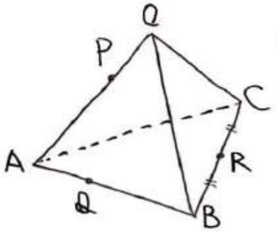
2018 北里大(薬)

I.



$\triangle OPQ = \frac{1}{3}\cdot\frac{1}{3}\cdot\triangle OAB$

$= \frac{1}{9}\cdot\frac{1}{2}\cdot 3\cdot 3\sin 60^\circ$

$= \frac{\sqrt{3}}{4} \cdots \underline{9}$

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$= \frac{\vec{b}+\vec{c}}{2} - \frac{\vec{a}}{3}$

$= -\frac{1}{3}\vec{a} + \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$

$\underline{7}$ $\underline{3}$ $\underline{3}$

$\overrightarrow{PR}\cdot\overrightarrow{PQ}$

$= \left(-\frac{1}{3}\vec{a}+\frac{1}{2}\vec{b}+\frac{1}{2}\vec{c}\right)\cdot\left(\frac{1}{3}\vec{a}+\frac{1}{3}\vec{b}\right)$

$= -1 + \frac{1}{6}\cdot\frac{9}{2} + \frac{3}{4} - \frac{1}{2} + \frac{9}{6} + \frac{3}{4}$

$= \frac{9}{4} + \frac{3}{2} - \frac{3}{2}$

$= \frac{9}{4} \cdots \underline{4}$

$|\overrightarrow{PR}|^2$

$= 1 + \frac{9}{4} + \frac{9}{4} - \frac{1}{3}\cdot\frac{9}{2} + \frac{1}{2}\cdot\frac{9}{2} - \frac{1}{3}\cdot\frac{9}{2}$

$= \frac{11}{2} - 3 + \frac{9}{4}$

$= \frac{22-12+9}{4} = \frac{19}{4}$ $\therefore |\overrightarrow{PR}| = \frac{\sqrt{19}}{2}$

$|\overrightarrow{PQ}|^2$

$= 1 + 1 + \frac{2}{9}\cdot\frac{9}{2} = 3$ $\therefore |\overrightarrow{PQ}| = \sqrt{3}$

$\cos\theta = \dfrac{\frac{9}{4}}{\frac{\sqrt{19}}{2}\cdot\sqrt{3}}$

$= \frac{9}{2\sqrt{57}}$

$= \frac{3\sqrt{57}}{38} \cdots \underline{7}$

II

(1)

$f(x) = \cos 2x - \cos 4x$

$= \cos 2x - (2\cos^2 2x - 1)$

$= -2\left(\cos^2 2x - \frac{1}{2}\cos 2x\right) + 1$

$= -2\left(\cos 2x - \frac{1}{4}\right)^2 + \frac{9}{8}$

$\cos 2x = \frac{1}{4}$ のとき 最大値 $\frac{9}{8} \cdots \underline{2}$

$\cos 2x = -1$: 最小値 $-2 \cdots \underline{7}$

(2)

$\begin{cases} \sin\theta + \sqrt{3}\cos\theta = -2a \\ \sqrt{3}\sin\theta\cos\theta = b \end{cases}$

$\Leftrightarrow \begin{cases} 2\sin\left(\theta+\frac{\pi}{3}\right) = -2a \\ \frac{\sqrt{3}}{2}\sin 2\theta = b \end{cases}$

$\Leftrightarrow \begin{cases} a = -\sin\left(\theta+\frac{\pi}{3}\right) \\ b = \frac{\sqrt{3}}{2}\sin 2\theta \end{cases}$

$\therefore -1 \leqq a \leqq \frac{\sqrt{3}}{2} \cdots \underline{8}$

$-\frac{\sqrt{3}}{2} \leqq b \leqq \frac{\sqrt{3}}{2} \cdots \underline{10}$

III

(1) $a_{n+1} + b_{n+1} = 2(a_n + b_n)$

$\therefore c_n = a_n + b_n = (a_1 + b_1)2^{n-1}$

$= 3\cdot 2^{n-1} \cdots \underline{5}$

$d_n = a_{n+1} - 3a_n$

$= 3c_n$

$\therefore d_2 = 3c_2 = 18 \cdots \underline{2}$

$d_4 = 3c_4 = 72 \cdots \underline{9}$

$a_{n+1} - 3a_n = 9\cdot 2^{n-1}$

$\Leftrightarrow \frac{a_{n+1}}{2^n} = \frac{3}{2}\cdot\frac{a_n}{2^{n-1}} + \frac{9}{2}$

$\therefore e_{n+1} = \frac{3}{2}e_n + \frac{9}{2}$

$\Leftrightarrow e_{n+1} + 9 = \frac{3}{2}(e_n + 9)$

$e_n + 9 = (e_1 + 9)\left(\frac{3}{2}\right)^{n-1}$

$\Leftrightarrow e_n = 10\left(\frac{3}{2}\right)^{n-1} - 9$

$\therefore e_3 = 10\cdot\frac{9}{4} - 9 = \frac{27}{2} \cdots \underline{8}$

$e_7 = 10\left(\frac{3}{2}\right)^6 - 9$

$= \frac{5\cdot 3^6}{32} - \frac{288}{32} \cdots \underline{7}$

$a_n = e_n\cdot 2^{n-1}$

$= 10\cdot 3^{n-1} - 9\cdot 2^{n-1}$

$b_n = c_n - a_n$

$= 12\cdot 2^{n-1} - 10\cdot 3^{n-1}$

$\therefore b_9 = 12\cdot 2^8 - 10\cdot 3^8$

$= 3\cdot 1024 - 10\cdot 81^2 \cdots \underline{8}$

IV

(1)

P(すべて異なる)

$= \frac{6\cdot5\cdot4}{6^3} = \frac{5}{9}$ … $\underline{7}$

(2)

P(abcが偶数)

$= 1 -$ P(abcが奇数)

$= 1 - \frac{3^3}{6^3}$

$= \frac{7}{8}$ … $\underline{6}$

P(abcが4で割りきれる)

= P(abcが偶数)

− P(abcが偶数だが4で割れない)

$= \frac{7}{8} - \frac{{}_3C_1\cdot2\cdot3\cdot3}{6^3}$

$= \frac{5}{8}$ … $\underline{5}$

(3)

P(a+b+cが5で割りきれる)

$= P(a+b+c=5)$

$+ P(a+b+c=10)$

$+ P(a+b+c=15)$

組

(1,1,3) … 3　(3,6,6) … 3

(1,2,2) … 3　(4,5,6) … 6

(1,3,6) … 6　(5,5,5) … 1

(1,4,5) … 6

(2,2,6) … 3

(2,3,5) … 6

(2,4,4) … 3

(3,3,4) … 3

$= \frac{6+27+10}{216}$

$= \frac{43}{216}$ … $\underline{10}$

V

(1)

$f(x) = 2x^3 + 2x^2 + 3\alpha$ とする

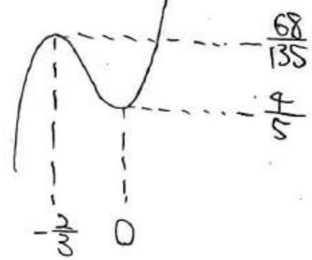
$\alpha = \int_{-1}^{1} (2t^3 + 2t^2 + 3\alpha)\,dt$

$= 2\int_0^1 (2t^2 + 3\alpha)\,dt$

$= \frac{4}{3} + 6\alpha$　$\therefore \alpha = -\frac{4}{15}$

$f'(x) = 6x^2 + 4x$

$= 2x(3x+2)$



$x = -\frac{2}{3}$ で極大値 $-\frac{68}{135}$ … $\underline{8}$, $\underline{10}$

$x = 0$ で極小値 $-\frac{4}{5}$ … $\underline{1}$, $\underline{6}$

$2x^3 + 2x^2 - \frac{4}{5} = 2x + a$

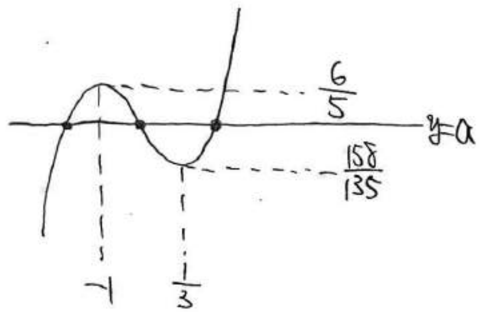
$\Leftrightarrow 2x^3 + 2x^2 - 2x - \frac{4}{5} = a$ … ①

左辺を $g(x)$ とおくと

$g'(x) = 6x^2 + 4x - 2$

$= 2(3x^2 + 2x - 1)$

$= 2(3x-1)(x+1)$



①が異なる3つの実数解をもつのは

$-\frac{158}{135} < a < \frac{6}{5}$ … $\underline{7}$

(2)

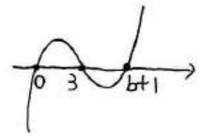
$x^3 + 2x^2 = 6x^2 - 3x + bx(x-3)$

$\Leftrightarrow x^3 - 4x^2 + 3x = bx(x-3)$

$\Leftrightarrow x(x-1)(x-3) = bx(x-3)$

$\Leftrightarrow x(x-3)(x-1-b) = 0$

$\Leftrightarrow x = 0, 3, 1+b$ … $\underline{7}$



$\int_0^3 x(x-3)(x-b-1)\,dx$

$= -\int_3^{b+1} x(x-3)(x-b-1)\,dx$

$\Leftrightarrow \int_0^3 x^2(x-3)\,dx - (b+1)\int_0^3 x(x-3)\,dx$

$= -\left\{\int_3^{b+1} (x-3)^2(x-b-1)\,dx\right.$

$\left. + \int_3^{b+1} 3(x-3)(x-b-1)\,dx\right\}$

$\Leftrightarrow -\frac{1}{12}3^4 - (b+1)\left(-\frac{1}{6}\right)3^3$

$= \frac{1}{12}(b-2)^4 + 3\cdot\frac{1}{6}(b-2)^3$

$\Leftrightarrow$ …

$\Leftrightarrow 27(2b-1) = (b+4)(b-2)^3$

これをみたすのは $b = 5$ … $\underline{2}$