2018 日本大学（医）

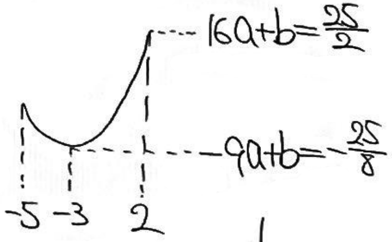
[1]

(1)

$y=ax^2+6ax+b$

$=a(x^2+6x)+b$

$=a(x+3)^2-9a+b$



$\downarrow$

$25a=\frac{125}{8}$

$\therefore a=\frac{5}{8},\ b=\frac{5}{2}$ //

(2)

$x=1\pm\sqrt{2}i$ を解にもつ.

実数解を α とすると

解と係数の関係より

$(1+\sqrt{2}i)(1-\sqrt{2}i)\alpha=8 \quad \therefore \alpha=\frac{8}{3}$

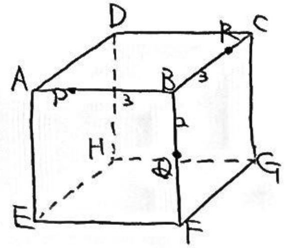
これより

$a(x^2-2x+3)(x-\frac{8}{3})$

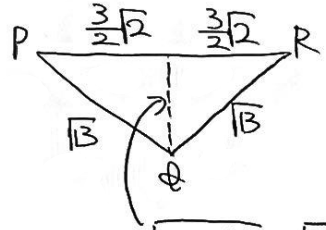
$=a(x^3-\frac{14}{3}x^2+\frac{25}{3}x-8)$

$\therefore \begin{cases} -\frac{14}{3}a=b \\ \frac{25}{3}a=25 \end{cases} \quad \therefore \begin{array}{l} a=3 \\ b=-14 \end{array}$ //

(3)



$PQ=\sqrt{13}$ // $RP=3\sqrt{2}$ //



$\sqrt{13-\frac{9}{2}}=\sqrt{\frac{17}{2}}$

$\triangle PQR=3\sqrt{2}\times\frac{\sqrt{17}}{\sqrt{2}}\times\frac{1}{2}=\frac{3}{2}\sqrt{17}$ //

(4)

P(4の倍数)

$=\frac{9}{36}=\frac{1}{4}$ //

下2桁が4の倍数	
12	44
6	52
24	6
32	64
6	

P(4の倍数かつ9の倍数)

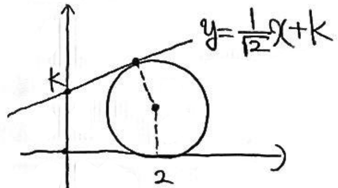
$=\frac{6}{216}$

$=\frac{1}{36}$ //

612	144
216	252
324	
432	

[2]

(1) $y-\frac{1}{\sqrt{2}}x=k$ とおく.



$y-\frac{1}{\sqrt{2}}x-k=0$ と $(2,1)$ の距離が半径以下なので

$\frac{|1-\sqrt{2}+1-k|}{\sqrt{\frac{1}{2}+1}}\leqq 1$

$\Leftrightarrow |k+\sqrt{2}-1|\leqq\frac{\sqrt{3}}{\sqrt{2}}$

$\Leftrightarrow -\frac{\sqrt{6}}{2}\leqq k+\sqrt{2}-1\leqq\frac{\sqrt{6}}{2}$

$\Leftrightarrow 1-\sqrt{2}-\frac{\sqrt{6}}{2}\leqq k\leqq 1-\sqrt{2}+\frac{\sqrt{6}}{2}$

$\therefore \max k=\max(y-\frac{1}{\sqrt{2}}x)$

$=1-\sqrt{2}+\frac{\sqrt{6}}{2}$ //

(2) $t=2^x-2^{-x}$ とおくと

$t^2=4^x+4^{-x}-2$ より

(方程)$=t^2+2-8t+14$

$=t^2-8t+16=0 \quad \therefore t=4$

$\downarrow$

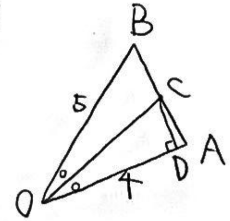
$(2^x)^2-1=4\cdot 2^x$

$\Leftrightarrow (2^x)^2-4\cdot 2^x-1=0$

$\therefore 2^x=2+\sqrt{5} \quad (\because 2^x>0)$

$\therefore x=\log_2(2+\sqrt{5})$ //

(3)



CはABを $4:5$ に内分

$C\left(\frac{5\cdot 2\sqrt{3}+4\cdot 3}{4+5},\ \frac{5\cdot 2+4\cdot 4}{4+5}\right)$

$\frac{10}{9}\sqrt{3}+\frac{4}{3}$, $\frac{26}{9}$

OAの傾きが $\frac{1}{\sqrt{3}}$ より

直線CDは

$y=-\sqrt{3}\left(x-\frac{10}{9}\sqrt{3}-\frac{4}{3}\right)+\frac{26}{9}$

$=-\sqrt{3}x+\frac{56}{9}+\frac{4}{3}\sqrt{3}$

直線OAと連立

$\frac{1}{\sqrt{3}}x=-\sqrt{3}x+\frac{56+12\sqrt{3}}{9}$

$\Leftrightarrow 4x=\frac{56+12\sqrt{3}}{3\sqrt{3}}$

$\therefore x=\frac{14+3\sqrt{3}}{3\sqrt{3}}$

$\therefore k=\frac{\frac{14+3\sqrt{3}}{3\sqrt{3}}}{2\sqrt{3}}=\frac{7}{9}+\frac{\sqrt{3}}{6}$ //

(4)

$$a_{n+1}=\frac{7a_n}{8a_n+3}$$

$\downarrow$ $n=1$ 代入

$$a_2=\frac{\frac{7}{3}}{\frac{8}{3}+3}=\frac{7}{17}$$

$\downarrow$

$b_1=3 \quad b_2=\frac{17}{7}$

ここで $b_n=\alpha+\beta^{n-1}$ とおくと

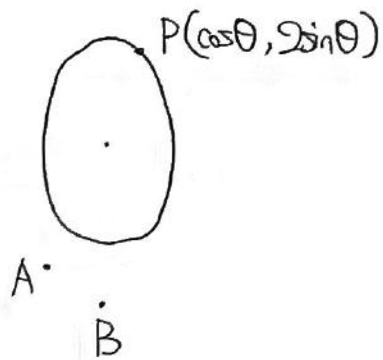
$$\begin{cases} b_1=\alpha+1=3 \\ b_2=\alpha+\beta=\frac{17}{7} \end{cases}$$

$\therefore \alpha=2,\ \beta=\frac{3}{7}$

$\therefore$ $b_n=2+\left(\frac{3}{7}\right)^{n-1}$ //

※当然天下り的な解き方で
正攻法ではありません.

[3]



$$\overrightarrow{AP}=\begin{pmatrix}\cos\theta+1\\2\sin\theta+\frac{3}{2}\sqrt{2}\end{pmatrix} \quad \overrightarrow{AB}=\begin{pmatrix}1\\-\frac{1}{2}\sqrt{2}\end{pmatrix}$$

$\triangle ABP$

$$=\frac{1}{2}\left|-\frac{\sqrt{2}}{2}\cos\theta-\frac{\sqrt{2}}{2}-2\sin\theta-\frac{3}{2}\sqrt{2}\right|$$

$$=\frac{\sqrt{2}}{2}\left|\sqrt{2}\sin\theta+\frac{1}{2}\cos\theta+2\right|$$

$$=\frac{\sqrt{2}}{4}\left|2\sqrt{2}\sin\theta+\cos\theta+4\right|$$

$$=\frac{\sqrt{2}}{4}\left|3\Big(\sin\theta\cdot\underbrace{\frac{2\sqrt{2}}{3}}_{\cos\alpha}+\cos\theta\cdot\underbrace{\frac{1}{3}}_{\sin\alpha}\Big)+4\right|$$

$$=\frac{\sqrt{2}}{4}\left|3\sin(\theta+\alpha)+4\right|$$

$\theta=-\alpha+\frac{\pi}{2}$ のとき

$\cos\left(-\alpha+\frac{\pi}{2}\right)=\sin\alpha=\frac{1}{3}$

$\sin\left(-\alpha+\frac{\pi}{2}\right)=\cos\alpha=\frac{2\sqrt{2}}{3}$

$P\left(\frac{1}{3}, \frac{4}{3}\sqrt{2}\right)$ // のとき

$\max\triangle ABP=\frac{7}{4}\sqrt{2}$ //

(2) Bを通る傾きmの直線

$y=mx-2\sqrt{2}$

$\downarrow$ $4x^2+y^2=4$

$4x^2+(mx-2\sqrt{2})^2=4$

$\Leftrightarrow (4+m^2)x^2-4\sqrt{2}mx+4=0$

$\frac{D}{4}=8m^2-(4+m^2)4$

$=4m^2-16=0 \quad \therefore m=\pm 2$

第4象限で接するのは

$y=2x-2\sqrt{2}$ //

接点は

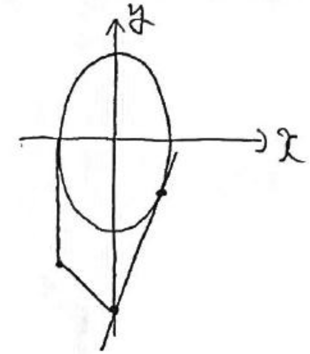
$8x^2-8\sqrt{2}x+4=0$

$\Leftrightarrow x^2-\sqrt{2}x+\frac{1}{2}=0$

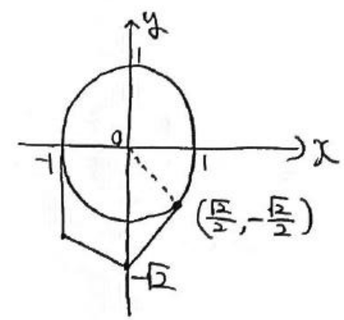
$\therefore x=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\therefore$ 接点 $\left(\frac{\sqrt{2}}{2}, -\sqrt{2}\right)$ //

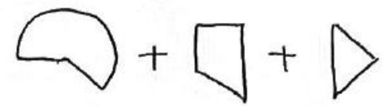
(3)



$\downarrow$ y軸方向 $\frac{1}{2}$ 倍



これの面積は



$=\frac{5}{8}\pi+\left(\frac{3}{4}\sqrt{2}+\sqrt{2}\right)\cdot 1\cdot\frac{1}{2}+\frac{1}{2}$

$=\frac{5}{8}\pi+\frac{7}{8}\sqrt{2}+\frac{1}{2}$

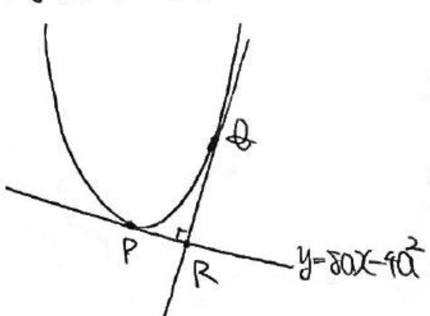
求める面積は上を2倍

$\frac{5}{4}\pi+\frac{7}{4}\sqrt{2}+1$

$=\frac{7\sqrt{2}+4+5\pi}{4}$ //

[4]

(1)



$y=8b(x-b)+4b^2$
$=8bx-4b^2$

交点は

$8bx-4b^2=8ax-4a^2$

$\Leftrightarrow 8(b-a)x=4b^2-4a^2$

$\therefore x=\dfrac{a+b}{2}$

$(R\text{の}y\text{座標})=8b\cdot\dfrac{a+b}{2}-4b^2$

$=4ab$

$=4\left(-\dfrac{1}{64}\right)=-\dfrac{1}{16}$

$\boxed{\begin{array}{l} l_P\text{と}l_Q\text{が垂直より} \\ 8a\cdot 8b=-1 \\ \therefore ab=-\frac{1}{64}\end{array}}$

(2) $P(a,4a^2)$, $Q(b,4b^2)$

$R\left(\dfrac{a+b}{2},-\dfrac{1}{16}\right)$

$\overrightarrow{RP}=\begin{pmatrix}\frac{a-b}{2}\\ 4a^2+\frac{1}{16}\end{pmatrix}$ $\overrightarrow{RQ}=\begin{pmatrix}\frac{b-a}{2}\\ 4b^2+\frac{1}{16}\end{pmatrix}$

$\triangle PQR$

$=\dfrac{1}{2}\left|\dfrac{a-b}{2}\left(4b^2+\dfrac{1}{16}\right)-\dfrac{b-a}{2}\left(4a^2+\dfrac{1}{16}\right)\right|$

$=\dfrac{1}{4}\left|(a-b)(4b^2+4a^2)+(a-b)\dfrac{1}{8}\right|$

$=\dfrac{1}{4}\left|(a-b)\left(4b^2+4a^2+\dfrac{1}{8}\right)\right|$

$=\dfrac{1}{4}\left|(a-b)(4b^2+4a^2-8ab)\right|$

$=\left|(a-b)(b-a)^2\right|=(b-a)^3$

(3)

$\triangle PQR$

$=(b-a)^3$

$=\left(b+\dfrac{1}{64b}\right)^3$

$\geqq\left(2\sqrt{b\cdot\dfrac{1}{64b}}\right)^3$

$=\left(\dfrac{1}{4}\right)^3$

$=\dfrac{1}{64}$

$\boxed{a=-\dfrac{1}{64b}}$

$\boxed{\begin{array}{l} b>a\text{より } b>0 \\ (\text{相加平均})\geqq(\text{相乗平均})\end{array}}$

等号成立は

$b=\dfrac{1}{64b}\Leftrightarrow b=\dfrac{1}{8}$ のとき

$\min\triangle PQR=\dfrac{1}{64}$ //

[5]

(1) 平面ABC: $x+\dfrac{y}{2}+\dfrac{z}{4}=1$

$\Leftrightarrow 4x+2y+z-4=0$

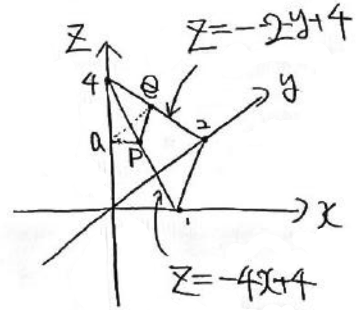
直線OH: $\begin{cases}x=4t\\ y=2t\\ z=t\end{cases}$

連立すると $16t+4t+t-4=0$

$\therefore t=\dfrac{4}{21}$

$H\left(\dfrac{16}{21},\dfrac{8}{21},\dfrac{4}{21}\right)$ //

(2)



図のPのx座標は

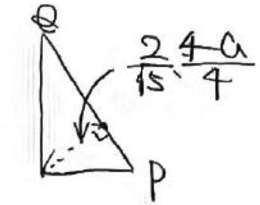
$a=-4x+4$

$\Leftrightarrow 4x=4-a$ $\therefore \dfrac{4-a}{4}$

図のQのy座標は

$a=-2y+4$

$\Leftrightarrow 2y=4-a$ $\therefore \dfrac{8-2a}{2}$



$S(a)$

$=\left(\dfrac{8-2a}{4}\right)^2\pi-\left(\dfrac{8-2a}{4\sqrt{5}}\right)^2\pi$

$=\left(\dfrac{1}{16}-\dfrac{1}{80}\right)(8-2a)^2\pi$

$=\dfrac{(4-a)^2}{5}\pi$

(3)

$\displaystyle\int_1^3\dfrac{S'(z)}{S(z)}dz$

$=\Big[\log S(z)\Big]_1^3$

$=\log\dfrac{S(3)}{S(1)}$

$=\log\dfrac{\frac{1}{5}\pi}{\frac{9}{5}\pi}$

$=\log\dfrac{1}{9}$

$=-2\log 3$ //