2018 金沢医科(後期)

$\boxed{1}$

$$1=a(-1-m)^2$$
$$2=a(1-m)^2$$
$$\Downarrow$$
$$2(-1-m)^2=(1-m)^2$$
$$\Leftrightarrow m^2+6m+1=0$$
$$\therefore m=-3\pm2\sqrt{2}$$

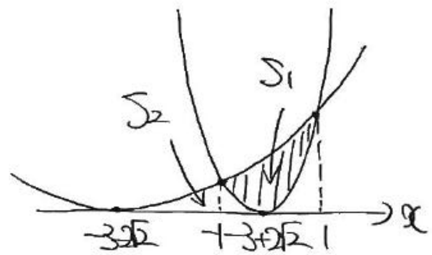
$$m_1=-3-2\sqrt{2},\ m_2=-3+2\sqrt{2}$$

m_1のとき

$$1=a_1(2+2\sqrt{2})^2$$
$$=a_1(12+8\sqrt{2})$$
$$\Leftrightarrow a_1=\frac{1}{12+8\sqrt{2}}$$
$$=\frac{12-8\sqrt{2}}{16}$$
$$=\frac{3-2\sqrt{2}}{4}$$

m_2のとき

$$1=a_2(2-2\sqrt{2})^2$$
$$=a_2(12-8\sqrt{2})$$
$$\therefore a_2=\frac{1}{12-8\sqrt{2}}$$
$$=\frac{12+8\sqrt{2}}{16}$$
$$=\frac{3+2\sqrt{2}}{4}$$



$$S_1=\int_{-1}^{1}\{a_1(x-m_1)^2-a_2(x-m_2)^2\}dx$$
$$=(a_1-a_2)\int_{-1}^{1}(x+1)(x-1)dx$$
$$=(a_1-a_2)\left\{-\frac{1}{6}2^3\right\}$$
$$=\frac{4}{3}(a_2-a_1)=\frac{4\sqrt{2}}{3}$$

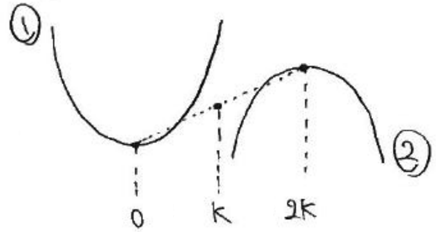
$$S_2=\int_{m_1}^{-1}a_1(x-m_1)^2dx$$
$$+\int_{-1}^{m_2}a_2(x-m_2)^2dx$$
$$=\left[\frac{a_1}{3}(x-m_1)^3\right]_{m_1}^{-1}$$
$$+\left[\frac{a_2}{3}(x-m_2)^3\right]_{-1}^{m_2}$$
$$=\frac{a_1}{3}(-1-m_1)^3-\frac{a_2}{3}(-1-m_2)^3$$
$$=\frac{a_1}{3}(2+2\sqrt{2})^3-\frac{a_2}{3}(2-2\sqrt{2})^3$$
$$=\frac{1}{3}(2+2\sqrt{2})-\frac{1}{3}(2-2\sqrt{2})$$
$$=\frac{4}{3}\sqrt{2}$$

$\boxed{2}$

$$a_{n+1}=16a_n^2$$
$$\Leftrightarrow \log_2 a_{n+1}=\log_2 16a_n^2$$
$$=2\log_2 a_n+4$$
$$\Downarrow\ b_n=\log_2 a_n \qquad b_1=1$$
$$b_{n+1}=2b_n+4$$
$$\Leftrightarrow b_{n+1}+4=2(b_n+4)$$
$$\Downarrow$$
$$b_n+4=(b_1+4)2^{n-1}$$
$$\therefore b_n=5\cdot2^{n-1}-4$$

$$\log_2(a_1\cdot a_2\cdots a_n)$$
$$=\sum_{k=1}^{n}\log a_k$$
$$=\sum_{k=1}^{n}(5\cdot2^{k-1}-4)$$
$$=\frac{5-5\cdot2^n}{1-2}-4n$$
$$=5\cdot2^n-4n-5$$

$\boxed{3}$ (1)



②の頂点$(2k,2)$

②のx^2の係数は$-\frac{1}{2}$

(2)

①と②を連立

$$\frac{1}{2}x^2=-\frac{1}{2}(x-2k)^2+2$$
$$=-\frac{1}{2}x^2+2kx-2k^2+2$$
$$\Leftrightarrow x^2-2kx+2k^2-2=0$$
$$\frac{D}{4}=k^2-(2k^2-2)$$
$$=2-k^2=0 \quad \therefore k=\sqrt{2}$$

このとき①.②の交点は

$$x^2-2\sqrt{2}x+2=0 \quad \therefore x=\sqrt{2}$$
$$C(\sqrt{2},1)$$

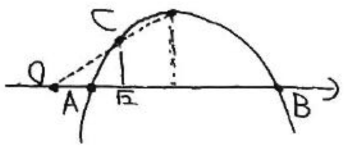
A.Bの座標は

$$0=-\frac{1}{2}(x-2\sqrt{2})^2+2$$
$$\Leftrightarrow (x-2\sqrt{2})^2=4$$

$\therefore x-2\sqrt{2}=\pm 2$

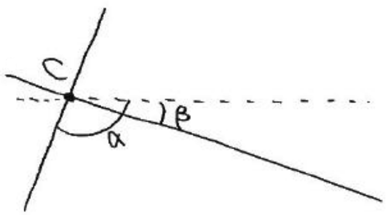
$\underline{A(2\sqrt{2}-2,0),\ B(2\sqrt{2}+2,0)}$ //

(3) $k=\sqrt{2}$ のとき



$\triangle OCA=(2\sqrt{2}-2)\times 1\times\frac{1}{2}$

$=\underline{\sqrt{2}-1}$ //



$\tan\alpha=\frac{1}{2-\sqrt{2}}=\frac{2+\sqrt{2}}{2}$

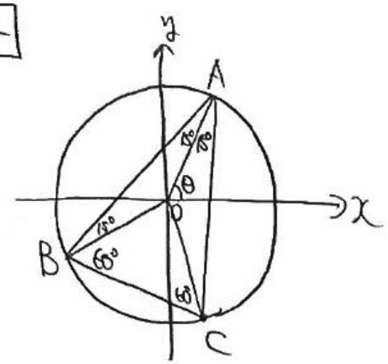
$\tan\beta=\frac{-1}{\sqrt{2}+2}=\frac{\sqrt{2}-2}{2}$

$\tan(\beta-\alpha)=\frac{\tan\beta-\tan\alpha}{1+\tan\beta\tan\alpha}$

$=\frac{-2}{1+\frac{-2}{4}}$

$=\underline{-4}$ //

$\boxed{4}$



$\angle AOB=\frac{5}{6}\pi$

$\therefore x_2=\underline{\cos\left(\theta+\frac{5}{6}\pi\right)}$ //

$y_2=\sin\left(\theta+\frac{5}{6}\pi\right)$

$\angle BOC=\frac{\pi}{3}$

$\therefore x_3=\underline{\cos\left(\theta+\frac{7}{6}\pi\right)}$ //

$y_3=\sin\left(\theta+\frac{7}{6}\pi\right)$

$\sum_{k=1}^{3}(x_k+y_k)$

$=\cos\theta+\cos\left(\theta+\frac{5}{6}\pi\right)+\cos\left(\theta+\frac{7}{6}\pi\right)$

$+\sin\theta+\sin\left(\theta+\frac{5}{6}\pi\right)+\sin\left(\theta+\frac{7}{6}\pi\right)$

$=\cos\theta+\cos\left(\theta+\pi+\frac{\pi}{6}\right)+\cos\left(\theta+\pi-\frac{\pi}{6}\right)$

$+\sin\theta+\sin\left(\theta+\pi+\frac{\pi}{6}\right)+\sin\left(\theta+\pi-\frac{\pi}{6}\right)$

$=\cos\theta+2\cos(\theta+\pi)\cos\frac{\pi}{6}$

$+\sin\theta+2\sin(\theta+\pi)\cos\frac{\pi}{6}$

$=\cos\theta-\sqrt{3}\cos\theta$

$+\sin\theta-\sqrt{3}\sin\theta$

$=(1-\sqrt{3})(\sin\theta+\cos\theta)$

$=\underline{(\sqrt{2}-\sqrt{6})}\sin\left(\theta+\frac{\pi}{4}\right)$ //

$\theta=\frac{5}{4}\pi$ のとき最大値 $\underline{\sqrt{6}-\sqrt{2}}$ //

$\sum_{k=1}^{3}(x_k^2+y_k^2)$

$=1+1+1=\underline{3}$ //