2019 北里大(薬)

I.

(1)

$a^2+b^2+c^2=(a+b+c)^2-2(ab+bc+ca)$

$\Leftrightarrow\ 2=0^2-2(ab+bc+ca)$

$\therefore ab+bc+ca=-1 \cdots \underline{2}$

$\underbrace{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}_{3}abc=\underbrace{ab+bc+ca}_{-1}$

$\therefore abc=-\frac{1}{3} \cdots \underline{5}$

$a^3+b^3+c^3$

$=(a^2+b^2+c^2)(a+b+c)$

$\quad -\{(b+c)a^2+\underbrace{(b^2+c^2)}_{(b+c)^2-2bc}a+bc(b+c)\}$

$=-(b+c)\{a^2+(b+c)a+bc\}$

$\quad +2abc$

$=-\underbrace{(b+c)}_{-a}\underbrace{(c+a)}_{-b}\underbrace{(a+b)}_{-c}+2abc$

$=3abc=-1 \cdots \underline{2}$

(2)

剰余の定理より

$f(1)=1$

$f(2)=7$

$f(-1)=3$

$f(x)=(x-1)(x-2)(x+1)Q(x)$

$\quad +ax^2+bx+c$ とおくと

$f(1)=a+b+c=1$

$f(2)=4a+2b+c=7$

$f(-1)=a-b+c=3$

$\downarrow b=-1$

$a+c=2$

$4a+c=9$

$\therefore a=\frac{7}{3}\quad c=-\frac{1}{3}$

$\therefore$ 余り $\frac{7}{3}x^2-x-\frac{1}{3} \cdots \underline{9}$

$g(x)=(x-1)^2Q_1(x)+4$

$\quad =(x-1)^2\{(x-2)^2Q_2(x)+ax+b\}+4$

$\quad =(x-1)^2(x-2)^2Q_2(x)$

$\qquad +(x-1)^2(ax+b)+4$

$g(x)=(x-2)^2Q_3(x)+1$

$\quad =(x-2)^2\{(x-1)^2Q_4(x)+cx+d\}+1$

$\quad =(x-2)^2(x-1)^2Q_4(x)$

$\qquad +(x-2)^2(cx+d)+1$

$Q_2(x)=Q_4(x)$

$(x-1)^2(ax+b)+4=(x-2)^2(cx+d)+1$

$\downarrow$

$$\begin{cases} a=c \\ b+4=4d+1 \\ 2a+b+4=1 \Leftrightarrow b=-2a-3 \\ 4=c+d+1 \Leftrightarrow d=-c+3 \end{cases}$$

$\downarrow$

$b+4=4d+1$

$\Leftrightarrow -2a+1=-4c+13$

$\Leftrightarrow\ 2a=12$

$\therefore a=c=6\quad \therefore d=-3$

余り: $(x-2)^2(6x-3)+1$

$\quad =6x^3-27x^2+36x-11 \cdots \underline{7}$

II

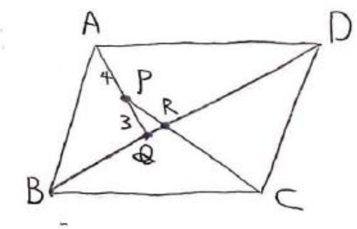
$8\vec{AP}+3(\vec{AP}-\vec{AB})+2(\vec{AP}-\vec{AC})+\vec{AP}-\vec{AD}$

$=14\vec{AP}-3\vec{AB}-2(\vec{AB}+\vec{AD})-\vec{AD}$

$=14\vec{AP}-5\vec{AB}-3\vec{AD}=\vec{0}$

$\therefore \vec{AP}=\frac{5}{14}\vec{AB}+\frac{3}{14}\vec{AD}$

$\underline{9}\qquad \underline{8}$



$\vec{AP}=\frac{8}{14}\cdot\frac{5\vec{AB}+3\vec{AD}}{3+5}$

$\frac{BQ}{BD}=\frac{3}{8} \cdots \underline{7}$

$\vec{CP}=\vec{AP}-(\vec{AB}+\vec{AD})$

$\quad =-\frac{9}{14}\vec{AB}-\frac{11}{14}\vec{AD}$

$\quad =\frac{9}{14}(-\vec{AB})+\frac{11}{14}(-\vec{AD})$

$\quad =\frac{20}{14}\cdot\frac{9\vec{CD}+11\vec{CB}}{11+9}$

$\therefore BR:RD=9:11$

$\therefore \frac{BR}{BD}=\frac{9}{20} \cdots \underline{9}$

以上より $BQ:QR:RD=15:3:22$

$\frac{T}{S}=\frac{1}{2}\cdot\frac{3}{40}\cdot\frac{3}{7}$

$\quad =\frac{9}{560} \cdots \underline{9}$

III

(1) $x=\frac{\pi}{6}$のとき

$\frac{\sqrt{3}}{2}=\sin 2t \quad (0 \leqq 2t \leqq \pi)$

$\therefore 2t=\frac{\pi}{3}, \frac{2}{3}\pi$

$\therefore t=\frac{\pi}{6}, \frac{\pi}{3} \cdots \underline{5}$

(2) $x=t$のとき

$\cos t=\sin 2t$

$\Leftrightarrow 0=\cos t(2\sin t-1)$

$\therefore \cos t=0, \sin t=\frac{1}{2}$

$\therefore t=\frac{\pi}{6}, \frac{\pi}{2} \cdots \underline{7}$

(3)

$\cos x=\sin 2t$

$=\cos\left(\frac{\pi}{2}-2t\right)$

$\therefore x=\pm\left(\frac{\pi}{2}-2t\right)$

$\therefore \pm 2t=-x\pm\frac{\pi}{2}$

$\therefore t=\mp\frac{1}{2}x+\frac{\pi}{4}$

$\therefore t_1=-\frac{1}{2}x+\frac{\pi}{4}$

$\underline{5} \quad \underline{7}$

$t_2=\frac{1}{2}x+\frac{\pi}{4}$

$\cos\left(x-\frac{t_1}{2}-\frac{t_2}{3}\right)$

$=\cos\left(x+\frac{x}{4}-\frac{\pi}{8}-\frac{x}{6}-\frac{\pi}{12}\right)$

$=\cos\left(\frac{13}{12}x-\frac{5}{24}\pi\right)$

$\frac{13}{12}x-\frac{\pi}{24}=0 \Leftrightarrow x=\frac{5}{26}\pi \cdots \underline{9}$ で最小

IV

(1) P(1234になる)

$=\frac{{}_3C_1 \cdot {}_2C_1 \cdot 1 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4}$

$=\frac{1}{140} \cdots \underline{5}$

(2) P(各位の数字異なる)

$=\frac{4! \cdot {}_3C_1 \cdot {}_2C_1 \cdot 1 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4}$

$=\frac{24}{140}=\frac{6}{35} \cdots \underline{5}$

(3)

P(2341以上)

$=P(4\sim)+P(3\sim)$

$+P(24\sim)+P(2342)+P(2341)$

$=\frac{1 \cdot 6 \cdot 5 \cdot 4+1 \cdot 6 \cdot 5 \cdot 4+2 \cdot 1 \cdot 5 \cdot 4+2+2 \cdot 3}{7 \cdot 6 \cdot 5 \cdot 4}$

$=\frac{60+60+20+1+3}{7 \cdot 6 \cdot 5 \cdot 2}$

$=\frac{144}{7 \cdot 6 \cdot 5 \cdot 2}$

$=\frac{12}{35} \cdots \underline{4}$

(4)

P(同じ数字が隣り合わない)

$=P(1212)+P(2121)$

$+P((1,1,2,3))+P((1,1,2,4))$

$+P((1,1,3,4))+P((2,2,1,3))$

$+P((2,2,1,4))+P((2,2,3,4))$

$+P((1,2,3,4))$

$=\frac{3 \cdot 2 \cdot 2}{7 \cdot 6 \cdot 5 \cdot 4}+\frac{3 \cdot 2 \cdot 2}{7 \cdot 6 \cdot 5 \cdot 4}$

$+\frac{3 \cdot 2(4!-2 \cdot 3!)}{7 \cdot 6 \cdot 5 \cdot 4} \times 2$

$+\frac{3 \cdot (4!-2 \cdot 3!)}{7 \cdot 6 \cdot 5 \cdot 4}+\frac{3(4!-2 \cdot 3!)}{7 \cdot 6 \cdot 5 \cdot 4} \times 2$

$+\frac{1 \cdot (4!-2 \cdot 3!)}{7 \cdot 6 \cdot 5 \cdot 4}+\frac{3 \cdot 2 \cdot 4!}{7 \cdot 6 \cdot 5 \cdot 4}$

$=\frac{24}{7 \cdot 6 \cdot 5 \cdot 4}+\frac{22(4!-2 \cdot 3!)}{7 \cdot 6 \cdot 5 \cdot 4}+\frac{3 \cdot 2 \cdot 4!}{7 \cdot 6 \cdot 5 \cdot 4}$

$=\frac{24+22 \cdot 12+6 \cdot 4!}{7 \cdot 6 \cdot 5 \cdot 4}$

$=\frac{1+11+6}{7 \cdot 5}=\frac{18}{35} \cdots \underline{4}$

V $f(x)=x^3-4x^2+5x$

(1) $f'(x)=3x^2-8x+5$

$=(3x-5)(x-1)$

$f'(x)=0 \Leftrightarrow x=\frac{5}{3}, 1$

$x=1$で極大値$2 \cdots \underline{2}$

$x=\frac{5}{3}$で極小値

$\frac{125}{27}-\frac{100}{9}+\frac{25}{3}=\frac{125-300+225}{27}$

$=\frac{50}{27} \cdots \underline{9}$

(2)

$l: y=(3a^2-8a+5)(x-a)+a^3-4a^2+5a$

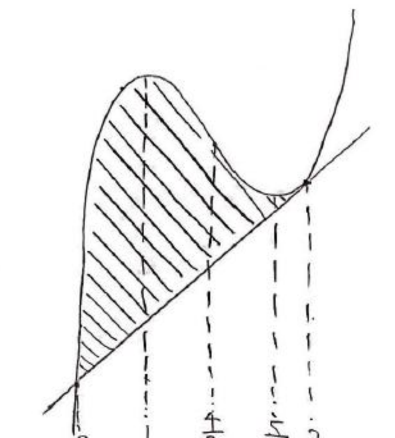
$=(3a^2-8a+5)x-2a^3+4a^2$

↓原点通る

$0=-2a^3+4a^2=2a^2(2-a)$

$\therefore a=2$

$\therefore l: y=x \cdots \underline{1}$

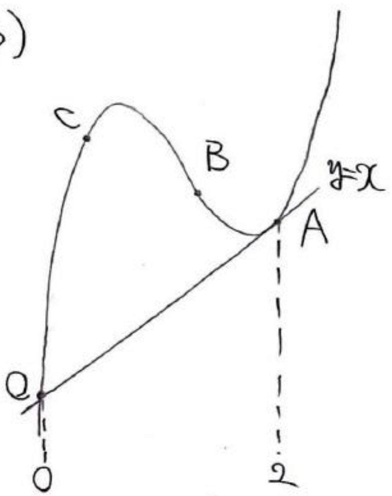


斜線部分の面積は

$$\frac{1}{12}(2-0)^4$$

$$= \frac{4}{3} \cdots 7$$

(3)



$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \overrightarrow{OB} = \begin{pmatrix} b \\ b^3-4b^2+5b \end{pmatrix}$$

$$\overrightarrow{OC} = \begin{pmatrix} \frac{b}{2} \\ \frac{b^3}{8}-b^2+\frac{5}{2}b \end{pmatrix}$$

S

$$= \frac{1}{2}\left|2(b^3-4b^2+5b)-2b\right|$$

$$+\frac{1}{2}\left|\frac{b^4}{8}-b^3+\frac{5}{2}b^2-\frac{b^4}{2}+2b^3-\frac{5}{2}b^2\right|$$

$$= \left|b^3-4b^2+4b\right|$$

$$+\frac{1}{2}\left|-\frac{3}{8}b^4+b^3\right|$$

$$= b\left|(b-2)^2\right|$$

$$+\frac{1}{2}\left|b^3\left(1-\frac{3}{8}b\right)\right|$$

$$= b(b-2)^2+\frac{1}{2}b^3\left(1-\frac{3}{8}b\right)$$

$$= -\frac{3}{16}b^4+\frac{3}{2}b^3-4b^2+4b \cdots 10$$

$$\frac{dS}{db}$$

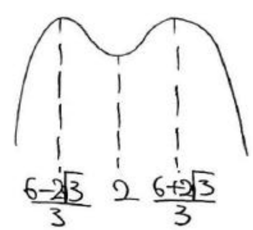
$$= -\frac{3}{4}b^3+\frac{9}{2}b^2-8b+4$$

$$= -\frac{1}{4}(3b^3-18b^2+32b-16)$$

$$= -\frac{1}{4}(b-2)(3b^2-12b+8)$$

$$\frac{dS}{db}=0$$

$$\Leftrightarrow b=2,\ \frac{6\pm\sqrt{12}}{3}$$



Sが最大となるとき $b=\frac{6-2\sqrt{3}}{3} \cdots 10$