2018 金沢医科大(前期)

$\boxed{1}$

(1) ①, ②が$(2,0)$通る

$\frac{2}{a}=1$, $\frac{2}{c+1}=1$

$\therefore a=2, c=1$

$P(a=2, c=1)$

$= \frac{1}{6} \cdot {}_3C_1 \cdot \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = \underline{\frac{1}{16}}$ //

(2) ①, ②が一致

$\to$ $a=c+1$, $b=d+1$

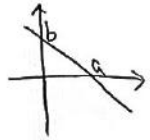
P(①, ②が一致)

$= P(a=1, c=0, b=4)$
$+ P(a=2, c=1, b=3)$
$+ P(a=3, c=2, b=2)$
$+ P(a=4, c=3, b=1)$

$= \frac{1}{6} \cdot \frac{1}{6} \left({}_3C_0 + {}_3C_1 + {}_3C_2 + {}_3C_3\right) \left(\frac{1}{2}\right)^3$

$= \underline{\frac{1}{36}}$ //

(3)



$S_1 = \frac{1}{2} ab$

$S_2 = \frac{1}{2}(c+1)(d+1)$

$P(S_1 = S_2)$

$= P(ab = (c+1)(d+1))$

$= P(ab = 4 \cap c = 0)$
$+ P(ab = 6 \cap c = 1)$
$+ P(ab = 6 \cap c = 2)$
$+ P(ab = 4 \cap c = 3)$

$= P((a,b) = (1,4), (2,2), (4,1))$
$\times {}_3C_0 \cdot \left(\frac{1}{2}\right)^3 \times 2$

$+ P((a,b) = (1,6), (2,3), (3,2), (6,1))$
$\times {}_3C_1 \cdot \left(\frac{1}{2}\right)^3 \times 2$

$= \frac{3}{36} \cdot \frac{1}{8} \cdot 2 + \frac{4}{36} \cdot 3 \cdot \frac{1}{8} \cdot 2$

$= \frac{1}{12} \cdot \frac{1}{4} + \frac{4}{12} \cdot \frac{1}{4} = \underline{\frac{5}{48}}$ //

(4) ①, ②が平行

$\to \begin{cases} ak = c+1 \\ bk = d+1 \end{cases}$ $(k \in \mathbb{Q})$

P(①, ②が平行 $\cap$ (2)以外)

$= P(a=4, b=6, c=1)$
$+ P(a=6, b=4, c=2)$

$= \frac{1}{6} \cdot \frac{1}{6} \left({}_3C_1 + {}_3C_2\right) \left(\frac{1}{2}\right)^3 = \underline{\frac{1}{48}}$ //

$\boxed{2}$

$z^2 = a^2 - b^2 + 2abi$

$3\bar{z} = 3a + 3bi$

↓ 互いに共役

$\begin{cases} a^2 - b^2 = 3a \\ 2ab = -3b \end{cases}$

$\therefore \underline{a = -\frac{3}{2}}$ //

$\frac{9}{4} - b^2 = -\frac{9}{2} \iff \frac{27}{4} = b^2$

$\therefore \underline{b = -\frac{3\sqrt{3}}{2}}$ // $(\because b<0)$

$z = -\frac{3}{2} - \frac{3\sqrt{3}}{2} i$

$= 3\left\{\cos\left(-\frac{2}{3}\pi\right) + i\sin\left(-\frac{2}{3}\pi\right)\right\}$

$z^3 = \underline{27}$ //

3次方程式でzが解なら$\bar{z}$も解.

$z^2 = 9\left\{\cos\left(-\frac{4}{3}\pi\right) + i\sin\left(-\frac{4}{3}\pi\right)\right\}$

$= 9\left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right)$

$= -\frac{9}{2} + \frac{9\sqrt{3}}{2} i$

$\bar{z}^2 = -\frac{9}{2} - \frac{9\sqrt{3}}{2} i$

唯一の実数解をαとおくと

解と係数の関係より

$\underbrace{z^2 + \bar{z}^2}_{-9} + \alpha = -10$

$\therefore \alpha = -1$

他には

$s = z^2 \bar{z}^2 + \bar{z}^2(-1) + (-1) z^2$

$= \underbrace{|z|^4}_{81} - \underbrace{(z^2 + \bar{z}^2)}_{-9} = \underline{90}$ //

$t = -z^2 \bar{z}^2 (-1) = \underline{81}$ //

3次方程式の実数解 $\alpha = \underline{-1}$ //

$\boxed{3}$

$\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$とおく

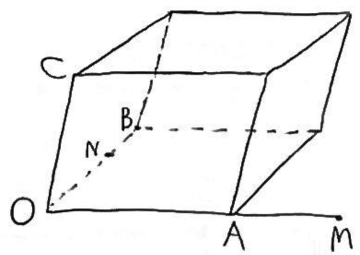
$\overrightarrow{OK}$

$= \frac{1}{3}\left(\overrightarrow{OF} + \overrightarrow{OI} + \overrightarrow{OJ}\right)$

$= \frac{1}{3}\left(\vec{b} + \vec{c} + \vec{a} + \frac{1}{2}\vec{b} + \vec{a} + \vec{b} + \frac{1}{2}\vec{c}\right)$

$= \frac{1}{6}\left(4\vec{a} + 5\vec{b} + 3\vec{c}\right)$

$= \underline{\frac{4\overrightarrow{OA} + 5\overrightarrow{OB} + 3\overrightarrow{OC}}{6}}$ //



$\overrightarrow{OP} = t\overrightarrow{OK}$ とおく.

$= \frac{4}{6}t\vec{a} + \frac{5}{6}t\vec{b} + \frac{3}{6}t\vec{c}$

$= \frac{8}{18}t\left(\frac{3}{2}\vec{a}\right) + \frac{25}{18}t\left(\frac{3}{5}\vec{b}\right) + \frac{9}{18}t\vec{c}$

↓ Pは平面CMN上

$\frac{8}{18}t + \frac{25}{18}t + \frac{9}{18}t = 1$

$\therefore t = \frac{18}{42} = \frac{3}{7}$

$\therefore \overrightarrow{OP} = \frac{12\overrightarrow{OA} + 15\overrightarrow{OB} + 9\overrightarrow{OC}}{42}$

$= \frac{4\overrightarrow{OA} + 5\overrightarrow{OB} + 3\overrightarrow{OC}}{14}$

$OP : OK = 3 : 7$

$\overrightarrow{OQ} = u\overrightarrow{OK}$ とおく

$= \frac{4}{6}u\vec{a} + \frac{5}{6}u\vec{b} + \frac{3}{6}u\vec{c}$

またQは平面BDGF上なので

$\overrightarrow{OQ} = \vec{b} + \alpha\vec{a} + \beta\vec{c}$ とおける.

$= \alpha\vec{a} + \vec{b} + \beta\vec{c}$

$$\begin{cases} \frac{4}{6}u = \alpha \\ \frac{5}{6}u = 1 \\ \frac{3}{6}u = \beta \end{cases} \quad \therefore \quad u = \frac{6}{5},\ \alpha = \frac{4}{5},\ \beta = \frac{3}{5}$$

$\therefore \overrightarrow{OQ} = \frac{4\overrightarrow{OA} + 5\overrightarrow{OB} + 3\overrightarrow{OC}}{5}$

$OQ : OK = 6 : 5$ なので

OP	OQ	OK
3		7
	6	5
15	42	35

$\therefore OP : OQ = 15 : 42$

$= 5 : 14$

$\boxed{4}$

$f(x) = \frac{ax^2 + bx + c}{x+2}$

$f'(x) = \frac{(2ax+b)(x+2) - (ax^2+bx+c)}{(x+2)^2}$

$= \frac{ax^2 + 4ax + 2b - c}{(x+2)^2}$

$$\begin{cases} f'(-3) = -3a + 2b - c = 0 \cdots (\text{i}) \\ f(-3) = -9a + 3b - c = -1 \cdots (\text{ii}) \end{cases}$$

$\downarrow 2b - c = 3a$

$f'(x) = \frac{ax^2 + 4ax + 3a}{(x+2)^2}$

$= \frac{a(x+1)(x+3)}{(x+2)^2}$

$\downarrow k = -1$

$f(-1) = a - b + c = 3 \cdots$

(i)−(ii) $\quad$ (i)+(iii)

$6a - b = 1 \quad -2a + b = 3$

$\therefore a = 1,\ b = 5,\ c = 7$

y切片 $d = \frac{c}{2} = \frac{7}{2}$

$f(x) = \frac{x^2 + 5x + 7}{x+2} = \frac{7}{2}$

$\Leftrightarrow 2x^2 + 10x + 14 = 7x + 14$

$\Leftrightarrow (2x+3)x = 0 \Leftrightarrow x = -\frac{3}{2}, 0$

$\therefore E\left(-\frac{3}{2}, \frac{7}{2}\right)$

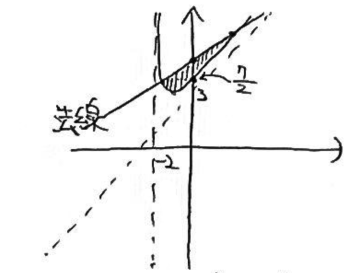
Eでの法線は

$y = -\frac{1}{f'\left(-\frac{3}{2}\right)}\left(x + \frac{3}{2}\right) + \frac{7}{2}$

$= -\frac{1}{-3}\left(x + \frac{3}{2}\right) + \frac{7}{2}$

$= \frac{1}{3}x + 4$

$f(x) = x + 3 + \frac{1}{x+2}$



$\frac{1}{3}x + 4 = \frac{x^2 + 5x + 7}{x+2}$

$\Leftrightarrow (x+12)(x+2) = 3x^2 + 15x + 21$

$\Leftrightarrow 0 = 2x^2 + x - 3$

$\Leftrightarrow 0 = (2x+3)(x-1) \quad \therefore x = -\frac{3}{2}, 1$

(面積)

$= \int_{-\frac{3}{2}}^{1} \left\{\frac{1}{3}x + 4 - \left(x + 3 + \frac{1}{x+2}\right)\right\} dx$

$= \int_{-\frac{3}{2}}^{1} \left(-\frac{2}{3}x + 1 - \frac{1}{x+2}\right) dx$

$= \left[-\frac{1}{3}x^2 + x - \log|x+2|\right]_{-\frac{3}{2}}^{1}$

$= \frac{35}{12} - \log_e 6$