2018 埼玉医科大(前期)

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問1.

$q=\sqrt{2}-1$ より

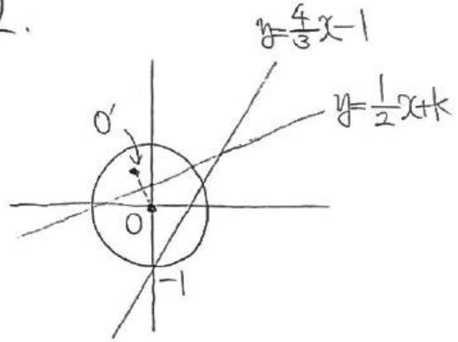
$\frac{1}{q}-q=\frac{1}{\sqrt{2}-1}-(\sqrt{2}-1)$

$=\underline{2}$ //

$\frac{1}{q^2}+q^2=\left(\frac{1}{q}-q\right)^2+2$

$=\underline{6}$ //

問2.



Oを $y=\frac{1}{2}x+k$ に関して対称に移した点を $O'(\alpha,\beta)$ とおく

$\begin{cases}\frac{\beta}{2}=\frac{1}{2}\cdot\frac{\alpha}{2}+k & \leftarrow 中点が\ y=\frac{1}{2}x+k\ 上\\ \frac{\beta}{\alpha}=-2 & \leftarrow OO'の傾き\end{cases}$

⇓

$\frac{-2\alpha}{2}=\frac{\alpha}{4}+k$

$\Leftrightarrow -\frac{5}{4}\alpha=k$

$\Leftrightarrow \alpha=-\frac{4}{5}k,\ \beta=\frac{8}{5}k$

$\left(-\frac{4}{5}k,\frac{8}{5}k\right)$ と $4x-3y-3=0$ の距離が1より

$\frac{\left|-\frac{16}{5}k-\frac{24}{5}k-3\right|}{\sqrt{16+9}}=1$

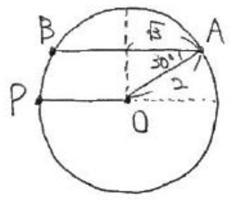
$\Leftrightarrow |-8k-3|=5$

$\Leftrightarrow 64k^2+48k+9=25$

$\Leftrightarrow 4k^2+3k-1=0$

$\therefore\ \underline{k=\frac{1}{4}\ または\ -1}$ //

問3



$A(\sqrt{3},1),\ B(-\sqrt{3},1),\ P(-2,0)$ とおける.

$\overrightarrow{AB}\cdot\overrightarrow{AP}=\begin{pmatrix}-2\sqrt{3}\\0\end{pmatrix}\cdot\begin{pmatrix}-2-\sqrt{3}\\-1\end{pmatrix}$

$=\underline{4\sqrt{3}+6}$ //

問4.

$\cos 4x=\cos 2x$

$=\cos(2x+2k\pi)$ $(k\in\mathbb{Z})$

$=\cos(-2x+2k\pi)$

⇓

$\begin{cases}4x=2x+2k\pi\\4x=-2x+2k\pi\end{cases}$

$\therefore\ \begin{cases}x=k\pi\\x=\frac{k}{3}\pi\end{cases}$

$\underline{x=0,\frac{\pi}{3},\frac{2}{3}\pi,\pi}$ で交わる.

(面積) $F(x)=\frac{1}{2}\sin 2x-\frac{1}{4}\sin 4x$

$=\int_0^{\frac{\pi}{3}}\underbrace{(\cos 2x-\cos 4x)}_{f(x)}dx$

$+\int_{\frac{\pi}{3}}^{\frac{2}{3}\pi}(\cos 4x-\cos 2x)dx$

$+\int_{\frac{2}{3}\pi}^{\pi}(\cos 2x-\cos 4x)dx$

$=F\left(\frac{\pi}{3}\right)-\underbrace{F(0)}_{0}-F\left(\frac{2}{3}\pi\right)+F\left(\frac{\pi}{3}\right)$

$+\underbrace{F(\pi)}_{0}-F\left(\frac{2}{3}\pi\right)$

$=2F\left(\frac{\pi}{3}\right)-2F\left(\frac{2}{3}\pi\right)$

$=\frac{\sqrt{3}}{2}-\frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)$

$-\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right)=\underline{\frac{3\sqrt{3}}{2}}$ //

2

問1.

正弦定理

$\frac{AB}{\sin\theta}=\frac{PB}{\sin\phi}=\frac{PA}{\sin(\pi-\theta-\phi)}=2$

$\therefore\ AB=2\sin\theta \cdots$ ⑤ //

$PA=2\sin(\theta+\phi)\cdots$ ⑦ //

$PB=2\sin\phi\cdots$ ⑥ //

問2.

$PA+PB$

$=2\{\sin(\theta+\phi)+\sin\phi\}$

$=2\left\{\sin\left(\frac{\theta+2\phi}{2}+\frac{\theta}{2}\right)\right.$

$\left.+\sin\left(\frac{\theta+2\phi}{2}-\frac{\theta}{2}\right)\right\}$

$=\underline{4\sin\frac{1\cdot\theta+2\phi}{2}\cos\frac{1\cdot\theta+0\cdot\phi}{2}}$ //

問3

$PA+PB$

$=f(\phi)=2\{\sin(\theta+\phi)+\sin\phi\}$

とおく.

$f'(\phi) = 2\{\cos(\theta+\phi) + \cos\phi\}$

$= 2\left[\cos\left(\frac{\theta+2\phi}{2} + \frac{\theta}{2}\right) + \cos\left(\frac{\theta+2\phi}{2} - \frac{\theta}{2}\right)\right\}$

$= 4\cos\frac{\theta+2\phi}{2}\cos\frac{\theta}{2}$

$f'(\phi) = 0 \iff \frac{\theta+2\phi}{2} = \frac{\pi}{2}$

$\iff \phi = \frac{1\cdot\pi - 1\cdot\theta}{2}$ //

問4.

$(PA+PB)AB$

$= 4\{\sin(\theta+\phi) + \sin\phi\}\sin\theta$

$= 8\sin\frac{\theta+2\phi}{2}\cos\frac{\theta}{2}\sin\theta$

$\leq 8\sin\frac{\pi}{2}\cos\frac{\theta}{2}\sin\theta$ ← $\phi = \frac{\pi-\theta}{2}$

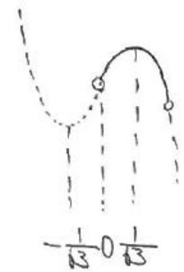
$= 8\cos\frac{\theta}{2}\sin\theta$

$= 16\cos^2\frac{\theta}{2}\sin\frac{\theta}{2}$

$= 16\sin\frac{\theta}{2} - 16\sin^3\frac{\theta}{2}$

$= \underbrace{16t - 16t^3}_{g(t)}$ ← $\sin\frac{\theta}{2} = t$, $0 < t < 1$

$g'(t) = 16(1 - 3t^2)$



$t = \sin\frac{\theta}{2} = \frac{1}{\sqrt{3}}$ のとき

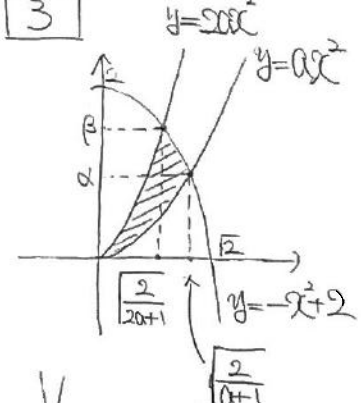
$\max g(t) = 16\cdot\frac{2}{3\sqrt{3}}$

$= \frac{32\sqrt{3}}{9}$

よって $\sin\frac{\theta}{2} = \frac{1}{\sqrt{3}}$, $\phi = \frac{\pi-\theta}{2}$ のとき

最大値 $\frac{32\sqrt{3}}{9}$ //

$\boxed{3}$



V

$= \int_0^{\alpha}\left(\frac{y}{a}\pi - \frac{y}{2a}\pi\right)dy$

$+ \int_{\alpha}^{\beta}\left((2-y)\pi - \frac{y}{2a}\pi\right)dy$

$= \left[\frac{1}{2a}y^2\pi\right]_0^{\alpha} + \left[\left(2y - \frac{1}{2}y^2\right)\pi\right]_{\alpha}^{\beta}$

$- \left[\frac{1}{4a}y^2\pi\right]_0^{\beta}$

$= \frac{\pi}{2a}\alpha^2 + \left(2\beta - \frac{1}{2}\beta^2\right)\pi$

$- \left(2\alpha - \frac{1}{2}\alpha^2\right)\pi - \frac{\pi}{4a}\beta^2$

ここで

$\alpha = \frac{2a}{a+1}$　$\beta = \frac{4a}{2a+1}$　より

V

$= \frac{\pi}{2a}\alpha^2(1+a) - \frac{\pi}{4a}\beta^2(1+2a)$

$+ 2\pi(\beta - \alpha)$

$= \pi\alpha^2\frac{1}{\alpha} - \pi\beta^2\frac{1}{\beta} + 2\pi(\beta-\alpha)$

$= \pi(\alpha-\beta) + 2\pi(\beta-\alpha)$

$= \pi(\beta-\alpha)$

$= \pi\cdot 2a\left(\frac{2}{2a+1} - \frac{1}{a+1}\right)$

$= \pi\cdot 2a\,\frac{2a+2-(2a+1)}{(2a+1)(a+1)}$

$= \frac{2a}{2a^2+3a+1}\pi$ //

問2.

V

$= \frac{1}{a + \frac{1}{2a} + \frac{3}{2}}\pi$

$\leq \frac{1}{2\sqrt{a\frac{1}{2a}} + \frac{3}{2}}\pi$

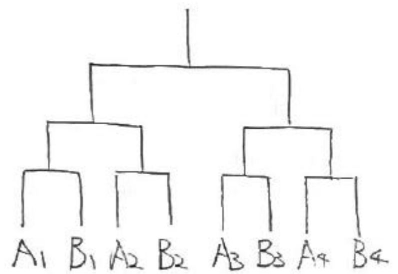
$a>0, \frac{1}{2a}>0$
(相加平均)
≧(相乗平均)

$= \frac{2}{2\sqrt{2}+3}\pi$

$= (6 - 4\sqrt{2})\pi$ //

等号成立は $a = \frac{1}{2a}$ ∴ $a = \frac{1}{\sqrt{2}}$ のとき

$\boxed{4}$



問1

$P(A_1$が決勝に行く$)$

$= P($準決でA_2に勝つ$)$
$\quad + P($準決でB_2 〃 $)$

$= \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}$

$= \frac{6+4}{27} = \underline{\frac{10}{27}}$

問2

$P(B_3$が決勝に行く$)$

$= P($準決でA_4に勝つ$)$
$\quad + P($準決でB_4に勝つ$)$

$= \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}$

$= \frac{4+3}{54} = \frac{7}{54}$

$P(A$組が優勝$)$

$= P(A_i$とA_jが決勝に行く$)$
$\quad + P(A_i$とB_j 〃 $)\cdot\frac{2}{3}$
$\quad + P(B_i$とA_j 〃 $)\cdot\frac{2}{3}$

$(i=1,2 \quad j=3,4)$

$= \frac{10}{27}\cdot\frac{10}{27}\cdot 2^2 + \frac{10}{27}\cdot\frac{7}{54}\cdot 2^2\cdot\frac{2}{3}$
$\quad + \frac{7}{54}\cdot\frac{10}{27}\cdot 2^2\cdot\frac{2}{3}$

$= \frac{400}{27^2} + \frac{10}{27}\cdot\frac{7}{27}\cdot 4\cdot\frac{2}{3}$

$= \frac{1200+560}{27^2\cdot 3}$

$= \underline{\frac{1760}{2187}}$

問3.

$P_{A_1\text{優勝}}(A_i$とA_jが決勝に行く$)$

$= \frac{P(A_1\text{と}A_j\text{が決勝に行き}A_1\text{優勝})}{P(A_1\text{優勝})}$

$= \frac{\frac{10}{27}\cdot\frac{10}{27}\cdot 2\cdot\frac{1}{2}}{\frac{10}{27}\cdot\frac{10}{27}\cdot 2\cdot\frac{1}{2} + \frac{10}{27}\cdot\frac{7}{54}\cdot 2\cdot\frac{2}{3}}$

$= \frac{\frac{10^2}{27^2}}{\frac{10^2}{27^2} + \frac{10}{27^2}\cdot\frac{14}{3}}$

$= \frac{1}{1+\frac{14}{30}}$

$= \frac{30}{44} = \underline{\frac{15}{22}}$