2018 東邦大（医）

1

$P(1人勝ち)$

$= \dfrac{3\times 4}{3^4}$ 誰が？

グチチチ
チパパパ
パググ

$= \dfrac{4}{27}$ //

$P(2人勝ち)$

$= \dfrac{3\times {}_4C_2}{3^4} = \dfrac{2}{9}$ //

2

$x=2$ を代入

$16+8a+40-24+b=0$

$\Leftrightarrow b=-8a-32$

$x^4+ax^3+10x^2-12x-8a-32=0$

$$\begin{array}{ccccc|c} 1 & a & 10 & -12 & -8a-32 & \\ & 2 & 2a+4 & 4a+28 & & 2 \\ \hline 1 & a+2 & 2a+14 & 4a+16 & & \end{array}$$

$(x-2)\{x^3+(a+2)x^2+(2a+14)x+4a+16\}$
$=0$

右のカッコに $x=2$ を代入すると

$8+4a+8+4a+28+4a+16=0$

$\Leftrightarrow 12a+60=0 \quad \therefore a=-5$ //

すなわち

$(x-2)(x^3-3x^2+4x-4)=0$

$\Leftrightarrow (x-2)^2(x^2-x+2)=0$

虚数解は $x=\dfrac{1\pm\sqrt{7}i}{2}$ //

3

$(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta=\dfrac{1}{3}$

$\therefore \sin\theta\cos\theta=-\dfrac{1}{3}$

$\left(\sin\theta-\dfrac{1}{\sin\theta}\right)\left(\cos\theta-\dfrac{1}{\cos\theta}\right)$

$=-\dfrac{1}{3}-\dfrac{\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\sin\theta}+\dfrac{1}{\sin\theta\cos\theta}$

$=-\dfrac{1}{3}-\dfrac{1}{\sin\theta\cos\theta}+\dfrac{1}{\sin\theta\cos\theta}$

$=-\dfrac{1}{3}=\dfrac{-1}{3}$ //

$\left(\sin\theta+\dfrac{1}{\sin\theta}\right)\left(\cos\theta+\dfrac{1}{\cos\theta}\right)$

$=-\dfrac{1}{3}+\dfrac{2}{\sin\theta\cos\theta}$

$=-\dfrac{1}{3}-6$

$=-\dfrac{19}{3}$

$\left(\sin\theta-\dfrac{1}{\sin\theta}\right)\left(\cos\theta-\dfrac{1}{\cos\theta}\right)$

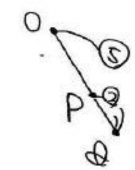
$=\dfrac{-1}{3}\cdot\dfrac{-19}{3}$

$=\dfrac{19}{9}$ //

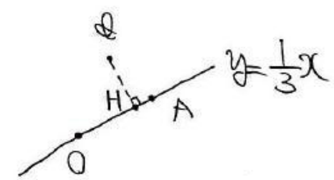
4

$\overrightarrow{OP}=\dfrac{1\cdot\overrightarrow{OA}+9\overrightarrow{OB}}{9+1}$

$=\begin{pmatrix}\frac{15}{10}\\ \frac{11}{10}\end{pmatrix}$



$\overrightarrow{OQ}=\dfrac{5}{3}\overrightarrow{OP}=\begin{pmatrix}\frac{5}{2}\\ \frac{11}{6}\end{pmatrix}$ //



$H(6t, 2t)$ とおくと

$\overrightarrow{QH}\cdot\overrightarrow{OA}=\begin{pmatrix}6t-\frac{5}{2}\\ 2t-\frac{11}{6}\end{pmatrix}\cdot\begin{pmatrix}3\\ 1\end{pmatrix}2=0$

$\Leftrightarrow 20t-\dfrac{56}{6}=0$

$\therefore t=\dfrac{7}{15}$

$\therefore \overrightarrow{OH}=\dfrac{7}{15}\overrightarrow{OA}$ //

5

$\displaystyle\sum_{n=5}^{10} 104(n)$

$=\displaystyle\sum_{n=5}^{10}(1\cdot n^2+0\cdot n+4)$

$=\displaystyle\sum_{n=1}^{10}(n^2+4)-\sum_{n=1}^{4}(n^2+4)$

$=\dfrac{1}{6}\cdot 10\cdot 11\cdot 21-\dfrac{1}{6}4\cdot 5\cdot 9+24$

$=\dfrac{2310-180}{6}+\dfrac{144}{6}$

$=\dfrac{2274}{6}=379$ //

$\displaystyle\sum_{n=5}^{10}\dfrac{1(n)}{401(n)-104(n)}$

$=\displaystyle\sum_{n=5}^{10}\dfrac{1}{4n^2+1-(n^2+4)}$

$=\dfrac{1}{3}\displaystyle\sum_{n=5}^{10}\dfrac{1}{(n-1)(n+1)}$

$=\dfrac{1}{6}\displaystyle\sum_{n=5}^{10}\left(\dfrac{1}{n-1}-\dfrac{1}{n+1}\right)$

$=\dfrac{1}{6}\left(\dfrac{1}{4}+\dfrac{1}{5}-\dfrac{1}{10}-\dfrac{1}{11}\right)$

$=\dfrac{1}{6}\cdot\dfrac{55+44-22-20}{220}$

$=\dfrac{19}{440}$ //

$\boxed{6}$

$$y=a\left(x-\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c$$

$$\Downarrow$$

$$\begin{cases}\frac{3}{2}<\frac{b}{2a}<2\\ 1<-\frac{b^2}{4a}+c<2\end{cases}$$

$$3a<b<4a$$

$$\Downarrow$$

$$a=2,\ b=7$$

$$\Downarrow$$

$$1<-\frac{49}{8}+c<2$$

$$\Leftrightarrow \frac{57}{8}<c<\frac{65}{8}$$

$$\therefore c=8$$

$$\therefore (a,b,c)=(2,7,8)$$

$$y=2\left(x-\frac{7}{4}\right)^2+\frac{15}{8}$$

$$\Leftrightarrow \left(x-\frac{7}{4}\right)^2=\frac{1}{2}\left(y-\frac{15}{8}\right)$$

$$=4\cdot\frac{1}{8}\left(y-\frac{15}{8}\right)$$

$$\Downarrow$$

$$X^2=4\cdot\frac{1}{8}\cdot Y$$

準線 $Y=-\frac{1}{8}$

$$\therefore y=\frac{14}{8}=\frac{7}{4}$$

$\boxed{7}$

$$\begin{cases}x^2+(y-b)^2=1 \\ y=x^2\end{cases}$$ とする

$\Downarrow$ 連立

$$y+y^2-2by+b^2=1$$

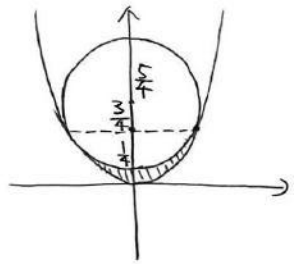
$$\Leftrightarrow y^2+(1-2b)y+b^2-1=0$$

$$D=(1-2b)^2-4(b^2-1)$$

$$=-4b+5=0$$

$$\therefore b=\frac{5}{4}$$

$$y^2-\frac{3}{2}y+\frac{9}{16}=0$$



求める体積は

$$\int_0^{\frac{3}{4}} x^2\pi\,dy-\int_{\frac{1}{4}}^{\frac{3}{4}} x^2\pi\,dy$$

$$=\int_0^{\frac{3}{4}} y\pi\,dy-\int_{\frac{1}{4}}^{\frac{3}{4}}\{1-(y-b)^2\}\pi\,dy$$

$$=\pi\left[\frac{1}{2}y^2\right]_0^{\frac{3}{4}}-\pi\left[y-\frac{1}{3}(y-b)^3\right]_{\frac{1}{4}}^{\frac{3}{4}}$$

$$=\frac{9}{32}\pi-\pi\left(\frac{3}{4}+\frac{1}{24}-\frac{1}{4}-\frac{1}{3}\right)$$

$$=\frac{9}{32}\pi-\frac{5}{24}\pi=\frac{7}{96}\pi$$

$\boxed{8}$

$$x\bar{z}+\bar{x}z=6 \qquad x\bar{x}=1,\ z\bar{z}=25$$

$$\Leftrightarrow \frac{25x}{z}+\frac{z}{x}=6$$

$$\Leftrightarrow 25\left(\frac{x}{z}\right)^2-6\frac{x}{z}+1=0$$

$$\Leftrightarrow \frac{x}{z}=\frac{3\pm4i}{25}$$

$$\Leftrightarrow x=\frac{3\pm4i}{25}z$$

同様に

$$y\bar{z}+\bar{y}z=16 \qquad y\bar{y}=4,\ z\bar{z}=25$$

$$\Leftrightarrow \frac{25y}{z}+\frac{4z}{y}=16$$

$$\Leftrightarrow 25\left(\frac{y}{z}\right)^2-16\frac{y}{z}+4=0$$

$$\Leftrightarrow \frac{y}{z}=\frac{8\pm6i}{25}$$

$$\Leftrightarrow y=\frac{8\pm6i}{25}z$$

$$x-y=\left(\frac{3\pm4i}{25}-\frac{8\pm6i}{25}\right)z$$

$$=\frac{-5\mp2i}{25}z$$ のとき

$$\therefore |x-y|=\left|\frac{-5\mp2i}{25}\right||z|$$

$$=\frac{\sqrt{29}}{5}$$

$$x-y=\left(\frac{3\pm4i}{25}-\frac{8\mp6i}{25}\right)z$$

$$=\frac{-5\pm10i}{25}z$$

$$=\frac{-1\pm2i}{5}z$$ のとき

$$\therefore |x-y|=\left|\frac{-1\pm2i}{5}\right||z|$$

$$=\sqrt{5}$$

$|x-y|=\sqrt{5}$ のとき

$$\frac{z-x}{y-x}=\frac{z-\frac{3\pm4i}{25}z}{\frac{1\mp2i}{5}z}$$

$$=\frac{\frac{22\mp4i}{25}}{\frac{1\mp2i}{5}}$$

$$=\frac{1}{5}\cdot\frac{22\mp4i}{1\mp2i}$$

$$=\frac{1}{5}\cdot\frac{(22\mp4i)(1\pm2i)}{5}$$

$$=\frac{1}{25}(22\pm44i\mp4i-8i^2)$$

$$=\frac{1}{25}(30\pm40i)$$

$$=\frac{6\pm8i}{5}=2\cdot\frac{3\pm4i}{5}$$

$$\therefore \cos\theta=\frac{3}{5}$$

$\boxed{9}$

$f(xy)-f(x)-f(y)$

$=xy-x-y$

$\downarrow x=y=2$

$f(4)-2f(2)=0$

$\therefore f(4)=\underline{6}$ //

$x=4,\ y=2$

$f(8)-f(4)-f(2)=2$

$\therefore f(8)=\underline{11}$ //

$x \to x+2,\ y \to x-2$

$f(x^2-4)-f(x+2)-f(x-2)$

$=x^2-4-(x+2)-(x-2)$

$\Leftrightarrow f(x^2-4)-x^2+2x+4$

$=f(x+2)+f(x-2)$

$f(x+2)+f(x-2)+(x-5)(x+3)$

$=f(x^2-4)-11=0$

$\Leftrightarrow f(x^2-4)=11$

$f(x)$は単調増加なので

11をとる値は8しかない。

$x^2-4=8$

$\therefore x=\underline{2\sqrt{3}}$ // $(x>0)$

$\boxed{10}$

$\tan(\alpha+\beta+\gamma)$

$=\dfrac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}$

$=\dfrac{-\frac{9}{19}+3}{1-\frac{5+4}{1-5\cdot4}3}$

$=\dfrac{-9+57}{19+9\cdot3}$

$=\dfrac{48}{46}=\underline{\dfrac{24}{23}}$ //

$\tan(\alpha+\beta+\gamma)$

$=\dfrac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}=1$

$\Leftrightarrow \tan(\alpha+\beta)+\tan\gamma=1-\tan(\alpha+\beta)\tan\gamma$

$\Leftrightarrow \{\tan(\alpha+\beta)+1\}(\tan\gamma+1)=2$

$1\leqq\tan\gamma\leqq7$で場合分け.

$\tan\alpha=x,\ \tan\beta=y$とおく.

(i) $\tan\gamma=1$のとき

$\tan(\alpha+\beta)=\dfrac{x+y}{1-xy}=0$ NG

(ii) $\tan\gamma=2$のとき

$\dfrac{x+y}{1-xy}=-\dfrac{1}{3}$

$\Leftrightarrow 3x+3y=xy-1$

$\Leftrightarrow xy-3x-3y-1=0$

$\Leftrightarrow (x-3)(y-3)=10$

$(x-3,\ y-3)=(5,2)$

$\therefore x=8,\ y=5$

$\therefore (\tan\alpha,\tan\beta,\tan\gamma)=\underline{(8,5,2)}$ //

記述式ならこの先の場合分けも必要.
あくまで解答欄にそった天下り的な
解答です.