2019 センター数学II・B

第1問

[1]

(1) $f(0)=-1$

$f\left(\frac{\pi}{3}\right)=3\cdot\frac{3}{4}+4\cdot\frac{\sqrt{3}}{4}-\frac{1}{4}$

$=2+\sqrt{3}$

(2)

$\cos^2\theta=\frac{\cos 2\theta+1}{2}$

$f(\theta)=3\cdot\frac{1-\cos 2\theta}{2}+2\sin 2\theta-\frac{\cos 2\theta+1}{2}$

$=2\sin 2\theta-2\cos 2\theta+1$

$=2\sqrt{2}\sin\left(2\theta-\frac{\pi}{4}\right)+1$

$0\leqq\theta\leqq\pi \Leftrightarrow -\frac{\pi}{4}\leqq 2\theta-\frac{\pi}{4}\leqq\frac{7}{4}\pi$

$2\theta-\frac{\pi}{4}=\frac{\pi}{4},\ \frac{3}{4}\pi$

で $M=3$ このとき

$\theta=\frac{\pi}{4},\ \frac{\pi}{2}$

[2]

真数条件

$\begin{cases}x+2>0\\ y+3>0\end{cases}\Leftrightarrow\begin{cases}x>-2\\ y>-3\end{cases}$ ②

$\log_4(y+3)=\frac{\log_2(y+3)}{2}$

より

$\log_2(x+2)-\log_2(y+3)=-1$

$\Leftrightarrow \log_2\frac{x+2}{y+3}=\log_2\frac{1}{2}$

$\Leftrightarrow \cdots \Leftrightarrow y+3=2(x+2)$

$\therefore y=2x+1$

$\left(\frac{1}{3}\right)^{2x+1}-11\left(\frac{1}{3}\right)^{x+1}+6=0$　$t=\left(\frac{1}{3}\right)^x$

$\Leftrightarrow \frac{1}{3}t^2-\frac{11}{3}t+6=0$

$\Leftrightarrow t^2-11t+18=0\cdots$⑤

$x>-2$ より

$0<t<9$

この範囲だと⑤は $t=2$

$\Leftrightarrow \frac{1}{3^x}=2$

$\Leftrightarrow 3^x=\frac{1}{2}$　$\boxed{2\log_3\frac{1}{2}+\log_3 3}$

$\therefore x=\log_3\frac{1}{2}$　$y=\log_3\frac{3}{4}$

第2問

(1) $f'(-1)=0$ である.

$f'(x)=3x^2+2px+q$

$f'(-1)=3-2p+q=0$

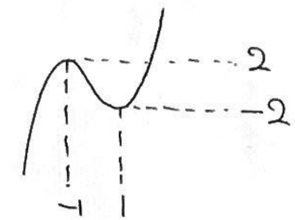
$f(-1)=-1+p-q=2$

$2-p=2\quad\therefore p=0$

$q=-3$

$f(x)=x^3-3x$

$f'(x)=3(x+1)(x-1)$



$x=1$ で極小値 -2

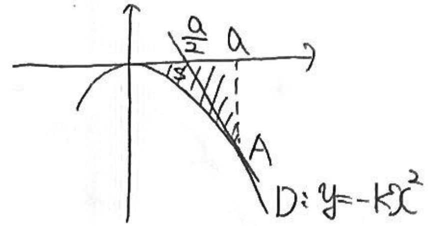
(2) $A(a,-ka^2)$ での接線 l は

$l:\ y=-2ka(x-a)-ka^2$

$=-2kax+ka^2$

$=ka(a-2x)$

x軸との交点は $x=\frac{a}{2}$



$\int_0^a kx^2dx$

$=\left[\frac{k}{3}x^3\right]_0^a=\frac{k}{3}a^3$

$S=\frac{k}{3}a^3-\frac{a}{2}\cdot ka^2\cdot\frac{1}{2}$

$=\frac{k}{12}a^3$

(3)

AがC上にあると

$-ka^2=a^3-3a$

$\Leftrightarrow k=\frac{3}{a}-a$

$x=b$ での C の接線は l より

$l:\ y=(3b^2-3)(x-b)+b^3-3b$

$=3(b^2-1)x-2b^3=g(x)$

$f(x)-g(x)$

$=x^3-3x-\{3(b^2-1)x-2b^3\}$

$=(x-b)^2(x+2b)$

$\boxed{x=bで接する}$

①,②より

$-2ka=3(b^2-1)$

$\downarrow k=\frac{3}{a}-a,\ a=-2b$

$-2\left(\frac{3}{a}-a\right)a=3\left(\frac{a^2}{4}-1\right)$

$\Leftrightarrow -6+2a^2=\frac{3}{4}a^2-3$

$\Leftrightarrow \frac{5}{4}a^2=3$

$\Leftrightarrow a^2=\frac{12}{5}$

$S=\frac{1}{12}\left(\frac{3}{a}-a\right)a^3$

$=\frac{1}{4}a^2-\frac{1}{12}a^4$

$=\frac{3}{5}-\frac{12}{25}=\frac{3}{25}$

第3問

(1) $S_n=\frac{3-3\cdot4^{n-1}\times4}{1-4}$

$=4^n-1$ ∴ $S_2=15$

$T_n=-1+\sum_{k=1}^{n-1}(4^k-1)$

$=-1+\frac{4-4^n}{1-4}-(n-1)$

$=\frac{1}{3}4^n-n-\frac{4}{3}$

$T_2=2$

(2) $S_n=4^n-1$ オ…①

$T_n=\frac{4^n}{3}-n-\frac{4}{3}$ ク…①

(3) $b_1=\frac{a_1+2T_1}{1}=\frac{-3-2}{1}=-5$

$T_{n+1}=4\cdot\frac{1}{3}4^n-n-1-\frac{4}{3}$

$=4\left(T_n+n+\frac{4}{3}\right)-n-\frac{7}{3}$

$=4T_n+3n+3$

$b_{n+1}=\frac{a_{n+1}+2T_{n+1}}{n+1}$

$=\frac{4(n+1)a_n+8T_n+2n(4T_n+3n+3)}{n(n+1)}$

$=\frac{4(n+1)a_n+8(n+1)T_n}{n(n+1)}+\frac{2n(3n+3)}{n(n+1)}$

$=4b_n+6$

$b_n=\alpha\cdot4^{n-1}-2$ とおく

$\downarrow b_1=-5$ より $\alpha=-3$

∴ $b_n=-3\cdot4^{n-1}-2$ タ…②

$nb_n=a_n+2T_n$

$\Leftrightarrow a_n=n(-3\cdot4^{n-1}-2)-2\left(\frac{4^n}{3}-n-\frac{4}{3}\right)$

$=\frac{-9n\cdot4^{n-1}-8\cdot4^{n-1}+8}{3}$

$=\frac{-(9n+8)4^{n-1}+8}{3}$

第4問

(1) $\vec{a}\cdot\vec{c}=0 \Leftrightarrow \angle AOC=90^\circ$

$\triangle OAC=\frac{\sqrt{5}}{2}$

(2) $\overrightarrow{BA}\cdot\overrightarrow{BC}$

$=(\vec{a}-\vec{b})\cdot(\vec{c}-\vec{b})$

$=-1-3+3=-1$

$|\overrightarrow{BA}|^2=|\vec{a}-\vec{b}|^2$

$=|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$

$=2$ ∴ $|\overrightarrow{BA}|=\sqrt{2}$

$|\overrightarrow{BC}|^2=|\vec{c}-\vec{b}|^2$

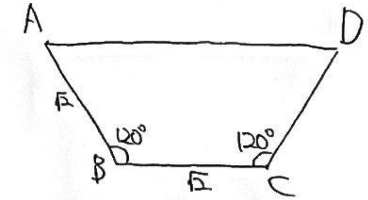
$=|\vec{c}|^2-2\vec{c}\cdot\vec{b}+|\vec{b}|^2$

$=5-6+3$ ∴ $|\overrightarrow{BC}|=\sqrt{2}$

$\cos\angle ABC=\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$

$=\frac{-1}{2}$

∴ $\angle ABC=120^\circ$



$\angle BAD=\angle ADC=60^\circ$

図より $\overrightarrow{AD}=2\overrightarrow{BC}$

$\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{AD}$

$=\vec{a}+2\overrightarrow{BC}$

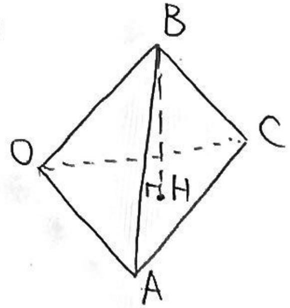
$=\vec{a}-2\vec{b}+2\vec{c}$

(四角形ABCD)

$=(\sqrt{2}+2\sqrt{2})\times\frac{\sqrt{3}}{2}\sqrt{2}\times\frac{1}{2}$

$=\frac{6\sqrt{3}}{4}=\frac{3\sqrt{3}}{2}$

(3)



$\overrightarrow{BH}\cdot\vec{a}=0,\ \overrightarrow{BH}\cdot\vec{c}=0$ より

↓

$$\begin{cases}(s\vec{a}+t\vec{c}-\vec{b})\cdot\vec{a}=0\\(s\vec{a}+t\vec{c}-\vec{b})\cdot\vec{c}=0\end{cases}$$

$$\Leftrightarrow\begin{cases}s-1=0\\5t-3=0\end{cases}\quad\therefore\ s=1,\ t=\frac{3}{5}$$

$$|\overrightarrow{BH}|^2=\left|\vec{a}+\frac{3}{5}\vec{c}-\vec{b}\right|^2$$

$$=1+\frac{9}{25}5+3-\frac{6}{5}3-2$$

$$=2+\frac{9}{5}-\frac{18}{5}=\frac{1}{5}$$

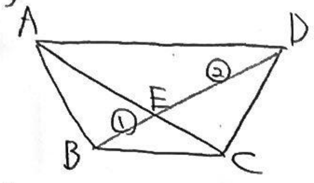
$$\therefore\ |\overrightarrow{BH}|=\frac{\sqrt{5}}{5}$$

$$\therefore\ V=\triangle OAC\times|\overrightarrow{BH}|\times\frac{1}{3}$$

$$=\frac{\sqrt{5}}{2}\times\frac{\sqrt{5}}{5}\times\frac{1}{3}$$

$$=\frac{1}{6}$$

(4)



V：四角錐OABCD＝1：3

↓

$3V$

最後の高さをhとおくと

$$\frac{3\sqrt{3}}{2}\times h\times\frac{1}{3}=\frac{1}{2}$$

$$\therefore\ h=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

第5問

(1)

$$\underset{25}{V(X)}=E(X^2)-\underset{49}{E(X)^2}$$

$$\Leftrightarrow E(X^2)=74$$

$$E(W)=E(1000X)$$

$$=1000E(X)$$

$$=-7\times10^3$$

$$V(W)=V(1000X)$$

$$=10^6V(X)$$

$$=5^2\times10^6$$

(2)

$$P(X\geqq0)=P\left(\frac{X+7}{5}\geqq1.4\right)$$

$$=P(Z\geqq1.4)$$

$$=0.5-0.4192$$

$$=0.0808$$

$$\fallingdotseq0.08$$

MはB(50, 0.08)に従うので

$$E(M)=50\times0.08=4.0$$

$$\sigma(M)=\sqrt{50\times0.08\times0.92}$$

$$=\sqrt{3.68}\fallingdotseq\sqrt{3.7}$$

(3)

$$\sigma(\bar{Y})=\sqrt{V(\bar{Y})}$$

$$=\sqrt{\frac{V(Y)}{n}}=\sqrt{\frac{36}{100}}$$

$$=0.6$$

$Z=\dfrac{\bar{Y}-m}{0.6}$ は $N(0,1)$ に従う。

$$P(|Z|\leqq1.64)$$

$$=0.4495\times2$$

$$=0.899\fallingdotseq0.90$$

mの90%信頼区間は

$$\left|\frac{\bar{Y}-m}{0.6}\right|\leqq1.64$$

$$\Leftrightarrow-1.64\leqq\frac{\bar{Y}-m}{0.6}\leqq1.64$$

$$\Leftrightarrow-0.984\leqq\bar{Y}-m\leqq0.984$$

$$\Leftrightarrow-0.984\leqq m-\bar{Y}\leqq0.984$$

$$\Leftrightarrow\underset{-10.2}{\bar{Y}}-0.98\leqq m\leqq\underset{-10.2}{\bar{Y}}+0.98$$

$$\Leftrightarrow-11.2\leqq m\leqq-9.2$$

より…②