2019 日本大(医)

[1]

(1)

(与式)

$=\left\{2(5-2\sqrt{6})(5+2\sqrt{6})+\frac{5+3\sqrt{3}}{7+4\sqrt{3}}\right\}^2$

$=\left(2+\frac{5+3\sqrt{3}}{7+4\sqrt{3}}\right)^2$

$=\left(\frac{19+11\sqrt{3}}{7+4\sqrt{3}}\right)^2$

$=\left\{(19+11\sqrt{3})(7-4\sqrt{3})\right\}^2$

$=(1+\sqrt{3})^2=\underline{4+2\sqrt{3}}$ //

(2)

(与式)

$\Leftrightarrow (x^2+2x-8)(x^2+2x-3)=36$

$\Leftrightarrow (x^2+2x)^2-11(x^2+2x)-12=0$

$\Leftrightarrow (x^2+2x-12)(x^2+2x+1)=0$

$\therefore \underline{x=-1,\ -1\pm\sqrt{13}}$ //

(3)

$\frac{{}_8C_2\cdot{}_4C_1}{{}_{12}C_3}=\frac{28\cdot 4}{220}=\underline{\frac{28}{55}}$ //

(4)

真数条件

$\begin{cases} x+4>0 \\ 10-x>0 \end{cases}$ $\therefore -4<x<10$

$\frac{\log_{10}(x+4)}{\log_{10}\frac{1}{10}} \leqq \log_{10}(10-x)$

$\Leftrightarrow \log_{10}(x+4)^{-1} \leqq \log_{10}(10-x)$

$\Leftrightarrow \frac{1}{x+4} \leqq 10-x$

$\Leftrightarrow 1 \leqq -x^2+6x+40$

$\Leftrightarrow x^2-6x-39 \leqq 0$

$\Leftrightarrow 3-4\sqrt{3} \leqq x \leqq 3+\underbrace{4\sqrt{3}}_{\fallingdotseq 6.8}$

これは $-4<x<10$ をみたす.

$\therefore \underline{3-4\sqrt{3} \leqq x \leqq 3+4\sqrt{3}}$ //

[2]

(1) 楕円上の $(s,-2s)$ であれば

接線の傾きが2になる.

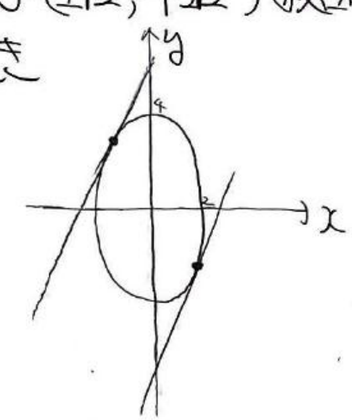
$4s^2+4s^2=16$

$\Leftrightarrow s^2=2$

$\Leftrightarrow s=\pm\sqrt{2}$

接点が $(\pm\sqrt{2}, \mp 2\sqrt{2})$ (複合同順)

のとき



このとき

$\mp 2\sqrt{2}=\pm 2\sqrt{2}+k$

$\therefore k=\mp 4\sqrt{2}$

図より $\underline{-4\sqrt{2}<k<4\sqrt{2}}$ //

(2)

$n \geqq 2$ のとき

a_n

$=S_n-S_{n-1}$

$=\frac{1}{24}\{2n^3-75n^2+97n$

$-2(n-1)^3+75(n-1)^2-97(n-1)\}$

$=\frac{1}{24}\{2(3n^2-3n+1)+75(-2n+1)$

$+97\}$

$=\frac{1}{24}(6n^2-156n+174)$

$=\frac{1}{4}(n^2-26n+29)$

$=\frac{1}{4}(n-13)^2-\frac{169}{4}+\frac{29}{4}$

$n=\underline{13}$ // のとき最小値 $\underline{-35}$ //

(3)

必要条件 $\sqrt{2+a}-b=0$

$\Leftrightarrow b=\sqrt{2+a}$

$\lim_{x\to 2}\frac{\sqrt{x+a}-\sqrt{2+a}}{x-2}$

$=\lim_{x\to 2}\frac{x-2}{(x-2)(\sqrt{x+a}+\sqrt{2+a})}$

$=\frac{1}{2\sqrt{2+a}}=2$

$\Leftrightarrow \sqrt{2+a}=\frac{1}{4}$

$\therefore 2+a=\frac{1}{16}$

$\therefore \underline{a=-\frac{31}{16}\quad b=\frac{1}{4}}$ //

(4) $-x^2+3=x^2-5x$

$\Leftrightarrow -2x^2+5x+3=0$

$\Leftrightarrow x=-\frac{1}{2}, 3$

これらで囲まれた面積は

$\frac{2}{6}\left(3+\frac{1}{2}\right)^3=\frac{1}{3}\cdot\left(\frac{7}{2}\right)^3$

$=\underline{\frac{343}{24}}$ //

2つの曲線を通る図形

$-x^2-y+3+k(x^2-y-5x)=0.$

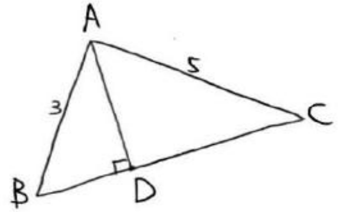
これに $k=1$ を代入すると

$-2y-5x+3=0$

$\Leftrightarrow \underline{5x+2y=3}$

[3]

(1)



$\triangle ABC$ に余弦定理

$\cos A=\dfrac{9+25-36}{2\cdot3\cdot5}=-\dfrac{1}{15}$

$\therefore \sin A=\dfrac{\sqrt{224}}{15}$

$\triangle ABC$ の面積において

$\dfrac{1}{2}\cdot3\cdot5\cdot\dfrac{\sqrt{224}}{15}=6\cdot AD\cdot\dfrac{1}{2}$

$\Leftrightarrow \sqrt{56}=3AD$

$\therefore AD=\dfrac{1}{3}\sqrt{56}=\underline{\dfrac{2}{3}\sqrt{14}}$

三平方の定理より

$BD=\sqrt{9-\dfrac{56}{9}}=\sqrt{\dfrac{25}{9}}=\dfrac{5}{3}$

つまり

$\dfrac{r_1}{2}\left(3+\dfrac{5}{3}+\dfrac{\sqrt{56}}{3}\right)=\dfrac{5}{3}\cdot\dfrac{\sqrt{56}}{3}\cdot\dfrac{1}{2}$

$\Leftrightarrow r_1\cdot\dfrac{14+\sqrt{56}}{3}=\dfrac{5\sqrt{56}}{9}$

$\Leftrightarrow 3(14+2\sqrt{14})r_1=10\sqrt{14}$

$\Leftrightarrow 3(\sqrt{14}+2)r_1=10$

$\Leftrightarrow 3\cdot10\cdot r_1=10(\sqrt{14}-2)$

$\therefore r_1=\underline{\dfrac{\sqrt{14}-2}{3}}$

$\triangle ADC$ の面積において

$\dfrac{r_2}{2}\left(\dfrac{\sqrt{56}}{3}+\dfrac{13}{3}+5\right)=\dfrac{13}{3}\cdot\dfrac{\sqrt{56}}{3}\cdot\dfrac{1}{2}$

$\Leftrightarrow r_2(\sqrt{56}+28)=\dfrac{13}{3}\sqrt{56}$

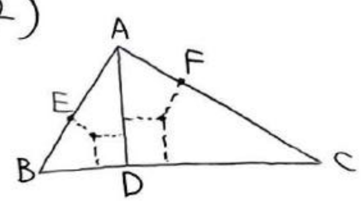
$\Leftrightarrow (\sqrt{14}+14)r_2=\dfrac{13}{3}\sqrt{14}$

$\Leftrightarrow (\sqrt{14}+1)r_2=\dfrac{13}{3}$

$\therefore r_2=\dfrac{\sqrt{14}-1}{3}$

$\therefore r_2=\underline{r_1+\dfrac{1}{3}}$

(2)



$AE+BD=\dfrac{1}{2}(AB+BD+AD)$

$=\dfrac{14+2\sqrt{14}}{6}$

$\therefore AE=\dfrac{7+\sqrt{14}}{3}-\dfrac{5}{3}=\underline{\dfrac{\sqrt{14}+2}{3}}$

$AF+DC=\dfrac{1}{2}(AD+DC+AC)$

$=\dfrac{1}{2}\cdot\dfrac{2\sqrt{14}+28}{3}$

$\therefore AF=\dfrac{\sqrt{14}+14}{3}-\dfrac{13}{3}=\dfrac{\sqrt{14}+1}{3}$

$\therefore \underline{AF=AE-\dfrac{1}{3}}$

(3) 内心の定義より

$\theta=\dfrac{1}{2}\angle BAC$

$\therefore \cos2\theta=-\dfrac{1}{15}$

$\Leftrightarrow 2\cos^2\theta-1=-\dfrac{1}{15}$

$\Leftrightarrow \cos^2\theta=\dfrac{7}{15}$

$\therefore \cos\theta=\dfrac{\sqrt{7}}{\sqrt{15}}=\underline{\dfrac{\sqrt{105}}{15}}$

$AO_1=\sqrt{AE^2+r_1^2}$ $\quad AO_2=\sqrt{AF^2+r_2^2}$

$=\sqrt{\dfrac{36}{9}}$ $\quad =\sqrt{\dfrac{30}{9}}$

$=2$ $\quad =\dfrac{\sqrt{30}}{3}$

$\triangle AO_1O_2$

$=\dfrac{1}{2}\cdot2\cdot\dfrac{\sqrt{30}}{3}\cdot\dfrac{\sqrt{120}}{15}$

$=\dfrac{\sqrt{30}\cdot2\sqrt{30}}{3\cdot15}=\underline{\dfrac{4}{3}}$

[4]

(1) 接点の x 座標を t とおくと

$y=-\dfrac{1}{2}(t-1)^{-\frac{3}{2}}(x-t)+(t-1)^{-\frac{1}{2}}$

$\downarrow$ (3,0)通る

$0=-\dfrac{1}{2}(t-1)^{-\frac{3}{2}}(3-t)+(t-1)^{-\frac{1}{2}}$

$\Leftrightarrow 0=-\dfrac{1}{2}(3-t)+t-1$

$\Leftrightarrow 0=\dfrac{3}{2}t-\dfrac{5}{2}$ $\quad\therefore t=\dfrac{5}{3}$

つまり

$y=-\dfrac{1}{2}\left(\dfrac{2}{3}\right)^{-\frac{3}{2}}\left(x-\dfrac{5}{3}\right)+\left(\dfrac{2}{3}\right)^{-\frac{1}{2}}$

$=-\dfrac{1}{2}\cdot\dfrac{3\sqrt{3}}{2\sqrt{2}}\left(x-\dfrac{5}{3}\right)+\dfrac{\sqrt{3}}{\sqrt{2}}$

$=-\dfrac{3\sqrt{3}}{4\sqrt{2}}x+\dfrac{5\sqrt{3}}{4\sqrt{2}}+\dfrac{4\sqrt{3}}{4\sqrt{2}}$

$=-\dfrac{3\sqrt{6}}{8}x+\dfrac{9\sqrt{6}}{8}$

$\therefore \underline{a=-\dfrac{3\sqrt{6}}{8}\quad b=\dfrac{9\sqrt{6}}{8}}$

(2)

$-x+3=\frac{1}{\sqrt{x-1}}>0$

$\Downarrow$

乗　$1<x<3 \cdots ①$

$x^2-6x+9=\frac{1}{x-1}$

$\Leftrightarrow x^3-7x^2+15x-9=1$

$\Leftrightarrow x^3-7x^2+15x-10=0$

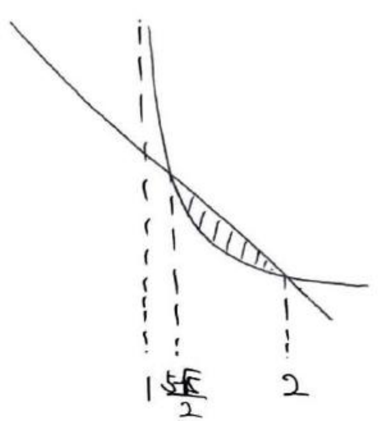
$\Leftrightarrow (x-2)(x^2-5x+5)=0$

$\Leftrightarrow x=2, \frac{5\pm\sqrt{5}}{2}$

①をみたすのは

$x=2, \frac{5-\sqrt{5}}{2}$ //

(3)



$\int_\alpha^2\left(-x+3-\frac{1}{\sqrt{x-1}}\right)dx$

$=\left[-\frac{1}{2}x^2+3x-2(x-1)^{\frac{1}{2}}\right]_\alpha^2$

$=-2+6-2+\frac{1}{2}\alpha^2-3\alpha+2\sqrt{\alpha-1}$

$=2+\frac{1}{2}(5\alpha-5)-3\alpha+2\sqrt{\frac{3-\sqrt{5}}{2}}$

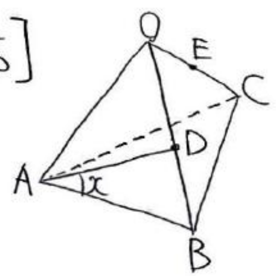
$=2+\frac{1}{2}\cdot\frac{15-5\sqrt{5}}{2}-\frac{15-3\sqrt{5}}{2}+2\sqrt{\frac{6-2\sqrt{5}}{4}}$

$=2+\frac{15-5\sqrt{5}}{4}-\frac{30-6\sqrt{5}}{4}+\sqrt{5}-1$

$=\frac{8}{4}-\frac{15}{4}+\frac{\sqrt{5}}{4}+\sqrt{5}-1$

$=\frac{5}{4}\sqrt{5}-\frac{11}{4}$ //

[5]



△ABDに正弦定理

$\frac{BD}{\sin x}=\frac{1}{\sin\left(\frac{2}{3}\pi-x\right)}$

$\Leftrightarrow BD\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)=\sin x$

$\Leftrightarrow BD=\frac{2\sin x}{\sqrt{3}\cos x+\sin x}$

$=\frac{2\tan x}{\sqrt{3}+\tan x}$

$DO=1-BD=\frac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x}$

$\therefore \vec{AD}=\frac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x}\vec{a}+\frac{2\tan x}{\sqrt{3}+\tan x}\vec{c}$ //

(2) △ABDに正弦定理

$\frac{AD}{\sin\frac{\pi}{3}}=\frac{BD}{\sin x}$

$\therefore AD=\frac{\sqrt{3}}{2}\cdot\frac{2}{\sqrt{3}\cos x+\sin x}$

$=\frac{\sqrt{3}}{\sqrt{3}\cos x+\sin x}$

$\cos\theta$

$=\frac{\vec{AD}\cdot\vec{AE}}{AD\cdot AE}$

$=\frac{\vec{AD}\cdot\frac{\vec{b}+\vec{c}}{2}}{AD\cdot\frac{\sqrt{3}}{2}}$

$=\frac{\frac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x}(\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c})+\frac{2\tan x}{\sqrt{3}+\tan x}(\vec{b}\cdot\vec{c}+1)}{\frac{3}{\sqrt{3}\cos x+\sin x}}$

$=\frac{(\sqrt{3}\cos x-\sin x)+2\sin x\cdot\frac{3}{2}}{3}$

$=\frac{2}{3}\sin x+\frac{\sqrt{3}}{3}\cos x$

$=\frac{\sqrt{7}}{3}\left(\sin x\cdot\underbrace{\frac{2}{\sqrt{7}}}_{\cos\alpha}+\cos x\cdot\underbrace{\frac{\sqrt{3}}{\sqrt{7}}}_{\sin\alpha}\right)$

$=\frac{\sqrt{7}}{3}\sin(x+\alpha)$

$x=\frac{\pi}{2}-\alpha$ のとき

$\max\cos\theta=\frac{\sqrt{7}}{3}$ //

(3)

△ADE

$=\frac{1}{2}AD\cdot AE\sin\theta$

$=\frac{\sqrt{3}}{4}\cdot\frac{\sqrt{3}}{\sqrt{3}\cos x+\sin x}\sqrt{1-\cos^2\theta}$

$=\frac{3}{4(\sqrt{3}\cos x+\sin x)}\sqrt{1-\left(\frac{3}{9}+\frac{4\sqrt{3}}{9}\sin x\cos x+\frac{1}{9}\sin^2 x\right)}$

$=\frac{3}{4(\sqrt{3}\cos x+\sin x)}\sqrt{\frac{2}{3}-\frac{4\sqrt{3}}{9}\sin x\cos x-\frac{1}{9}\sin^2 x}$

$=\frac{\sqrt{\frac{6}{\cos^2 x}-4\sqrt{3}\tan x-\tan^2 x}}{4(\sqrt{3}+\tan x)}$

$=\frac{\sqrt{5\tan^2 x-4\sqrt{3}\tan x+6}}{4(\sqrt{3}+\tan x)}$

$\sqrt{3}+\tan x = t$ とおくと

$\triangle ADE$

$$= \frac{\sqrt{5(t-\sqrt{3})^2 - 4\sqrt{3}(t-\sqrt{3}) + 6}}{4t}$$

$$= \frac{\sqrt{5t^2 - 14\sqrt{3}t + 33}}{4t}$$

$$= \frac{1}{4}\sqrt{5 - 14\sqrt{3}\cdot\frac{1}{t} + 33\left(\frac{1}{t}\right)^2}$$

$$= \frac{1}{4}\sqrt{33\left(\frac{1}{t} - \frac{7}{33}\sqrt{3}\right)^2 + \frac{6}{11}}$$

$0 \leqq x \leqq \frac{\pi}{3}$ より $\sqrt{3} \leqq t \leqq 2\sqrt{3}$.

$$\frac{1}{t} = \frac{7}{33}\sqrt{3} \iff t = \frac{33}{7\sqrt{3}}$$

$$= \frac{11}{7}\sqrt{3}$$

のとき最小.

つまり

$$\sqrt{3}+\tan x = \frac{11}{7}\sqrt{3}$$

$\iff \tan x = \frac{4}{7}\sqrt{3}$ のとき