2018 近畿大(医)(前期)

1

(1) $3^x = t$ とおく.

$f(x) = t^3 - t^2 - 3t + 3$

$= (t-1)(t^2-3) = 0$

$t = 3^x = 1, \sqrt{3}$

$\therefore \underline{x = 0, \frac{1}{2}}$ //

$g(x)$

$= t^3 - t^2 - 3t + 3 + \frac{1}{t^3} - \frac{1}{t^2} - \frac{3}{t} + 3$

$= t^3 - t^2 - 3t + 6 + \frac{1}{t^3} - \frac{1}{t^2} - \frac{3}{t}$

$t + \frac{1}{t} = s$ とおくと

$t^2 + \frac{1}{t^2} = s^2 - 2$

$t^3 + \frac{1}{t^3} = \left(t + \frac{1}{t}\right)\left(t^2 - 1 + \frac{1}{t^2}\right)$

$= s(s^2-3)$

$= s^3 - 3s - (s^2-2) - 3s + 6$

$= s^3 - s^2 - 6s + 8 \quad (s \geqq 2)$

相加相乗

$\frac{dg(x)}{ds} = 3s^2 - 2s - 6$

$\frac{dg(x)}{ds} = 0 \iff s = \frac{1 \pm \sqrt{19}}{3} < 2$

$\therefore \frac{dg(x)}{ds} > 0$

$g(x)$は単調増加より.

$\min g(x) = g(2) = \underline{0}$ //

(2)

$\begin{cases} \log_3 \frac{L^2}{G} = \log_3 27 \cdot 32 \\ \log_2 LG = \log_2 2^7 \cdot 3^3 \end{cases}$

$\downarrow$

$\begin{cases} \frac{L^2}{G} = 3^3 \cdot 2^5 \\ LG = 2^7 \cdot 3^3 \end{cases}$

$\downarrow$

$L^3 = 3^6 \cdot 2^{12}$

$\therefore L = 3^2 \cdot 2^4 = 9 \cdot 16 = 144$

$\therefore G = 2^3 \cdot 3 = 24$

$24 \,\underline{)\, m, \; n\,}$
$\quad\;\; m' \;\; n'$

$\downarrow$

$24 m'n' = 144$

$m'n' = 6$

$(m', n') = (2, 3)$

$(\because G < m < n < L)$

$\therefore \underline{m = 48, \; n = 72}$ //

(3)

$x + 4y = a(5x + 6y) + b$

$\iff 0 = (5a-1)x + (6a-4)y + b$

$= (5a-1)x + (6a-4)(ax+b) + b$

$= (6a^2 + a - 1)x$

$+ 6ab - 3b$

$\downarrow$ どんなxでも成立

$6a^2 + a - 1 = 0$

$\iff (2a+1)(3a-1) = 0$

$a = -\frac{1}{2}, \frac{1}{3}$

$6ab - 3b = (6a-3)b = 0$

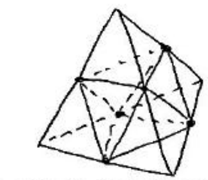
$\therefore (a, b) = \underline{\left(-\frac{1}{2}, 0\right), \left(\frac{1}{3}, 0\right)}$ //

2

(1) 相似比 1:3

体積比 1:27 $\quad \underline{\frac{1}{27}倍}$ //

(2)



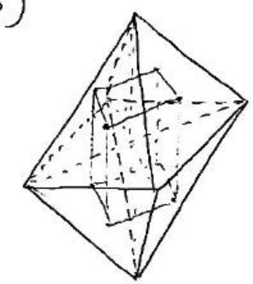
正四面体ABCDを1とする.

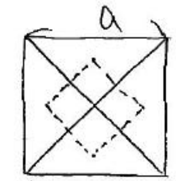
正八面体

$= 1 - \frac{1}{8} \times 4 = \frac{1}{2}$

$\therefore \underline{\frac{1}{2}倍}$ //

(3)





正八面体

$= a^2 \times \frac{a}{\sqrt{2}} \times \frac{1}{3} \times 2$

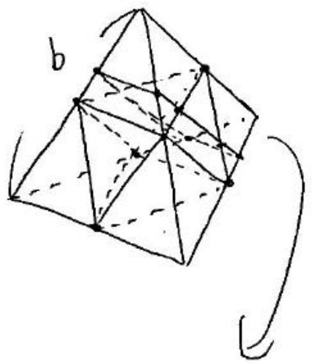
$= \frac{\sqrt{2}a^3}{3}$

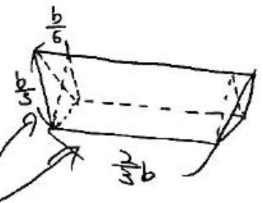
立方体 $= \left(\frac{\sqrt{2}}{3}a\right)^3 = \frac{\sqrt{8}}{27}a^3$

(答) $= \frac{\frac{\sqrt{8}}{27}}{\frac{\sqrt{2}}{3} \times 2} = \frac{\sqrt{8}}{27} \times \frac{3}{\sqrt{2}} \times \frac{1}{2}$

$= \underline{\frac{1}{9}倍}$ //

(4)





正八面体の体積をαとおく.

(先端の三角錐)$=\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{6}\alpha$

$=\frac{\alpha}{54}$

(三角柱)

$=\frac{\alpha}{54}\times 3\times 4=\frac{12}{54}\alpha$

これより

(A,B含む立体)

$=\frac{\alpha}{54}\times 2+\frac{12}{54}\alpha=\frac{14}{54}\alpha$

(答え)$=\underline{\frac{7}{27}倍}$ //

$\boxed{3}$

(1) $a+b=c^2+1>0$

$ab=6$

より $\underline{a>0,\ b>0}$ //

(2) $a+b=c^2+1$

$a-b=\frac{2}{c}\quad(c\neq 0)$

$\downarrow$

$a=\frac{1}{2}\left(c^2+1+\frac{2}{c}\right)$

$b=\frac{1}{2}\left(c^2+1-\frac{2}{c}\right)$

$\downarrow ab=6$

$\frac{1}{4}\left\{(c^2+1)^2-\frac{4}{c^2}\right\}=6$

$\Leftrightarrow c^4+2c^2+1-\frac{4}{c^2}=24$

$\Leftrightarrow c^6+2c^4-23c^2-4=0$

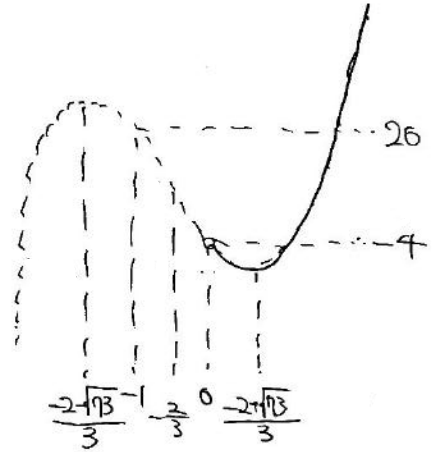
$\downarrow c^2=x$

$f(x)=\underline{x^3+2x^2-23x-4}$ //

(3)

$f'(x)=3x^2+4x-23$

$f'(x)=0\Leftrightarrow x=\frac{-2\pm\sqrt{73}}{3}$



正の実数解$\underline{1コ}$ //

(4)

$f(x)=0\Leftrightarrow x=4$

∘ $c=2$のとき

$(a,b,c)=(3,2,2)$

∘ $c=-2$のとき

$\underline{(a,b,c)=(2,3,-2)}$ //