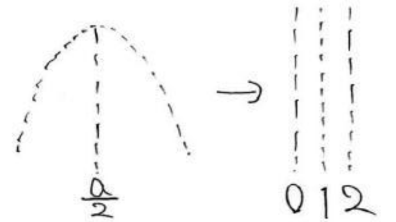
2019 国際医療福祉大(医)

第1問

(A)

$f(x)=-x^2+ax+a^2-6a+9$

$=-\left(x-\frac{a}{2}\right)^2+\frac{5}{4}a^2-6a+9$



(i) $\frac{a}{2}\leqq 1 \iff a\leqq 2$ のとき

$m=f(2)=\underline{a^2-4a+5}$

(ii) $\frac{a}{2}\geqq 1 \iff a\geqq 2$ のとき

$m=f(0)=\underline{a^2-6a+9}$

(B)

$f(x)$

$=\dfrac{\tan x}{\frac{1}{\cos^2 x}}-\sqrt{3}\sin^2 x$

$=\sin x\cos x-\sqrt{3}\,\dfrac{1-\cos 2x}{2}$

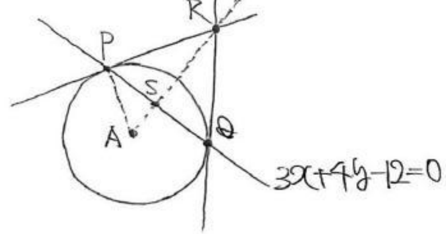
$=\frac{1}{2}\sin 2x+\frac{\sqrt{3}}{2}\cos 2x-\frac{\sqrt{3}}{2}$

$=\sin\left(2x+\frac{\pi}{3}\right)-\frac{\sqrt{3}}{2}$

$\left(\frac{\pi}{3}\leqq 2x+\frac{\pi}{3}\leqq\frac{5}{6}\pi\right)$

$\therefore\ \underline{\dfrac{1-\sqrt{3}}{2}\leqq f(x)\leqq\dfrac{2-\sqrt{3}}{2}}$

(C)



ASは(2,1)とlの距り

$AS=\dfrac{|-2|}{\sqrt{9+16}}=\underline{\dfrac{2}{5}}$

四角形PAQRは向かい合わせの角度の和が180°より円に内接する. 方べきの定理より

$PS\cdot QS=\frac{2}{5}\cdot\left(10-\frac{2}{5}\right)$

$\iff PS^2=\frac{96}{25}$

$\therefore\ PS=\frac{4\sqrt{6}}{5}$

三平方の定理より

$r=\sqrt{\left(\frac{4\sqrt{6}}{5}\right)^2+\left(\frac{2}{5}\right)^2}$

$=\dfrac{\sqrt{96+4}}{5}=\underline{2}$

$PQ=\underline{\dfrac{8\sqrt{6}}{5}}$

(D)

$(x-4)^2-8y^2=8$

$\iff \frac{(x-4)^2}{8}-y^2=1$

Kの焦点 $(4\pm3,0)$

$\downarrow$

$\underline{(1,0),\ (7,0)}$

$x-4=X,\ y=Y$ とおくと

$\frac{X^2}{8}-Y^2=1.$

(0,1)を通る直線は $Y=mX+1$ とおける. 連立すると

$\frac{X^2}{8}-(mX+1)^2=1$

$\iff X^2-8m^2X^2-16mX-16=0$

$\frac{D}{4}=64m^2-(1-8m^2)(-16)$

$=16(1-4m^2)$

$\iff m^2=\frac{1}{4}\iff m=\pm\frac{1}{2}$

$\therefore\ Y=\pm\frac{1}{2}X+1$

$\downarrow x-4=X,\ y=Y$

$y=\pm\frac{1}{2}(x-4)+1$

$\therefore\ \underline{y=\frac{1}{2}x-1}\quad\underline{y=-\frac{1}{2}x+3}$

第2問

(1)

P(すべて異なる)

$=1\cdot\frac{4}{5}\cdot\frac{3}{5}\cdot\frac{2}{5}\cdot\frac{1}{5}=\underline{\dfrac{24}{625}}$

P(数字3種類)

$=P(a\times3,\ b\times1,\ c\times1)$

$+P(a\times2,\ b\times2,\ c\times1)$

$=\dfrac{{}_5C_3\times3\times\frac{5!}{3!}}{5^5}$

$+\dfrac{{}_5C_3\times3\times\frac{5!}{2!2!}}{5^5}$

$=\dfrac{2\times3\times20}{5^4}+\dfrac{2\times3\times30}{5^4}$

$=\dfrac{2\times3\times2}{5^2}=\underline{\dfrac{12}{25}}$

$P_{\text{数3種類}}$(同じ数字連続しない)

$= \frac{25}{12}\{P(abaca)$

$+P(ababc)$

$+P(abacb)$

$+P(abcab)$

$+P(acbab)$

$+P(cabab)\}$

$= \frac{2+2+2+4+2+2}{50} = \underline{\frac{7}{25}}$ //

$P(x_3+x_4=x_5)$

| $x_3, x_4$の組 | $x_5$ |
|---|---|
| (1, 1) | 2 |
| (1, 2) | 3 |
| (1, 3) | 4 |
| (1, 4) | 5 |
| (2, 2) | 4 |
| (2, 3) | 5 |

$= \frac{1\times2+2\times4}{5^3} = \underline{\frac{2}{25}}$ //

(2)

$P(x_1<x_2<x_3)$

$= \frac{{}_5C_3\times1}{5^3} = \underline{\frac{2}{25}}$ //

$P(x_1 \leqq x_2 \leqq x_3 \cap x_3 > x_4)$

$= P(x_1 \sim x_4$で最大値が$2)$

$+P($ 〃 $3)$

$+P($ 〃 $4)$

$+P($ 〃 $5)$

$= \frac{{}_3C_2}{5^4} + \frac{{}_4C_2}{5^4}\times2$

$+ \frac{{}_5C_2}{5^4}\times3 + \frac{{}_6C_2}{5^4}\times4 = \underline{\frac{21}{125}}$ //

1 2
○||○

1 2 3
○||○||

1 2 3 4
||○|| ||○

1 2 3 4 5
||○|| || ||○

$P(x_1+x_2<x_3+x_4)$

$= \{1-P(x_1+x_2=x_3+x_4)\}\frac{1}{2}$

$= \left(1-\frac{1^2+2^2+3^2+4^2+5^2+4^2+3^2+2^2+1^2}{5^4}\right)\frac{1}{2}$

$= \left(1-\frac{85}{625}\right)\frac{1}{2}$

$= \frac{125-17}{125}\cdot\frac{1}{2} = \underline{\frac{54}{125}}$ //

第3問

$P(x_P, 0, 0)$とおくと

$\overrightarrow{OA}\cdot\overrightarrow{PD} = 0 \iff x_P = 7$

$\therefore \underline{P(7,0,0)}$ //

平面OAB: $z=0$ より

$\underline{Q(7,5,0)}$ //

直線OC: $\begin{cases} x=t \\ y=2t \\ z=t \end{cases}$ $(t\in\mathbb{R})$

$R(r, 2r, r)$とおくと

$\overrightarrow{OC}\cdot\overrightarrow{DR} = 0$

$\iff 7-r+2(5-2r)+1-r=0$

$\iff -6r=-18 \quad \therefore r=3$

$\therefore \underline{R(3,6,3)}$ //

$\overrightarrow{PQ} = \begin{pmatrix}0\\5\\0\end{pmatrix}$ $\overrightarrow{PR} = \begin{pmatrix}-4\\6\\3\end{pmatrix}$

$\triangle PQR = \frac{1}{2}\sqrt{|\overrightarrow{PQ}|^2|\overrightarrow{PR}|^2-(\overrightarrow{PQ}\cdot\overrightarrow{PR})^2}$

$= \frac{1}{2}\sqrt{25\cdot61-900}$

$= \underline{\frac{25}{2}}$ //

平面ABC: $z=ax+by+c$とおく.

$\begin{cases} 0=16a+c \\ 0=9a+7b+c \\ 4=4a+8b+c \end{cases}$

$\downarrow$

$0=-7a+7b \iff a=b$

$4=-12a+8b$

$\downarrow$

$a=b=-1,\ c=16$

$z=-x-y+16$

$\iff x+y+z-16=0$

直線DS: $\begin{cases} x=7+u \\ y=5+u \\ z=1+u \end{cases}$ $(u\in\mathbb{R})$

$S(7+s, 5+s, 1+s)$とおくと

平面ABCと連立して

$7+s+5+s+1+s-16=0$

$\therefore s=1$

$S(8,6,2)$

$\overrightarrow{PQ}$と$\overrightarrow{PR}$の両方に垂直なベクトルの1つを

$\begin{pmatrix}1\\ \alpha\\ \beta\end{pmatrix}$

とおくと

$\begin{pmatrix}0\\5\\0\end{pmatrix}\begin{pmatrix}1\\ \alpha\\ \beta\end{pmatrix}=5\alpha=0 \quad \therefore \alpha=0$

$\begin{pmatrix}-4\\6\\3\end{pmatrix}\begin{pmatrix}1\\ \alpha\\ \beta\end{pmatrix}=-4+3\beta=0$

$\therefore \beta=\frac{4}{3}$

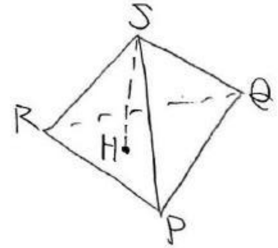
つまり

$\vec{n}=\frac{1}{5}(3,0,4)$

$\overrightarrow{PS}=\begin{pmatrix}1\\6\\2\end{pmatrix}$

$\therefore \overrightarrow{PS}\cdot\vec{n}=\frac{1}{5}(3+8)=\frac{11}{5}$

$\overrightarrow{PS}\cdot\vec{n}=|\overrightarrow{PS}||\cos\angle PSH|=SH$




(四面体PQRS)$=\frac{25}{2}\times\frac{11}{5}\times\frac{1}{3}$

$=\frac{55}{6}$

第4問

(1) $f(x)=x^2e^x$

$f'(x)=2xe^x+x^2e^x$

$=x(2+x)e^x$

$f''(x)=(2+x)e^x+xe^x+x(2+x)e^x$

$=(x^2+4x+2)e^x$

$f'(x)=0 \Leftrightarrow x=-2,0$

$x=-2$で極大値 $(f''(-2)<0)$

$x=0$で極小値 $(f''(0)>0)$

$f''(x)=0 \Leftrightarrow x=-2\pm\sqrt{2}$

(2)

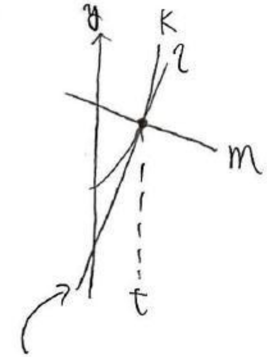
$\int x^2e^x dx$

$=x^2e^x-2xe^x+2e^x+C$

$\boxed{x^2e^x+2xe^x}\ \boxed{-2xe^x-2e^x}\ \boxed{+2e^x}$

$=(x^2-2x+2)e^x+C$

(3)




$y=t(2+t)e^t(x-t)+t^2e^t$

$=t(2+t)e^tx+(-t^3-t^2)e^t$

$m: y=-\frac{1}{t(2+t)e^t}(x-t)+t^2e^t$

$=-\frac{x}{t(2+t)e^t}+\frac{1}{(2+t)e^t}+t^2e^t$

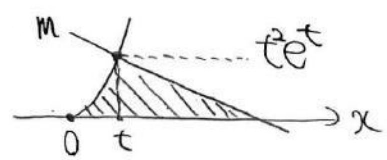
$S_1$

$=\int_0^t\{x^2e^x-t(2+t)e^tx+(t^3+t^2)e^t\}dx$

$=\left[(x^2-2x+2)e^x-\frac{t(2+t)}{2}e^tx^2+(t^3+t^2)e^tx\right]_0^t$

$=(t^2-2t+2)e^t-\frac{t^3(2+t)}{2}e^t+(t^4+t^3)e^t-2$

$=\left(\frac{1}{2}t^4+t^2-2t+2\right)e^t-2$




$S_2$

$=\int_0^t x^2e^xdx$

$+\underbrace{t^2e^t\times t(2+t)e^t}_{底辺}\times t^2e^t\times\frac{1}{2}$

$=(t^2-2t+2)e^t-2+\left(\frac{t^6}{2}+t^5\right)e^{3t}$

$S_1=S_2$

$\Leftrightarrow \frac{1}{2}t^4e^t=\left(\frac{t^6}{2}+t^5\right)e^{3t}$

$\Leftrightarrow \quad 1=(t^2+2t)e^{2t}$

$\therefore (\alpha^2+2\alpha)e^{2\alpha}=1$

(4) $f^{-1}(e)=1$, $f^{-1}(4e^2)=2$

$I=\int_e^{4e^2}f^{-1}(x)dx \quad x=y^2e^y$

$=\int_1^2 y\frac{dx}{dy}dy$

$=\int_1^2(2y^2+y^3)e^ydy$

$=\left[(2y^2+y^3)e^y-(4y+3y^2)e^y+(4+6y)e^y-6e^y\right]_1^2$

$=(16-20+16-6)e^2$

$-\{(3-7+10-6)e\}$

$=6e^2$