2018 昭和大(医) I期

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(1)

$$\cos\angle AOB = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{4}{2\cdot 4} = \frac{1}{2}$$

$$\therefore \angle AOB = \underline{\frac{\pi}{3}}$$

(2)

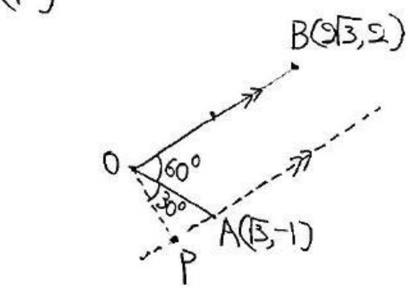
$$|\vec{P}|^2 = (\sqrt{3}+2\sqrt{3}t)^2 + (-1+2t)^2 = 16t^2+8t+4 = 16\left(t+\frac{1}{4}\right)^2+3$$

$\underline{t=-\frac{1}{4}}$ のとき $\min|\vec{P}| = \underline{\sqrt{3}}$

(3)

$$\vec{OP}\cdot\vec{AP} = \vec{OP}\cdot(\vec{OP}-\vec{OA}) = 16t^2+8t+4-\begin{pmatrix}\sqrt{3}+2\sqrt{3}t\\ -1+2t\end{pmatrix}\cdot\begin{pmatrix}\sqrt{3}\\ -1\end{pmatrix} = 16t^2+8t+4-(4t+4) = \underline{16t^2+4t}$$

(4)



$t=-\frac{1}{4}$ のとき P は図の位置にある.

$\triangle OAB : \triangle OAP = 4:1$

$\triangle OAB$ は $\triangle OAP$ の $\underline{4倍}$

2

(1)

$$z^4 = 16\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 16\left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right)$$

$$= 16\left\{\cos\left(\frac{2}{3}\pi+2k\pi\right)+i\sin\left(\frac{2}{3}\pi+2k\pi\right)\right\} \quad (k\in\mathbb{Z})$$

$$z = 2\left\{\cos\left(\frac{\pi}{6}+\frac{k}{2}\pi\right)+i\sin\left(\frac{\pi}{6}+\frac{k}{2}\pi\right)\right\} \quad (k=0,1,2,3)$$

$\mathrm{Re}\,z$ が最大なのは $k=0$ のとき

$$z = 2\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right) = \underline{\sqrt{3}+i}$$

(2)

$$z^p = 2^p\left(\cos\frac{\pi p}{6}+i\sin\frac{\pi p}{6}\right)$$

$$(1-i)^q = (\sqrt{2})^q\left\{\cos\left(-\frac{\pi q}{4}\right)+i\sin\left(-\frac{\pi q}{4}\right)\right\}$$

$z^p=(1-i)^q$ より $p=\frac{q}{2} \Leftrightarrow q=2p$ が必要.

$$\frac{\pi p}{6} = -\frac{\pi(2p)}{4}+2t\pi$$

または

$$-\frac{\pi p}{6} = -\frac{\pi(2p)}{4}+2t\pi$$

$$\Leftrightarrow \begin{cases} \dfrac{8p}{12} = 2t & \cdots\cdots ① \\ \text{または} \\ \dfrac{p}{3} = 2t \end{cases}$$

自然数 p が一番小さいのは ① で $t=1 \Leftrightarrow p=3$ のとき, つまり

$$(p, q) = \underline{(3, 6)}$$

3

(1) 以下100を法とする合同式.

$$2018^{2018} \equiv 18^{2018} \equiv 324^{1009} \equiv 24^{1009} \equiv 24\cdot 576^{504} \equiv 24\cdot(-24)^{504} \equiv 24\cdot 576^{252} \equiv 24\cdot(-24)^{252} \equiv 24\cdot 576^{126} \equiv 24\cdot 576^{63} \equiv 24\cdot(-24)^{63} \equiv -(-24)^{64} \equiv -576^{32} \equiv \cdots \equiv -576 \equiv 24$$

(2)

$${}_9C_2(3x^3)^2\left(\frac{1}{x}\right)^7 = 36\cdot 9\frac{1}{x} = \underline{324\frac{1}{x}}$$

(3) $3x+y=k$ とおく.

$$\log_3 x + \log_3 y = \log_3 xy = \log_3 9 \Leftrightarrow xy = 9$$

$$k = 3x+\frac{9}{x} \geq 2\sqrt{3x\cdot\frac{9}{x}} = 6\sqrt{3}$$

(相加平均) $\geq$ (相乗平均)

$3x=\frac{9}{x} \Leftrightarrow x^2=3 \quad \therefore x=\sqrt{3}$ のとき

$$\min k = \underline{6\sqrt{3}}$$

(4) 小さい順にソート

23, 31, 35, 37, 49, 57, 60, 67, 73, 81, 92

$Q_1 = 35$, $Q_3 = 73$

$(\text{四分位偏差}) = \dfrac{Q_3 - Q_1}{2}$

$= \underline{19}$

(5)

(5-1)

$P(\text{2種類の色})$

$= \dfrac{{}_4C_2 \cdot ({}_4C_1 \cdot {}_4C_3 \times 2 + {}_4C_2 \cdot {}_4C_2)}{{}_{16}C_4}$

$= \dfrac{6\,(32+36)}{{}_{16}C_4}$

$= \dfrac{6 \cdot 68 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{16 \cdot 15 \cdot 14 \cdot 13}$

$= \dfrac{102}{455}$

(5-2)

E

$= 1 \cdot P(\text{1種類}) + 2 \cdot P(\text{2種類}) + 3 \cdot P(\text{3種類}) + 4 \cdot P(\text{4種類})$

$= 1 \cdot \dfrac{4}{{}_{16}C_4} + 2\dfrac{102}{455} + 3P(\text{3種類}) + 4 \cdot \dfrac{4^4}{{}_{16}C_4}$

$= \dfrac{1}{455} + 2\dfrac{102}{455} + 3\left(1 - \dfrac{1}{455} - \dfrac{102}{455} - \dfrac{64}{455}\right) + 4\dfrac{64}{455}$

$= \dfrac{1}{455} + 2\dfrac{102}{455} + 3\dfrac{288}{455} + 4\dfrac{64}{455} = \underline{\dfrac{265}{91}}$

$\boxed{4}$

(1)

$\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2 + 2nk + k^2}$

$\displaystyle = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{1 + 2\frac{k}{n} + \left(\frac{k}{n}\right)^2}$

$\displaystyle = \int_0^1 \frac{1}{(x+1)^2}\,dx = \left[-(x+1)^{-1}\right]_0^1$

$= -\dfrac{1}{2} + 1 = \underline{\dfrac{1}{2}}$

(2)

接点 (p, q) とすると

接線：$px - qy = 1$

$\Downarrow (0, t)$ とる

$-qt = 1 \iff q = -\dfrac{1}{t}$

また $p^2 - q^2 = 1$ より

$p^2 = q^2 + 1 = \dfrac{1}{t^2} + 1$

$\iff p = \pm\dfrac{\sqrt{t^2+1}}{t}$

$A\left(\dfrac{\sqrt{t^2+1}}{t}, -\dfrac{1}{t}\right), B\left(-\dfrac{\sqrt{t^2+1}}{t}, -\dfrac{1}{t}\right)$

とする.

$\overrightarrow{PA} = \begin{pmatrix} \frac{\sqrt{t^2+1}}{t} \\ -\frac{1}{t} - t \end{pmatrix}$ $\overrightarrow{PB} = \begin{pmatrix} -\frac{\sqrt{t^2+1}}{t} \\ -\frac{1}{t} - t \end{pmatrix}$

$S(t)$

$= \dfrac{1}{2}\left|\dfrac{\sqrt{t^2+1}}{t}\left(-\dfrac{1}{t} - t\right) + \dfrac{\sqrt{t^2+1}}{t}\left(-\dfrac{1}{t} - t\right)\right|$

$= \left|\dfrac{\sqrt{t^2+1}}{t}\left(-\dfrac{1+t^2}{t}\right)\right|$

$= \dfrac{(t^2+1)^{\frac{3}{2}}}{t^2} = \dfrac{(X+1)^{\frac{3}{2}}}{X} = f(X)$ $(X > 0)$

$t^2 = X$

$\dfrac{df(X)}{dX} = \dfrac{\frac{3}{2}(X+1)^{\frac{1}{2}}X - (X+1)^{\frac{3}{2}}}{X^2}$

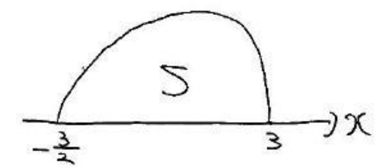
$= \dfrac{(X+1)^{\frac{1}{2}}(3X - 2(X+1))}{2X^2}$

$= \dfrac{(X+1)^{\frac{1}{2}}(X-2)}{2X^2}$

X	$0 \cdots 2 \cdots$
$\frac{df(X)}{dX}$	$\times \; - \; 0 \; +$
$f(X)$	$\times \searrow \frac{3\sqrt{3}}{2} \nearrow$

$\min S(t) = \min f(X) = \underline{\dfrac{3\sqrt{3}}{2}}$

(3)



$\displaystyle S = \int_{-\frac{3}{2}}^{3} y\,dx$

$\displaystyle = \int_{\frac{\pi}{3}}^{0} 2\sin 3t \cdot (-6\sin 2t)\,dt$

$\displaystyle = 12\int_0^{\frac{\pi}{3}} \sin 3t \sin 2t\,dt$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$-)\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$

$\displaystyle = 12\int_0^{\frac{\pi}{3}} \left\{-\frac{1}{2}(\cos 5t - \cos t)\right\} dt$

$= -6\left[\dfrac{1}{5}\sin 5t - \sin t\right]_0^{\frac{\pi}{3}}$

$= -6\left(-\dfrac{\sqrt{3}}{10} - \dfrac{\sqrt{3}}{2}\right)$

$= \underline{\dfrac{18}{5}\sqrt{3}}$