# 2019 岩手医科大（医）

## 第1問

問1.

P(○が1つもない)

$= P$(1回目の操作で○が0回)

$+P($　　〃　　1回$)$

$+P($　　〃　　2回$)$

$= \left(\frac{2}{3}\right)^3$

$+{}_3C_1\frac{1}{3}\cdot\left(\frac{2}{3}\right)^2\times\left(\frac{2}{3}\right)^3$

$+{}_3C_2\left(\frac{1}{3}\right)^2\frac{2}{3}\times\left(\frac{1}{3}\right)^3$

$= \left(\frac{2}{3}\right)^3+\left(\frac{2}{3}\right)^5+\frac{2}{3^5}$

$= \frac{72+32+2}{243} = \underline{\frac{106}{243}}$ //

P(すべて○)

$= P$(1回目の操作で○が1回)

$+P($　　〃　　2回$)$

$+P($　　〃　　3回$)$

$= {}_3C_1\frac{1}{3}\left(\frac{2}{3}\right)^2\times\left(\frac{1}{3}\right)^3$

$+{}_3C_2\left(\frac{1}{3}\right)^2\frac{2}{3}\times\left(\frac{2}{3}\right)^3$

$+\left(\frac{1}{3}\right)^3$

$= \frac{4}{3^5}+\frac{16}{3^5}+\frac{9}{3^5}$

$= \underline{\frac{29}{243}}$ //

問2

P(○が2コ→○がなし)

$= \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

P(○が2コ→○が2コ)

$= {}_3C_1\frac{1}{3}\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

P(○が2コ→○が4コ)

$= {}_3C_2\left(\frac{1}{3}\right)^2\frac{2}{3} = \frac{2}{9}$

P(○が2コ→○が6コ)

$= \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

P(○が4コ→○がなし)

$= \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

P(○が4コ→○が2コ)

$= {}_3C_1\frac{2}{3}\left(\frac{1}{3}\right)^2 = \frac{2}{9}$

P(○が4コ→○が4コ)

$= {}_3C_2\left(\frac{2}{3}\right)^2\frac{1}{3} = \frac{4}{9}$

P(○が4コ→○が6コ)

$= \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$p_{n+1} = \underline{\frac{4}{9}}p_n + \underline{\frac{2}{9}}q_n$

$q_{n+1} = \underline{\frac{2}{9}}p_n + \underline{\frac{4}{9}}q_n$

問3 $p_1 = \frac{4}{9}$　$q_1 = \frac{2}{9}$ である.

$p_{n+1}+q_{n+1} = \frac{2}{3}(p_n+q_n)$

$p_{n+1}-q_{n+1} = \frac{2}{9}(p_n-q_n)$

↓

$p_n+q_n = (p_1+q_1)\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^n$

$p_n-q_n = (p_1-q_1)\left(\frac{2}{9}\right)^{n-1} = \left(\frac{2}{9}\right)^n$

$\therefore\ \underline{p_n = \frac{1}{2}\left\{\left(\frac{2}{3}\right)^n+\left(\frac{2}{9}\right)^n\right\}}$ //

$\underline{q_n = \frac{1}{2}\left\{\left(\frac{2}{3}\right)^n-\left(\frac{2}{9}\right)^n\right\}}$ //

問4

$r_{n+1} = r_n+\frac{1}{27}p_n+\frac{8}{27}q_n$

$= r_n+\frac{9}{54}\left(\frac{2}{3}\right)^n-\frac{7}{54}\left(\frac{2}{9}\right)^n$

$n\geqq 2$のとき

$r_n$

$= r_1+\sum_{k=1}^{n-1}\frac{1}{6}\left(\frac{2}{3}\right)^k-\sum_{k=1}^{n-1}\frac{7}{54}\left(\frac{2}{9}\right)^k$

$= \frac{1}{27}+\frac{1}{6}\frac{\frac{2}{3}-\left(\frac{2}{3}\right)^n}{1-\frac{2}{3}}-\frac{7}{54}\frac{\frac{2}{9}-\left(\frac{2}{9}\right)^n}{1-\frac{2}{9}}$

$= \frac{1}{27}+\frac{1}{2}\left\{\frac{2}{3}-\left(\frac{2}{3}\right)^n\right\}$

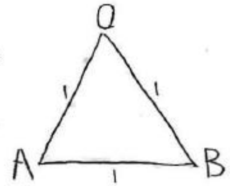
$-\frac{1}{6}\left\{\frac{2}{9}-\left(\frac{2}{9}\right)^n\right\}$

$= \underline{\frac{1}{3}-\frac{1}{2}\left(\frac{2}{3}\right)^n+\frac{1}{6}\left(\frac{2}{9}\right)^n}$ //

これは$n=1$のときも成立.

## 第2問

問1.

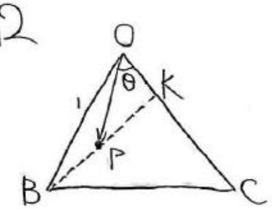



$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 60^\circ$

$= \underline{\frac{1}{2}}$ //

$\triangle OAB = 1\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \underline{\frac{\sqrt{3}}{4}}$ //

問2




$\vec{OP}\cdot\vec{c} = |\vec{OP}|\cdot\cos\theta\cdot|\vec{c}|$

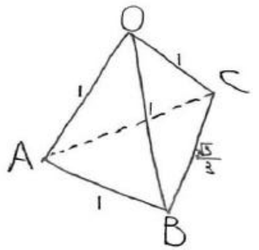
$= |\vec{OK}||\vec{c}|$

(余)

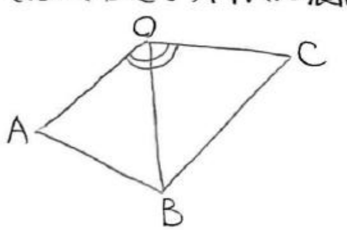
$\cos\angle BOC = \dfrac{1^2+1^2-\frac{4}{3}}{2\cdot 1\cdot 1} = \dfrac{1}{3}$

より $\overrightarrow{OP}\cdot\vec{c} = \dfrac{1}{3}\cdot 1 = \dfrac{1}{3}$ //

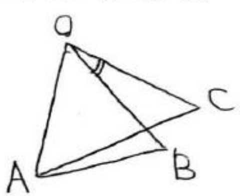
問3




$\cos\angle AOC$の下限は展開して




$\cos\angle AOC$の上限は




以上より

$\cos\left(\dfrac{\pi}{3}+\angle BOC\right) < \cos\angle AOC$

$< \cos\left(\angle BOC - \dfrac{\pi}{3}\right)$

$\Leftrightarrow \cos\dfrac{\pi}{3}\cos\angle BOC - \sin\dfrac{\pi}{3}\sin\angle BOC$

$< \cos\angle AOC < \cos\angle BOC\cos\dfrac{\pi}{3} + \sin\angle BOC\sin\dfrac{\pi}{3}$

$\Leftrightarrow \dfrac{1}{6} - \dfrac{\sqrt{3}}{2}\cdot\dfrac{2\sqrt{2}}{3} < \cos\angle BOC$

$< \dfrac{1}{6} + \dfrac{\sqrt{3}}{2}\cdot\dfrac{2\sqrt{2}}{3}$

$\therefore \dfrac{1-2\sqrt{6}}{6} < \vec{c}\cdot\vec{a} < \dfrac{1+2\sqrt{6}}{6}$ //

問4 体

四面体OABCが最大なのは

平面OAB⊥平面OBCのとき.

CからOBに下ろした垂線の足を

Hとすると

$\overrightarrow{CH} = \overrightarrow{OH} - \overrightarrow{OC}$

$= \dfrac{1}{3}\vec{b} - \vec{c}$ $\left(\because \cos\angle BOC = \dfrac{1}{3}\right)$

これが$\vec{a}$と垂直より

$\left(\dfrac{1}{3}\vec{b} - \vec{c}\right)\cdot\vec{a} = \dfrac{1}{6} - \vec{c}\cdot\vec{a} = 0$

$\therefore \vec{c}\cdot\vec{a} = \dfrac{1}{6}$ //

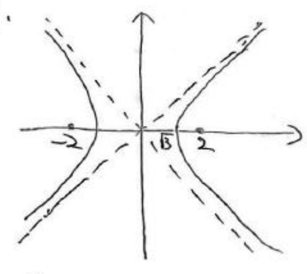
このとき体積は

$\triangle OAB \times CH \times \dfrac{1}{3}$

$= \dfrac{\sqrt{3}}{4}\cdot\dfrac{2\sqrt{2}}{3}\cdot\dfrac{1}{3} = \dfrac{\sqrt{6}}{18}$ //

# 第3問

問1.




$\dfrac{x^2}{3} - y^2 = 1$を原点を中心に

45°回転したものがC.

$\dfrac{x^2}{3} - y^2 = 1$上の$(x, y)$は

C上の$(X, Y)$を$-45°$回転

したものとすると

$x + yi = (X+Yi)\{\cos(-45°) + i\sin(-45°)\}$

$= (X+Yi)\left(\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}i\right)$

$= \dfrac{X+Y}{\sqrt{2}} + \dfrac{-X+Y}{\sqrt{2}}i$

$\therefore \begin{cases} x = \dfrac{X+Y}{\sqrt{2}} \\ y = \dfrac{-X+Y}{\sqrt{2}} \end{cases}$

$\therefore \dfrac{1}{3}\cdot\dfrac{(X+Y)^2}{2} - \dfrac{(-X+Y)^2}{2} = 1$

$\Leftrightarrow (X+Y)^2 - 3(-X+Y)^2 = 6$

$\Leftrightarrow -2X^2 + 8XY - 2Y^2 = 6$

$\Leftrightarrow X^2 - 4XY + Y^2 = -3$

$\therefore C: x^2 - 4xy + y^2 = -3$ //

問2

$l$とCの交点のx座標は

$x^2 - 4x(2\sqrt{6}-x) + (2\sqrt{6}-x)^2 = -3$

$\Leftrightarrow 6x^2 - 12\sqrt{6}x + 27 = 0$

$\Leftrightarrow 2x^2 - 4\sqrt{6}x + 9 = 0$

$\Leftrightarrow (\sqrt{2}x - \sqrt{3})(\sqrt{2}x - 3\sqrt{3}) = 0$

$\Leftrightarrow x = \dfrac{\sqrt{3}}{\sqrt{2}}, \dfrac{3\sqrt{3}}{\sqrt{2}}$

$\therefore A\left(\dfrac{3\sqrt{6}}{2}, \dfrac{\sqrt{6}}{2}\right)$ //

これを原点を中心に$\dfrac{\pi}{4}$回転させると

$\left(\dfrac{3\sqrt{6}}{2} + \dfrac{\sqrt{6}}{2}i\right)\left(\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i\right)$

$= \dfrac{3\sqrt{3}}{2} - \dfrac{\sqrt{3}}{2} + \dfrac{3\sqrt{3}}{2}i + \dfrac{\sqrt{3}}{2}i$

$= \sqrt{3} + 2\sqrt{3}i$

$\therefore (\sqrt{3}, 2\sqrt{3})$ //

問3

$\dfrac{1}{(1-x^2)^2}$

$= \dfrac{1}{(1+x)^2(1-x)^2}$

$= \left\{\dfrac{1}{(1-x)(1+x)}\right\}^2$

$$= \left\{ \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1+x} \right\}^2$$

$$= \frac{\frac{1}{4}}{(1-x)^2} + \frac{\frac{1}{2}}{(1-x)(1+x)} + \frac{\frac{1}{4}}{(1+x)^2}$$

$$= \frac{\frac{1}{4}}{(1-x)^2} + \frac{1}{2}\left\{ \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1+x} \right\} + \frac{\frac{1}{4}}{(1+x)^2}$$

$$= \frac{1}{4}\left\{ \frac{1}{(1-x)^2} + \frac{1}{1-x} + \frac{1}{1+x} + \frac{1}{(1+x)^2} \right\}$$

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{(1-x^2)^2}\,dx$$

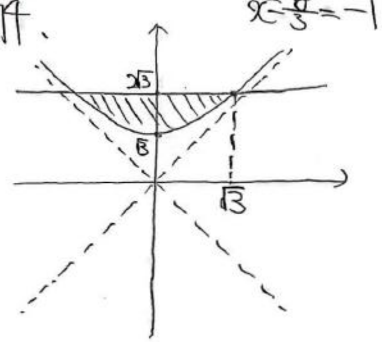
$$= \frac{1}{4}\Big[ -\frac{1}{x-1} - \log|1-x| + \log|1+x| - \frac{1}{1+x} \Big]_0^{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{4}\Big[ -\frac{2x}{x^2-1} + \log\left|\frac{x+1}{x-1}\right| \Big]_0^{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{4}\left( \frac{\sqrt{3}}{\frac{1}{4}} + \log\left|\frac{\sqrt{3}+2}{\sqrt{3}-2}\right| \right)$$

$$= \underline{\sqrt{3} + \frac{1}{2}\log(2+\sqrt{3})} \;/\!/$$

図1、




$C$ と $l$ を原点を中心に45°回転したものが上の図。上の斜線部分の面積を求めればよい。

$$x^2 - \frac{y^2}{3} = -1$$

$$\Leftrightarrow y^2 = 3(x^2+1)$$

$\therefore$ $y = \sqrt{3(x^2+1)}$ を使う。

求める面積を $S$ とおくと

$$S$$

$$= 2\sqrt{3}\cdot 2\sqrt{3} - 2\int_0^{\sqrt{3}} \sqrt{3(x^2+1)}\,dx$$

$$= 12 - 2\sqrt{3}\int_0^{\sqrt{3}} \sqrt{x^2+1}\,dx$$

$$= 12 - 2\sqrt{3}\int_0^{\frac{\pi}{3}} \sqrt{\tan^2\theta+1}\,\frac{1}{\cos^2\theta}\,d\theta$$

$$= 12 - 2\sqrt{3}\int_0^{\frac{\pi}{3}} \frac{1}{\cos^3\theta}\,d\theta$$

ちなみに

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{(1-x^2)^2}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{(1-\sin^2\theta)^2}\cos\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{\cos^3\theta}\,d\theta$$

なので

$$S$$

$$= 12 - 2\sqrt{3}\left(\sqrt{3} + \frac{1}{2}\log(2+\sqrt{3})\right)$$

$$= \underline{6 - \sqrt{3}\log(2+\sqrt{3})} \;/\!/$$