2019 埼玉医科大(前期)

1

問1.

$1 \leqq \log_e n \leqq 12$

$\Leftrightarrow 1 \leqq \dfrac{\log_{10} n}{\log_{10} e} \leqq 12$

$\Leftrightarrow \log_{10} e \leqq \log_{10} n \leqq 12 \log_{10} e$

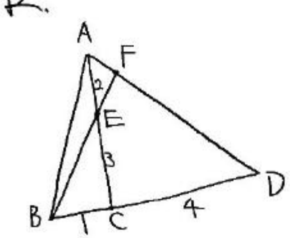
$$\begin{array}{r} 0.4343 \\ \times \quad 12 \\ \hline 8686 \\ 4343 \\ \hline 5.2116 \end{array}$$

$\therefore 0 < \log_{10} n < 6$

$\Leftrightarrow 1 < n < 10^6$

nの桁数Pは $1 \leqq P \leqq 6$ //

問2.



メネラウスの定理

$\dfrac{5}{1} \cdot \dfrac{3}{2} \cdot \dfrac{AF}{FD} = 1$

$\therefore AF : FD = 2 : 15$

(四角形ABCF)

$= \triangle ABC + \triangle AFC$

$= \quad " \quad + \dfrac{2}{17} \triangle ACD$

$= \quad " \quad + \dfrac{2}{17} \cdot 4 \cdot \triangle ABC$

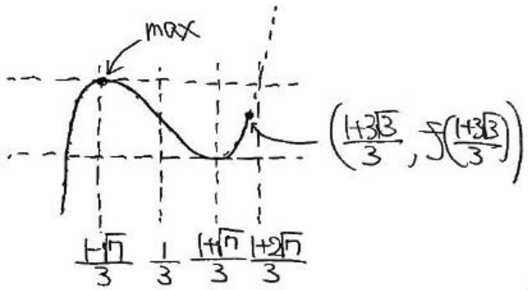
$= \dfrac{25}{17} \triangle ABC$ //

問3

$f(x) = x^3 - x^2 - 2x + 1$

$f'(x) = 3x^2 - 2x - 2$

$f'(x) = 0 \Leftrightarrow x = \dfrac{1 \pm \sqrt{7}}{3}$



$x = \dfrac{1-\sqrt{7}}{3}$ のとき最大

		$\frac{1}{3}$	$-\frac{1}{9}$	
$3\ -2\ -2$	1	-1	-2	1
	1	$-\frac{2}{3}$	$-\frac{2}{3}$	
		$-\frac{1}{3}$	$-\frac{4}{3}$	1
		$-\frac{1}{3}$	$\frac{2}{9}$	$\frac{2}{9}$
			$-\frac{14}{9}$	$\frac{7}{9}$

$f(x) = (3x^2 - 2x - 2)\left(\frac{1}{3}x - \frac{1}{9}\right) - \frac{14}{9}x + \frac{7}{9}$

$f\left(\frac{1-\sqrt{7}}{3}\right) = -\frac{14}{9} \cdot \frac{1-\sqrt{7}}{3} + \frac{7}{9}$

$= -\dfrac{14 - 14\sqrt{7}}{27} + \dfrac{21}{27}$

$= \dfrac{7 + 14\sqrt{7}}{27}$ //

2

問1.

(左辺)

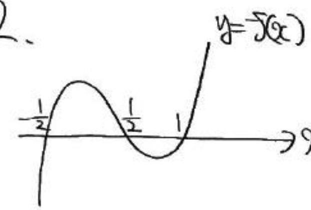
$= 4(1 - \cos^2\theta) + 2(2\cos^2\theta - 1)\cos\theta + \cos\theta - 3$

$= 4x^3 - 4x^2 - 1x + 1$ //

$= (x-1)(4x^2 \quad -1)$

$= (x-1)(2x+1)(2x-1) \geqq 0$

問2.



$-\dfrac{1}{2} \leqq \underset{\cos\theta}{x} \leqq \dfrac{1}{2}, \quad 1 \leqq \underset{\cos\theta}{x}$

$\therefore \frac{1}{3}\pi \leqq \theta \leqq \frac{2}{3}\pi, \ \frac{4}{3}\pi \leqq \theta \leqq \frac{5}{3}\pi$

$\theta = 0$ //

問3

$S = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx + \int_{\frac{1}{2}}^{1} (-f(x))\,dx$

$= 2\int_0^{\frac{1}{2}} (-4x^2 + 1)\,dx$

$+ \left[-x^4 + \frac{4}{3}x^3 + \frac{1}{2}x^2 - x\right]_{\frac{1}{2}}^{1}$

$= 2\left[-\frac{4}{3}x^3 + x\right]_0^{\frac{1}{2}}$

$+ \frac{1}{3} + \frac{1}{2} - 1 - \left(-\frac{1}{16} + \frac{1}{6} + \frac{1}{8} - \frac{1}{2}\right)$

$= -\frac{1}{3} + 1 - \frac{1}{6} - \left(\frac{-3+8+6-24}{48}\right)$

$= \frac{1}{2} + \frac{13}{48} = \frac{37}{48}$ //

3

$z = \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)$

問1.

$S_8 = 1 + z + z^2 + \cdots + z^7$

$= \dfrac{1 - z^8}{1 - z}$

$= \dfrac{1 - \cos\left(-\frac{8}{3}\pi\right) - i\sin\left(-\frac{8}{3}\pi\right)}{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$ （$\cos\left(-\frac{8}{3}\pi\right) = -\frac{1}{2}$, $\sin\left(-\frac{8}{3}\pi\right) = -\frac{\sqrt{3}}{2}$）

$= \dfrac{\frac{3}{2} + \frac{\sqrt{3}}{2}i}{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$

$= \dfrac{\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

$= \sqrt{3}\left\{\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right\}$

$= \frac{3}{2} + \frac{-\sqrt{3}}{2}i$ //

問2.

$1 + z + z^2 + \cdots + z^{n-1}$

$= \frac{1 - z^n}{1 - z}$

$= \frac{1 - \cos n\theta - i\sin n\theta}{1 - \cos\theta - i\sin\theta} \times \frac{1 - \cos\theta + i\sin\theta}{1 - \cos\theta + i\sin\theta}$

$= \frac{(1 - \cos n\theta - i\sin n\theta)\{(1 - \cos\theta) + i\sin\theta\}}{(1 - \cos\theta)^2 + \sin^2\theta}$

虚部をみると

$\sum_{k=0}^{n-1} \sin k\theta$

$= \frac{(1 - \cos n\theta)\sin\theta - \sin n\theta(1 - \cos\theta)}{2 - 2\cos\theta}$

$= \frac{-\sin n\theta + \sin n\theta\cos\theta - \cos n\theta\sin\theta + \sin\theta}{2 - 2\cos\theta}$

$= \frac{-\sin n\theta + 1\cdot\sin(n-1)\theta + 1\cdot\sin\theta}{2 - 2\cos\theta}$ //

問3 $\theta = \frac{\pi}{n}$ とおくと

$\sum_{k=0}^{n-1} \sin\frac{k\pi}{n}$

$= \frac{\sin(n-1)\frac{\pi}{n} + \sin\frac{\pi}{n}}{2\left(1 - \cos\frac{\pi}{n}\right)}$

$= \frac{\sin\left(\pi - \frac{\pi}{n}\right) + \sin\frac{\pi}{n}}{2\left(1 - \cos\frac{\pi}{n}\right)} \times \frac{1 + \cos\frac{\pi}{n}}{1 + \cos\frac{\pi}{n}}$

$= \frac{2\sin\frac{\pi}{n}}{2\left(1 - \cos^2\frac{\pi}{n}\right)} \times \left(1 + \cos\frac{\pi}{n}\right)$

$= \frac{1 + 1\cdot\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}$ //

これより

$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \sin\frac{k\pi}{n}$

$= \lim_{n\to\infty} \frac{1}{\pi} \cdot \frac{\frac{\pi}{n}}{\sin\frac{\pi}{n}} \left(1 + \cos\frac{\pi}{n}\right)$

$= \frac{2}{\pi}$ //

※当然区分求積でもいいでしょう.

4

問1.

P(黒石が5コ連続で並ぶ)

$= \frac{6! \times 5!}{10!}$ ← 黒5コを1つとみる．6コの順列と黒の並べ方

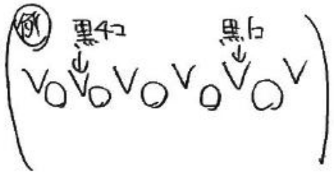
$= \frac{5\cdot4\cdot3\cdot2}{10\cdot9\cdot8\cdot7}$

$= \frac{1}{42}$ //

問2.

P(黒石が4コ連続で並ぶ)

= P(



)

$= \frac{5! \times {}_5P_4 \times {}_6P_2}{10!}$ （白の並べ方／黒4コの選び方／黒の並べ方）

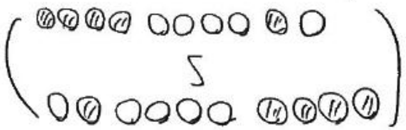
$= \frac{5\cdot4\cdot3\cdot2\cdot6\cdot5}{10\cdot9\cdot8\cdot7\cdot6}$

$= \frac{5}{42}$ //

問3

P(黒石と白石が4コずつ並ぶ)

= P(



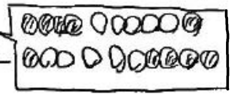
)

$= \frac{{}_5P_4 \cdot {}_5P_4 \cdot 2\cdot2\cdot2}{10!}$

$= \frac{2}{63}$

P(黒石が4コ連続かつ白石が5コ連続)

$= \frac{{}_5P_4 \cdot 5! \cdot 2}{10!}$



$= \frac{1}{126}$

P(白石が4コ連続かつ黒石が5コ連続)

$= \frac{1}{126}$

求める確率は

$\frac{5}{42} + \frac{5}{42} - \frac{2}{63} - \frac{1}{126} \times 2$

$= \frac{4}{21}$ //