$$

$$\begin{array}{r} 576 \\ 7\overline{)4038} \\ 35 \\ \hline 53 \\ 49 \\ \hline 48 \end{array} \qquad \begin{array}{r} 673 \\ 3\overline{)2019} \\ 18 \\ \hline 21 \end{array}$$

$\Leftrightarrow 576,\cdots < k < 673$

$\therefore 577 \leqq k \leqq 672$

$\therefore$ kは$\underline{96個}$ //

(2)

解と係数の関係

$$\begin{cases} \alpha+\beta+\gamma = -3 \\ \alpha\beta+\beta\gamma+\gamma\alpha = 0 \\ \alpha\beta\gamma = 1 \end{cases}$$

また

$\alpha^2+\beta^2+\gamma^2 = -a$

$\Leftrightarrow (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha) = -a$

$\therefore \underline{a=-9}$ //

$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2 = b$

$\Leftrightarrow b = (\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2\alpha\beta^2\gamma - 2\alpha\beta\gamma^2 - 2\alpha^2\beta\gamma$

$\Leftrightarrow b = 0^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma) = \underline{6}$ //

$\alpha^2\beta^2\gamma^2 = -c$

$\therefore \underline{c=-1}$ //

(3)

(3-1)

$P(1回) = \frac{2}{6} = \underline{\frac{1}{3}}$ //

(3-2)

$P(2回) = \frac{3+4+5+6}{36}$

$= \underline{\frac{1}{2}}$ //

(1,4) (1,5) (1,6) (2,3) ⋮ (4,1) ⋮ (4,6)

(3-3)

$P(4回) = \frac{23}{6^4}$

(1,1,1,回) ⋮ (2,1,1,□)

$P(5回) = \frac{6}{6^5}$

(1,1,1,1,□)

より

$E = \sum_{k=1}^{5} k \cdot P(k回)$

$= 1\cdot\frac{1}{3} + 2\cdot\frac{1}{2} + 3\cdot\left(1-\frac{1}{3}-\frac{1}{2}-\frac{23}{6^4}-\frac{1}{6^4}\right)$

$+ 4\cdot\frac{23}{6^4} + 5\cdot\frac{1}{6^4}$

$= \frac{4}{3} + 3\left(\frac{1}{6}-\frac{24}{6^4}\right) + \frac{97}{6^4}$

$= \frac{8}{6} + \frac{3}{6} + \frac{25}{6^4}$

$= \frac{11\cdot 216+25}{6^4} = \underline{\frac{2401}{1296}}$ //

$\boxed{4}$

(1)

$f'(x) = \frac{-\sin x(1+\sin x) - \cos^2 x}{(1+\sin x)^2}$

$= \frac{-\sin x - 1}{(1+\sin x)^2}$

$f'\left(\frac{\pi}{6}\right) = \frac{-\frac{1}{2}-1}{\frac{9}{4}} = \underline{-\frac{2}{3}}$ //

(2)

(i) $x \geqq 1,\ y \geqq 1$ のとき

$\log_3 x + \log_3 y = \log_3 9$

$\therefore y = \frac{9}{x}$

(ii) $0<x<1,\ y \geqq 1$ のとき

$-\log_3 x + \log_3 y = \log_3 9$

$\therefore y = 9x$

(iii) $x \geqq 1,\ 0<y<1$ のとき

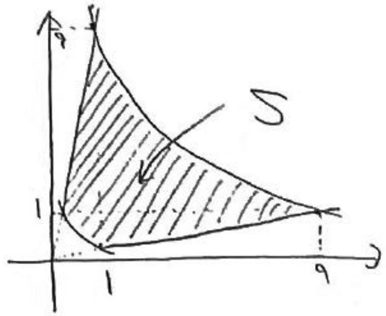
$\log_3 x - \log_3 y = \log_3 9$

$\therefore y = \frac{1}{9}x$

(iv) $0<x<1,\ 0<y<1$ のとき

$-\log_3 x - \log_3 y = \log_3 9$

$\therefore y = \frac{1}{9x}$



S

$= \int_{\frac{1}{9}}^{1} \left(9x - \frac{1}{9x}\right) dx + \int_{1}^{9} \left(\frac{9}{x} - \frac{1}{9}x\right) dx$

$= \left[\frac{9}{2}x^2 - \frac{1}{9}\log x\right]_{\frac{1}{9}}^{1} + \left[9\log x - \frac{1}{18}x^2\right]_{1}^{9}$

$= \frac{9}{2} - \left(\frac{1}{18} + \frac{1}{9}\log 9\right) + 9\log 9 - \frac{9}{2} + \frac{1}{18}$

$= \frac{80}{9}\log 9 = \underline{\frac{160}{9}\log 3}$ //

(3)

$\frac{dx}{d\theta} = -\sin\theta\cos\theta + (1+\cos\theta)(-\sin\theta)$

$= -\sin\theta(1+2\cos\theta)$

$\frac{dy}{d\theta} = -\sin^2\theta + (1+\cos\theta)\cos\theta$

$= 2\cos^2\theta + \cos\theta - 1$

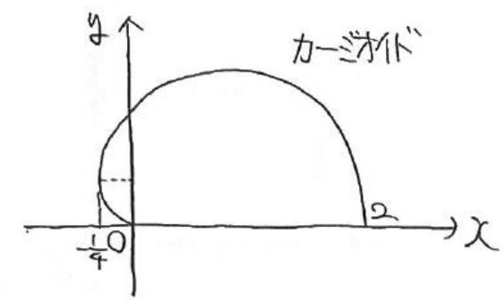
$= (2\cos\theta - 1)(\cos\theta + 1)$

ここで $(1+\cos(-\theta))\cos(-\theta) = x$

$(1+\cos(-\theta))\sin(-\theta) = -y$

より $0 \leqq \theta \leqq \pi$ のみ調べる。

θ	0	$\cdots$	$\frac{\pi}{3}$	$\cdots$	$\frac{2}{3}\pi$	$\cdots$	π
$\frac{dx}{d\theta}$	0	$-$	$-$	$-$	0	$+$	0
x	2	$\searrow$	$\frac{3}{4}$	$\searrow$	$-\frac{1}{4}$	$\nearrow$	0
$\frac{dy}{d\theta}$	$+$	$+$	0	$-$	$-$	$-$	0
y	0	$\nearrow$	$\frac{3\sqrt{3}}{4}$	$\searrow$	$\frac{\sqrt{3}}{4}$	$\searrow$	0



x軸に関して対称であるので
求める体積は

$\int_{-\frac{1}{4}}^{2} y^2\pi\, dx - \int_{-\frac{1}{4}}^{0} y^2\pi\, dx$

$= \pi\int_{\frac{2}{3}\pi}^{0} y^2 \frac{dx}{d\theta} d\theta - \pi\int_{\frac{2}{3}\pi}^{\pi} y^2 \frac{dx}{d\theta} d\theta$

$= \pi\int_{\pi}^{0} y^2 \frac{dx}{d\theta} d\theta$

$= \pi\int_{\pi}^{0} (1+\cos\theta)^2\sin^2\theta\,(-\sin\theta)(1+2\cos\theta)\, d\theta$

$= \pi\int_{0}^{\pi} (1+\cos\theta)^2\sin^3\theta\,(1+2\cos\theta)\, d\theta$

$= \pi\int_{0}^{\pi} (1+\cos\theta)^2(1-\cos^2\theta)(1+2\cos\theta)\sin\theta\, d\theta$

$\downarrow \cos\theta = t$

$= \pi\int_{1}^{-1} (1+t)^2(1-t^2)(1+2t)(-dt)$

$= 2\pi\int_{0}^{1} (1+4t^2-5t^4)\, dt$

$= \underline{\frac{8}{3}\pi}$ //