2018 昭和大(医) II期

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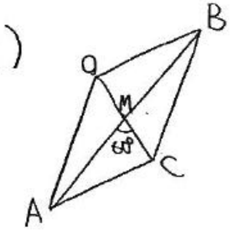
(1)



$\overrightarrow{OC}=\vec{a}+\vec{b}$, $\overrightarrow{BA}=\vec{a}-\vec{b}$

↓

$\overrightarrow{OC}\cdot\overrightarrow{BA}=|\overrightarrow{OC}||\overrightarrow{BA}|\cos 60°$

$\Leftrightarrow |\vec{a}|^2-|\vec{b}|^2=1$

また△AMCに余弦定理

$AC^2=1^2+\left(\frac{1}{2}\right)^2-2\cdot1\cdot\frac{1}{2}\cos 60°$

$\Leftrightarrow |\vec{b}|^2=\frac{3}{4}$ $\quad |\vec{a}|^2=\frac{7}{4}$

$\therefore |\vec{a}|=\frac{\sqrt{7}}{2}$ $\quad |\vec{b}|=\frac{\sqrt{3}}{2}$

$|\overrightarrow{OC}|^2=|\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2=1$

$\Leftrightarrow 2\vec{a}\cdot\vec{b}+\frac{10}{4}=1$

$\therefore \vec{a}\cdot\vec{b}=-\frac{3}{4}$

(2)

$$|\vec{a}+s\vec{b}|^2=|\vec{a}|^2+2s\vec{a}\cdot\vec{b}+s^2|\vec{b}|^2$$
$$=\frac{7}{4}-\frac{3s}{2}+\frac{3}{4}s^2$$
$$=\frac{3}{4}(s^2-2s)+\frac{7}{4}$$
$$=\frac{3}{4}(s-1)^2+1$$

$s=1$のとき最小. このとき

$$(\vec{a}+s\vec{b})\cdot\overrightarrow{BA}$$
$$=(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$$
$$=|\vec{a}|^2-|\vec{b}|^2=1$$

(3)

すべての実数tに対し

$|t\vec{a}+k\vec{b}|^2-|\vec{a}|^2>0$ である実数k

の範囲を考える.

$$|t\vec{a}+k\vec{b}|^2-|\vec{a}|^2$$
$$=\frac{7}{4}t^2-\frac{3}{2}kt+\frac{3}{4}k^2-\frac{7}{4}=f(t)$$

$f(t)=0$で$D<0$であればいい.

$$D=\frac{9}{4}k^2-4\cdot\frac{7}{4}\left(\frac{3}{4}k^2-\frac{7}{4}\right)$$
$$=-3k^2+\frac{49}{4}<0$$

$\Leftrightarrow k^2>\frac{49}{12}$

$\therefore k<-\frac{7\sqrt{3}}{6},\ \frac{7\sqrt{3}}{6}<k$

(4)

(平行四辺形OACB)

$$=\triangle OAB\times 2$$
$$=\frac{1}{2}\sqrt{|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2}\cdot 2$$
$$=\sqrt{\frac{7}{4}\cdot\frac{3}{4}-\frac{9}{16}}$$
$$=\sqrt{\frac{12}{16}}=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$$

2

(1)

$$\overline{\left(\frac{\alpha\beta}{\overline{\alpha\beta}}\right)}$$
$$=\frac{\overline{\alpha\beta}}{\alpha\beta}$$

(2)

$$\alpha=\sqrt{2}\,\frac{1}{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i}$$
$$=\sqrt{2}\,\frac{1}{\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)}$$
$$=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$\beta=\frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}-\frac{1}{2}i}$$
$$=\frac{2\sqrt{3}}{\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)}$$
$$=2\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

$$\frac{\alpha}{\beta}=\frac{2}{1-i}\cdot\frac{\sqrt{3}-i}{4\sqrt{3}}\times\frac{1+i}{1+i}$$
$$=\frac{\sqrt{3}+1+(\sqrt{3}-1)i}{4\sqrt{3}}$$
$$=\frac{\sqrt{2}}{2\sqrt{3}}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}+\frac{\sqrt{3}-1}{2\sqrt{2}}i\right)$$
$$=\frac{\sqrt{6}}{6}\left(\underbrace{\frac{\sqrt{6}+\sqrt{2}}{4}}_{\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}+\underbrace{\frac{\sqrt{6}-\sqrt{2}}{4}}_{\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}i\right)$$

$$\begin{cases}\cos\frac{\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4}\\ \sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}\end{cases}$$

$$\alpha\beta=(1+i)\sqrt{3}(\sqrt{3}+i)$$
$$=\sqrt{3}\left(\sqrt{3}-1+(\sqrt{3}+1)i\right)$$
$$=2\sqrt{6}\left\{\frac{3-\sqrt{3}}{2\sqrt{6}}+\frac{3+\sqrt{3}}{2\sqrt{6}}i\right\}$$
$$=2\sqrt{6}\left(\underbrace{\frac{\sqrt{6}-\sqrt{2}}{4}}_{\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}+\underbrace{\frac{\sqrt{6}+\sqrt{2}}{4}}_{\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}i\right)$$

$$\begin{cases}\cos\frac{5}{12}\pi=\frac{\sqrt{6}-\sqrt{2}}{4}\\ \sin\frac{5}{12}\pi=\frac{\sqrt{6}+\sqrt{2}}{4}\end{cases}$$

(3)

$$\left[\overline{\left(\frac{\alpha\beta}{\overline{\alpha\beta}}\right)}\right]^9$$

$$=\left(\frac{\overline{\alpha\beta}}{\alpha\overline{\beta}}\right)^9$$

$$=\left\{\left(\overline{\frac{\alpha}{\beta}}\right)\cdot\frac{\beta}{\alpha}\right\}^9$$

$$=\left\{\frac{1}{6}\left(\frac{\beta}{\alpha}\right)^2\right\}^9$$

$$\boxed{\begin{aligned}\frac{\alpha}{\beta}\cdot\left(\overline{\frac{\alpha}{\beta}}\right)&=\left|\frac{\alpha}{\beta}\right|^2\\&=\frac{1}{6}\\ \therefore\ \overline{\left(\frac{\alpha}{\beta}\right)}&=\frac{\beta}{6\alpha}\end{aligned}}$$

$$=\left\{\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right\}^9$$

$$=\cos\left(-\frac{3}{2}\pi\right)+i\sin\left(-\frac{3}{2}\pi\right)$$

$$=\underline{i}$$

$\boxed{3}$

(1)

$$\begin{cases}\frac{1}{2}2^x+\frac{1}{3}3^y=19\\ 4\cdot2^x-9\cdot3^y=47\end{cases}$$

↓

$$4\cdot2^x+\frac{8}{3}\cdot3^y=152$$
$$-)\ 4\cdot2^x-9\cdot3^y=47$$
$$\frac{35}{3}\cdot3^y=105$$

$\therefore 3^y=9 \quad \therefore y=2$

$\frac{1}{2}\cdot2^x=16 \quad \underline{\therefore x=5}$

(2)

$$(a+2b)(2b+3c)(3c+a)+6abc$$
$$=(2b+3c)(a^2+(2b+3c)a+6bc)+6abc$$
$$=(2b+3c)a^2+\{(2b+3c)^2+6bc\}a+(2b+3c)6bc$$
$$=\{(2b+3c)a+6bc\}\{a+(2b+3c)\}$$
$$=(2ab+6bc+3ca)(a+2b+3c)$$

(3)

P(1人がグーで勝つ)

+P(〃 チョキ 〃)

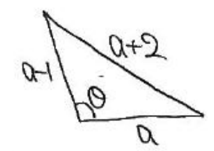
+P(〃 パー 〃)

$$=5\cdot\frac{1}{3^5}\times3$$
$$=\underline{\frac{5}{81}}$$

(4) 三角形の成立条件

$$\begin{cases}a-1+a>a+2 \iff a>3\\ a+a+2>a-1 \iff a>-3\\ a+2+a-1>a \iff a>-1\\ a-1>0 \iff a>1\end{cases}$$

$\therefore a>3$



余弦定理

$$\cos\theta=\frac{a^2+(a-1)^2-(a+2)^2}{2a(a-1)}<0$$

$\iff a^2-6a-3<0$

$\therefore \underline{3<a<3+2\sqrt{3}}$

(5)

$$S_n=\frac{1}{2}\cdot1\cdot1\cdot\sin\frac{2\pi}{n}$$

$$S_{1009}=\frac{1}{2}\sin\frac{2\pi}{1009}=\frac{1}{2}\sin\theta$$

$$S_{2018}=\frac{1}{2}\sin\frac{2\pi}{2018}=\frac{1}{2}\sin\frac{\theta}{2}$$

とおく

$$S_{2018}=\frac{1}{2}\sqrt{\sin^2\frac{\theta}{2}}$$
$$=\frac{1}{2}\sqrt{\frac{1-\cos\theta}{2}}$$
$$=\frac{1}{2}\sqrt{\frac{1-\sqrt{1-\sin^2\theta}}{2}}$$
$$=\frac{1}{2}\sqrt{\frac{1-\sqrt{1-4S_{1009}^2}}{2}}$$

$\boxed{4}$

(1)

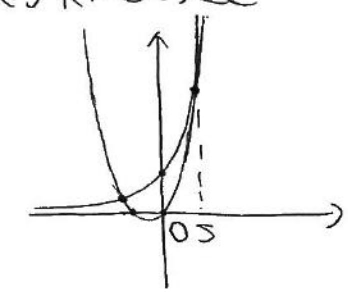
$$\int_1^e x(\log x)^2dx$$
$$=\left[\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{x^2}{4}\right]_1^e$$

$\boxed{x(\log x)^2+x\log x}\ \boxed{-x\log x-\frac{x}{2}}\ \boxed{+\frac{x}{2}}$

$$=\frac{e^2}{2}-\frac{e^2}{2}+\frac{e^2}{4}-\frac{1}{4}$$
$$=\frac{1}{4}(e^2-1)$$

(2)

(i) $k>0$ のとき



$x=s\ (s>0)$ で接するとすると

$$\begin{cases}傾き一致:\ 3e^{3s}=6ks+2k\\ y座標一致:\ e^{3s}=3ks^2+2ks\end{cases}$$

↓

$$6ks+2k=9ks^2+6ks$$

$\therefore s^2=\frac{2}{9} \quad \therefore s=\frac{\sqrt{2}}{3}$

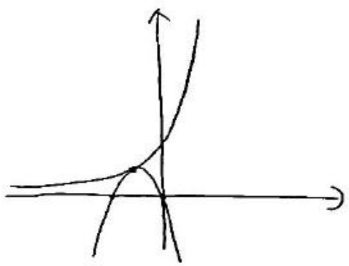
$3e^{\sqrt{2}}=(2\sqrt{2}+2)k$

$\therefore k=\dfrac{3e^{\sqrt{2}}}{2(\sqrt{2}+1)}=\dfrac{3(\sqrt{2}-1)e^{\sqrt{2}}}{2}$

kがこれ以上だと共有点を2つもつ.

$\therefore 0<k<\dfrac{3(\sqrt{2}-1)e^{\sqrt{2}}}{2}$

(ii) $k<0$のとき



(i)と同様に考えて, $x=s\ (s<0)$ で接するとすると $s=-\dfrac{\sqrt{2}}{3}$

このとき

$3e^{-\sqrt{2}}=(-2\sqrt{2}+2)k$

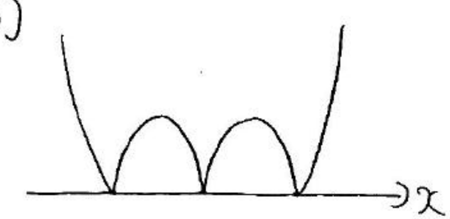
$\therefore k=\dfrac{3e^{-\sqrt{2}}}{2(1-\sqrt{2})}=-\dfrac{3(1+\sqrt{2})e^{-\sqrt{2}}}{2}$

これより大きいと共有しないし, 小さいと2点共有する.

(i)(ii)より

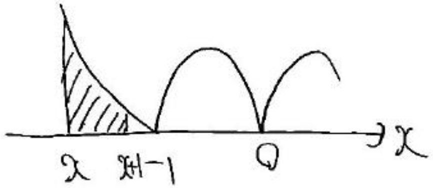
$k=-\dfrac{3(1+\sqrt{2})e^{-\sqrt{2}}}{2},\ 0<k<\dfrac{3(\sqrt{2}-1)e^{\sqrt{2}}}{2}$

(3)



$|t(t^2-1)|$は偶関数より $x<0$を調べる.

(i) $x\leqq -2$のとき



$f(x)=\int_x^{x+1}(-t^3+t)dt$

$f'(x)=-(x+1)^3+x+1-(-x^3+x)$

$=-3x^2-3x=-3x(x+1)<0$

(ii) $-2\leqq x\leqq -1$のとき

$f(x)=\int_x^{-1}(-t^3+t)dt$

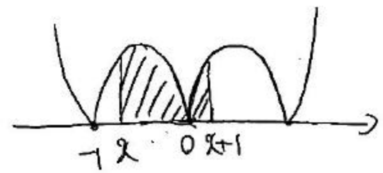
$+\int_{-1}^{x+1}(t^3-t)dt$

$f'(x)=x^3-x+(x+1)^3-(x+1)$

$=2x^3+3x^2+x$

$=x(2x+1)(x+1)\leqq 0$

(iii) $-1\leqq x\leqq 0$ のとき



$f(x)=\int_x^{0}(t^3-t)dt+\int_0^{x+1}(-t^3+t)dt$

$f'(x)=-x^3+x-(x+1)^3+x+1$

$=-2x^3-3x^2-x$

$=-x(x+1)(2x+1)$

x	$\cdots$	-1	$\cdots$	$-\frac{1}{2}$	$\cdots$
$f'(x)$	$-$	0	$-$	0	$-$
$f(x)$	↘		↘		↗

$\min f(x)=f(-\frac{1}{2})$ または $f(\frac{1}{2})$

$f(-\frac{1}{2})$

$=\int_{-\frac{1}{2}}^{0}(t^3-t)dt+\int_0^{\frac{1}{2}}(-t^3+t)dt$

$=2\left[-\frac{1}{4}t^4+\frac{1}{2}t^2\right]_0^{\frac{1}{2}}$

$=2\left(-\frac{1}{64}+\frac{1}{8}\right)$

$=\dfrac{7}{32}$