2019 慶應義塾大(薬)

[I]

(1)

$$\frac{3}{\sqrt{a^2+2a+1}}=\frac{1}{\sqrt{9a^2-6a+1}}$$

$\Leftrightarrow 3\sqrt{9a^2-6a+1}=\sqrt{a^2+2a+1}$

$\downarrow$ 2乗

$81a^2-54a+9=a^2+2a+1$

$\Leftrightarrow 80a^2-56a+8=0$

$\Leftrightarrow 10a^2-7a+1=0$

$\Leftrightarrow (2a-1)(5a-1)=0$

$\therefore\ \underline{a=\frac{1}{5}}$ // $(\because -1<a<\frac{1}{3})$

(2)

$$\begin{cases} 3^{\frac{1}{3}}<a^{\frac{1}{6}} \xrightarrow{6乗} 9<a \\ a^{\frac{3}{2}}<24\sqrt{3}=2^3\cdot 3^{\frac{3}{2}}=(12)^{\frac{3}{2}} \end{cases}$$

$\therefore\ a<12$

$\therefore\ \underline{a=10,11}$ //

(3)

$b_{n+1}=b_n+n+1$

$n\geqq 2$のとき

$b_n=1+\sum_{k=1}^{n-1}(k+1)$

$=1+\frac{1}{2}(n-1)n+n-1$

$=\frac{1}{2}n^2+\frac{1}{2}n=\frac{n(n+1)}{2}$

これは$n=1$も成立.

$\sum_{k=1}^{n}\frac{1}{b_k}=\sum_{k=1}^{n}\frac{2}{k(k+1)}$

$=2\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=2\left(1-\frac{1}{n+1}\right)$

$=\underline{\frac{2n}{n+1}}$ //

(4)

(i) $3k=15x+24z$より

$14y+3k=266$

$\therefore\ \underline{y=-\frac{3}{14}k+19}$ //

(ii) $k=14,28,42,56,70,84$

ここで

$5x+8z=k$

より $k=14$以外で 実現可能.

$(x,y,z)=(4,13,1),(2,10,4),$

$(8,7,2),(6,4,5)$

$\underline{(12,1,3)(4,1,8)}$ //

(5).

(i) $A(1,7)$

$B(t,-3t+10)$ とおく

$\vec{OA}\cdot\vec{OB}=|\vec{OA}||\vec{OB}|\cos 45^\circ$

$\Leftrightarrow -20t+70=5\sqrt{2}\cdot\sqrt{10t^2-60t+100}\,\frac{1}{\sqrt{2}}$

$\Leftrightarrow -4t+14=\sqrt{10t^2-60t+100}$

$\downarrow$ 2乗

$16t^2-112t+196=10t^2-60t+100$

$\Leftrightarrow 6t^2-52t+96=0$

$\Leftrightarrow 3t^2-26t+48=0$

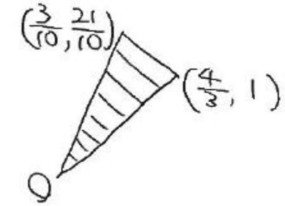
$\Leftrightarrow (3t-8)(t-6)=0$

$-4t+14>0$より $t=\frac{8}{3}$

$\therefore\ \underline{B\left(\frac{8}{3},2\right)}$ //

$\vec{OP}=\frac{10}{3}s\left(\frac{3}{10}\vec{OA}\right)+2t\left(\frac{1}{2}\vec{OB}\right)$

$=\frac{10}{3}s\begin{pmatrix}\frac{3}{10}\\ \frac{21}{10}\end{pmatrix}+2t\begin{pmatrix}\frac{4}{3}\\ 1\end{pmatrix}$



$\frac{10}{3}s+2t\leqq 1$

$S=\frac{1}{2}\left|\frac{3}{10}\cdot 1-\frac{21}{10}\cdot\frac{4}{3}\right|$

$=\frac{1}{2}\cdot\frac{25}{10}=\underline{\frac{5}{4}}$ //

(6) (i)

y

$=\sqrt{3}\sin^2\theta+2\sqrt{3}+\frac{1}{2}\sin 2\theta$

$=\frac{1}{2}\sin 2\theta-\frac{\sqrt{3}}{2}\cos 2\theta+\frac{5}{2}\sqrt{3}$

$\underline{a=\frac{1}{2}\quad b=-\frac{\sqrt{3}}{2}\quad c=\frac{5}{2}\sqrt{3}}$ //

(ii)

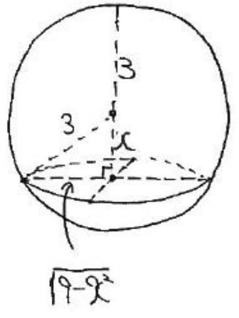
$y=\sin\left(2\theta-\frac{\pi}{3}\right)+\frac{5}{2}\sqrt{3}$

$2\theta-\frac{\pi}{3}=\frac{\pi}{2}\Leftrightarrow \underline{\theta=\frac{5}{12}\pi}$ のとき //

max $\underline{y=1+\frac{5}{2}\sqrt{3}}$ //

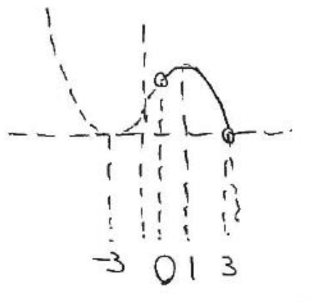
$\underline{\theta=0}$ // のとき min $\underline{y=2\sqrt{3}}$ //

(7)



$$V=\left(2\sqrt{9-x^2}\right)^2\frac{1}{2}\times(3+x)\times\frac{1}{3}$$
$$=\frac{2}{3}(9-x^2)(3+x)$$
$$=\frac{2}{3}\underbrace{(-x^3-3x^2+9x+27)}_{f(x)}$$

$$f'(x)=-3x^2-6x+9$$
$$=-3(x^2+2x-3)$$



$x=1$ のとき $\max V=\frac{64}{3}$

[II]

(1) $\frac{7!}{2!2!}=\underline{1260\text{通り}}$

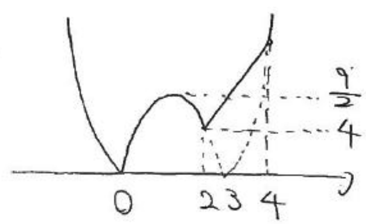
(2)
$C\sim\cdots\frac{6!}{2!}=360$
$EC\sim\cdots 5!=120$
$EECC\sim\cdots 3!=6$
$EECI\sim\cdots 3!=6$
$EECN\sim\cdots 3!=6$
EECSCIN
$\underset{\text{チ}\ \text{ッ}\ \text{デ}}{\text{EECSCNI}}$

(3)
$C\sim\cdots 360$
$E\sim\cdots 360$
$I\sim\cdots\frac{6!}{2!2!}=180$
$N\sim\cdots 180$
$SCC\sim\cdots\frac{4!}{2!}=12$
$SCE\sim\cdots 4!=24$
$SCIC\sim\frac{3!}{2!}=3$
$SCIEC\sim 2!=2$
$SCIEE\sim 2!=2$
SCIENCE $\longleftarrow$ $\underline{1124\text{番目}}$

[III]

(1) $a=4$ のとき

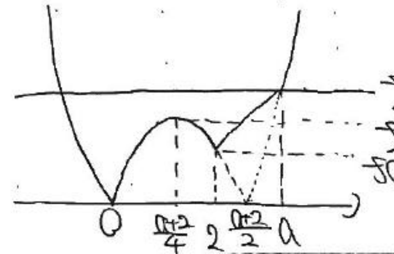
$$f(x)=\begin{cases}2x(x-3) & (x\geqq 4)\\ 2x & (2\leqq x<4)\\ 2x(3-x) & (0\leqq x\leqq 2)\\ 2x(x-3) & (x<0)\end{cases}$$



$y=k$ と異なる4点で交わるとき
$4<k<\frac{9}{2}$.

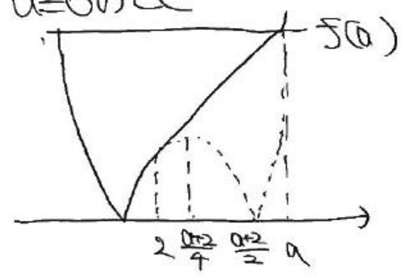
(2)
$$f(x)=\begin{cases}2x\left(x-\frac{a+2}{2}\right) & (x\geqq a)\\ (a-2)x & (2\leqq x<a)\\ 2x\left(\frac{a+2}{2}-x\right) & (0\leqq x<2)\\ 2x\left(x-\frac{a+2}{2}\right) & (x<0)\end{cases}$$

(i) $\frac{a+2}{4}\leqq 2$ つまり $2<a\leqq 6$ のとき



求める範囲は
$$\begin{cases}f(a)>f\left(\frac{a+2}{4}\right)\\ \text{または}\\ 0<f(a)<f(2)\end{cases}$$
$$\Leftrightarrow\begin{cases}a(a-2)>\frac{(a+2)^2}{8}\\ \text{または}\\ 0<a(a-2)<2(a-2)\leftarrow \text{NG}\end{cases}$$
$\downarrow$
$8a^2-16a>a^2+4a+4$
$\Leftrightarrow 7a^2-20a-4>0$
$\therefore\ a>\frac{10+8\sqrt{2}}{7}$
$\therefore\ \frac{10+8\sqrt{2}}{7}<a\leqq 6$

(ii) $a\geqq 6$ のとき



図から必ず2点で交わる. $\therefore\ a\geqq 6$

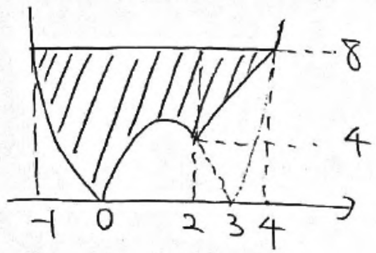
(i)(ii)より

$$a>\frac{10+8\sqrt{2}}{7}$$

(3)

$$\frac{10+8\sqrt{2}}{7} \fallingdotseq \frac{21.2}{7}$$

(2)をみたす最小の自然数は4



$$\int_{-1}^{4}\{8-f(x)\}dx$$

$$=\int_{-1}^{0}\{8-(2x^2-6x)\}dx$$

$$+\int_{0}^{2}\{8-(-2x^2+6x)\}dx$$

$$+2\cdot4\cdot\frac{1}{2}$$

$$=\left[-\frac{2}{3}x^3+3x^2+8x\right]_{-1}^{0}$$

$$+\left[\frac{2}{3}x^3-3x^2+8x\right]_{0}^{2}+4$$

$$=-\left(\frac{2}{3}+3-8\right)$$

$$+\frac{16}{3}-12+16+4$$

$$=-\frac{2}{3}+5+\frac{16}{3}+8$$

$$=\frac{14}{3}+\frac{39}{3}$$

$$=\frac{53}{3}$$