2019 金沢医科大(前期)

1

(1) $a=b=6,\ c=1$ のとき

$\max P=\dfrac{41}{100}$

$a=b=1,\ c=6$ のとき

$\min P=\dfrac{1}{100}$

(2) $P\left(P=\dfrac{19}{50}\right)$

$=P(5a+2b-c=38)$

a	b	c
6	6	4
	5	2

$=\dfrac{2}{6^3}=\dfrac{1}{108}$

(3) P(Pの分母が4)

$=P(5a+2b-c=25)$

a	b	c
5	3	6
	2	4
	1	2

a	b	c
4	5	5
	4	3
	3	1
3	6	2

$=\dfrac{7}{216}$

(4) P(Pの分母が5)

$=P(5a+2b-c=20,40)$

$5a+2b-c=20$ のとき

a	b	c
4	3	6
	2	4
	1	2

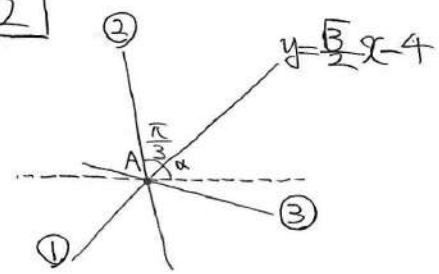
a	b	c
3	5	5
	4	3
	3	1
2	6	2

$5a+2b-c=40$ のとき

a	b	c
6	6	2

$=\dfrac{8}{216}=\dfrac{1}{27}$

2



$\tan\alpha=\dfrac{\sqrt3}{2}$ とする.

$\tan\left(\alpha\pm\dfrac{\pi}{3}\right)=\dfrac{\tan\alpha\pm\tan\frac{\pi}{3}}{1\mp\tan\alpha\tan\frac{\pi}{3}}$

$=\dfrac{\frac{\sqrt3}{2}\pm\sqrt3}{1\mp\frac{\sqrt3}{2}\sqrt3}$

$=\dfrac{\frac{3}{2}\sqrt3}{-\frac12},\ \dfrac{-\frac12\sqrt3}{\frac52}$

$=-3\sqrt3,\ -\dfrac{\sqrt3}{5}$

②は

$y=-3\sqrt3(x-2\sqrt3)-1$

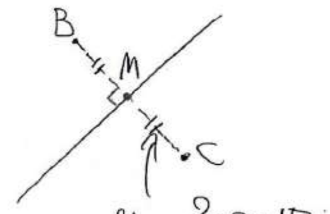
$=-3\sqrt3x+17$

③は

$y=-\dfrac{\sqrt3}{5}(x-2\sqrt3)-1$

$=-\dfrac{\sqrt3}{5}x+\dfrac15$

②とy軸の交点Bは$B(0,17)$



$y=-\dfrac{2}{\sqrt3}x+17$

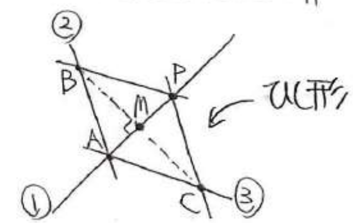
交点Mは

$\dfrac{\sqrt3}{2}x-4=-\dfrac{2}{\sqrt3}x+17$

$\Leftrightarrow 3x-8\sqrt3=-4x+34\sqrt3$

$\Leftrightarrow x=6\sqrt3 \quad M(6\sqrt3,5)$

$\therefore C(12\sqrt3,-7)$



平行四辺形

図より) $P(10\sqrt3,11)$

$AB=\sqrt{12+324}=\sqrt{336}$

(平行四辺形)$=\sqrt{336}\cdot\sqrt{336}\sin\dfrac{\pi}{3}$

$=168\sqrt3$

3

$0.11,\ 0.121,\ 0.1221,\cdots\cdots,a_n,\ a_{n+1},$

$\underset{b_1}{+0.011}\quad \underset{b_2}{+0.0011}\quad b_n$

階差数列をb_nとすると

$b_n=0.011\times\left(\dfrac{1}{10}\right)^{n-1}$

$=11\times\left(\dfrac{1}{10}\right)^{n+2}$

$\therefore a_{n+1}=a_n+\dfrac{11}{10^{n+2}}$

これより$n\geqq2$のとき

$a_n=0.11+\displaystyle\sum_{k=1}^{n-1}\dfrac{11}{10^{k+2}}$

$=\dfrac{11}{100}+\dfrac{\frac{11}{1000}-\frac{11}{10^{n+1}}\cdot\frac{1}{10}}{1-\frac{1}{10}}$

$=\dfrac{11}{100}+\dfrac{\frac{11}{100}-\frac{11}{10^{n+1}}}{9}$

$=\dfrac{11}{100}\left(\dfrac{10}{9}-\dfrac{1}{9\cdot10^{n-1}}\right)$

$=\dfrac{11}{90}\left(1-\dfrac{1}{10^n}\right)$

これは$n=1$を満たす.

$\therefore a_n=\dfrac{11}{90}\left(1-\dfrac{1}{10^n}\right)$

S_n

$= \sum_{k=1}^{n} \left(\frac{11}{90} - \frac{11}{90} \cdot \frac{1}{10^k} \right)$

$= \frac{11}{90} n - \frac{11}{90} \cdot \frac{\frac{1}{10} - \frac{1}{10^{n+1}}}{1 - \frac{1}{10}}$

$= \frac{11}{90} n - \frac{11}{90} \cdot \frac{1 - \frac{1}{10^n}}{9}$

$= \frac{11}{90} \left(n - \frac{1}{9} + \frac{1}{9 \cdot 10^n} \right) > 3$

$\Leftrightarrow n - \frac{1}{9} + \frac{1}{9 \cdot 10^n} > \frac{270}{11}$

$\Leftrightarrow n - \frac{1}{9} \left(1 - \frac{1}{10^n} \right) > 24 + \frac{6}{11}$

$\therefore \min n = \underline{25}$

$\boxed{4}$

$f(x) = ax + \sqrt{b - x^2}$

$f'(x) = a + \frac{1}{2}(b - x^2)^{-\frac{1}{2}}(-2x)$

$= a - \frac{x}{\sqrt{b - x^2}}$

必要条件 ↓

$\begin{cases} f\left(\frac{3}{2}\right) = \frac{3}{2}a + \sqrt{b - \frac{9}{4}} = 2\sqrt{3} \\ f'\left(\frac{3}{2}\right) = a - \frac{\frac{3}{2}}{\sqrt{b - \frac{9}{4}}} = 0 \end{cases}$

$a = \frac{\frac{3}{2}}{\sqrt{b - \frac{9}{4}}} \Leftrightarrow \sqrt{b - \frac{9}{4}} = \frac{\frac{3}{2}}{a}$

↓

$\frac{3}{2}a + \frac{\frac{3}{2}}{a} = 2\sqrt{3}$

$\Leftrightarrow 3a^2 + 3 = 4\sqrt{3}a$

$\Leftrightarrow 3a^2 - 4\sqrt{3}a + 3 = 0$

$\Leftrightarrow (\sqrt{3}a - 1)(\sqrt{3}a - 3) = 0$

$\Leftrightarrow a = \frac{1}{\sqrt{3}}, \frac{3}{\sqrt{3}}$

$a = \frac{1}{\sqrt{3}}$ のとき

$\sqrt{b - \frac{9}{4}} = \frac{3\sqrt{3}}{2}$

$b - \frac{9}{4} = \frac{27}{4} \quad \therefore b = 9$

$a = \sqrt{3}$ のとき

$\sqrt{b - \frac{9}{4}} = \frac{\sqrt{3}}{2}$

$b - \frac{9}{4} = \frac{3}{4} \quad \therefore b = 3$

ここで $b = 3$ とすると $f(x) = \sqrt{3}x + \sqrt{3 - x^2}$

より 定義域が $3 - x^2 \geqq 0 \Leftrightarrow -\sqrt{3} \leqq x \leqq \sqrt{3}$

このとき $x = -\sqrt{3}$ で最小をとる.

$f(-\sqrt{3}) = -3$

となり最小値が $-\sqrt{3}$ とならない.

$b = 9$ とすると $f(x) = \frac{1}{\sqrt{3}}x + \sqrt{9 - x^2}$

より 定義域が $9 - x^2 \geqq 0 \Leftrightarrow -3 \leqq x \leqq 3$.

$x = -3$ で最小値 $\frac{1}{\sqrt{3}}(-3) = -\sqrt{3}$

$\therefore a = \frac{1}{\sqrt{3}} = \underline{\frac{\sqrt{3}}{3}}, \ b = \underline{9}, \ k = \underline{-3}$

$f(x) = \frac{1}{\sqrt{3}}x + \sqrt{9 - x^2}$ より

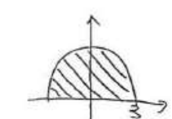
$M(-3, -\sqrt{3}), N(3, \sqrt{3})$

これを通る直線は $y = \underline{\frac{\sqrt{3}}{3}x}$

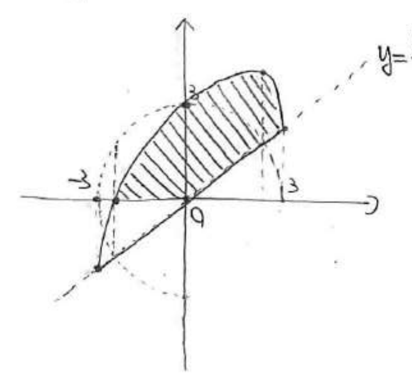
これと $y = f(x)$ で囲まれた面積は

$\int_{-3}^{3} \left(f(x) - \frac{\sqrt{3}}{3}x \right) dx$

$= \int_{-3}^{3} \sqrt{9 - x^2}\, dx$



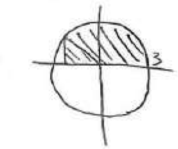
$= \underline{\frac{9}{2}\pi}$



求める面積は斜線部分.

$\int_{-\frac{3\sqrt{3}}{2}}^{0} f(x)\,dx + \int_{0}^{3} \sqrt{9 - x^2}\, dx$

$= \int_{-\frac{3\sqrt{3}}{2}}^{0} \frac{1}{\sqrt{3}} x\,dx + \int_{-\frac{3\sqrt{3}}{2}}^{3} \sqrt{9 - x^2}\, dx$

$= \left[\frac{1}{2\sqrt{3}} x^2 \right]_{-\frac{3\sqrt{3}}{2}}^{0} +$ 

$= -\frac{1}{2\sqrt{3}} \cdot \frac{27}{4} + 9\pi \times \frac{150}{360}$

$+ \frac{3\sqrt{3}}{2} \times \frac{3}{2} \times \frac{1}{2}$

$= \underline{\frac{15}{4}\pi}$