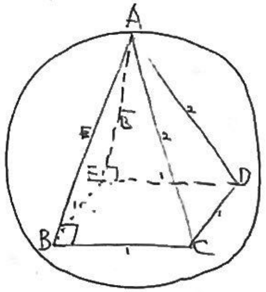
2018 岩手医科大

第1問

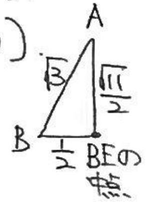


問1.

B(0, 0, 0), C(1, 0, 0),

D(1, 1, 0), E(0, 1, 0)

$A\left(0, \frac{1}{2}, \frac{\sqrt{11}}{2}\right) \leftarrow$



とおく.

図から球の中心を

$O\left(x, \frac{1}{2}, z\right)$ とおける.

$\downarrow$

$OB^2 = OA^2 = OD^2 = r^2$

$x^2 + \frac{1}{4} + z^2 = x^2 + \left(z - \frac{\sqrt{11}}{2}\right)^2$

$= (x-1)^2 + \frac{1}{4} + z^2$

$\downarrow$

$$\begin{cases} \frac{1}{4} = -\sqrt{11}z + \frac{11}{4} \\ x^2 = (x-1)^2 \end{cases}$$

解くと

$x = \frac{1}{2}$, $z = \frac{5}{2\sqrt{11}}$

$r^2 = \frac{1}{4} + \frac{1}{4} + \frac{25}{44} = \frac{47}{44}$

$\therefore r = \frac{\sqrt{47}}{2\sqrt{11}} = \underline{\frac{\sqrt{517}}{22}}$

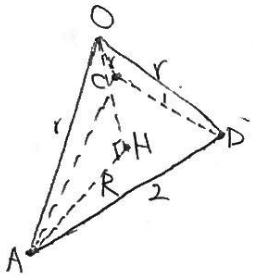
問2.

余弦定理

$\cos\angle CAD = \frac{4+4-1}{2 \cdot 2 \cdot 2} = \underline{\frac{7}{8}}$

$\triangle ACD = \frac{1}{2} 2 \cdot 2 \sin\angle CAD$

$= 2 \cdot \frac{\sqrt{15}}{8} = \underline{\frac{\sqrt{15}}{4}}$



OからΔ垂線の足をHとおく.

$\triangle ACD$の外接円の半径Rは

正 $2R = \frac{1}{\sin\angle CAD} = \frac{8}{\sqrt{15}}$

$\therefore R = \frac{4}{\sqrt{15}}$

$\triangle OAH \equiv \triangle ODH \equiv \triangle OCH$より

Hは$\triangle ACD$の外心である.

$\therefore OH = \sqrt{r^2 - R^2}$

$= \sqrt{\frac{47}{44} - \frac{16}{15}}$

四面体OACD

$= \frac{\sqrt{15}}{4} \times \sqrt{\frac{47}{44} - \frac{16}{15}} \times \frac{1}{3}$

$= \frac{1}{12}\sqrt{\frac{47 \cdot 15}{44} - 16}$

$= \frac{1}{24}\sqrt{\frac{47 \cdot 15}{11} - 64}$

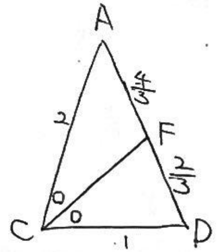
$= \frac{1}{24}\sqrt{\frac{47 \cdot 15 - 64 \cdot 11}{11}}$

$= \frac{1}{24} \cdot \frac{1}{\sqrt{11}} = \underline{\frac{\sqrt{11}}{264}}$

問3.

(四角錐ABCDE)

$= 1 \times \frac{\sqrt{11}}{2} \times \frac{1}{3} = \underline{\frac{\sqrt{11}}{6}}$



余

$CF^2 = 2^2 + \left(\frac{4}{3}\right)^2 - 2 \cdot 2 \cdot \frac{4}{3} \cos\angle CAD$

$= 4 + \frac{16}{9} - \frac{14}{3}$

$= \frac{36 + 16 - 42}{9} = \frac{10}{9}$

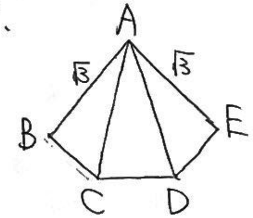
$\therefore CF = \underline{\frac{\sqrt{10}}{3}}$

(三角錐ABCF)

$= (\text{三角錐ABCD}) \times \frac{2}{3}$

$= \frac{\sqrt{11}}{6} \times \frac{1}{2} \times \frac{2}{3} = \underline{\frac{\sqrt{11}}{18}}$

問4.



$l = BP + PQ + QE$

$\geq BE$

余

$BE^2 = 3 + 3 - 2\sqrt{3}\sqrt{3}\cos\left(\angle CAD + \frac{\pi}{3}\right)$

$= 6 - 6\left(\frac{7}{8} \cdot \frac{1}{2} - \frac{\sqrt{15}}{8} \cdot \frac{\sqrt{3}}{2}\right)$

$= \frac{6}{16}\left(16 - 7 + \sqrt{45}\right)$

$= \frac{3}{16}\left(18 + 2\sqrt{45}\right)$

$\therefore BE = \frac{\sqrt{3}}{4}\left(\sqrt{15} + \sqrt{3}\right)$

$= \frac{3\sqrt{5} + 3}{4}$

第2問

問1.

1ケタ…4コ

2ケタ…$4^2=16$コ

3ケタ…$4^3=64$コ

⋮

$$\begin{array}{r} 4\,\underline{)\,30} \\ 4\,\underline{)\;\;7}\cdots 2 \\ 1\cdots 3 \end{array}$$

↓

$a_{30}=\underline{153}$

また77は2ケタ最後の数.

$a_n=77 \Leftrightarrow n=\underline{20}$

問2.

$a_n=7531$

$\Leftrightarrow n=4\cdot 4^3+3\cdot 4^2+2\cdot 4+1$

$=\underline{313}$

問3

$10<a_n<100$

↓

$11\leqq a_n\leqq 77$

(総和)

$=11+13+15+17$

$+31+33+35+37$

$+51+53+55+57$

$+71+73+75+77$

$=(1+3+5+7)\times 10\times 4+(1+3+5+7)\times 4$

$=16\times 44$

$=64\times 11=\underline{704}$

問4.

$a_m=777$

$\Leftrightarrow m=4+16+64=\underline{84}$

$a_n<1000$で100の位が1,3,5であるものは $4^2=\underline{16}$コある.

$\displaystyle\sum_{k=1}^{84} a_k$

$=$(1ケタの数の和)

$+$(2ケタ 〃)

$+$(3ケタ 〃)

$=16+704$

$+\{(1+3+5+7)\times 1600+704\times 4\}$

$=25616+704\cdot 5=29136$

これを素因数分解すると

$29136=\underline{2^4\times 3\times 607}$

第3問

問1.

$y^2=x^2(2-|x|)\geqq 0$

$\Leftrightarrow |x|\leqq 2$

$\therefore\ \underline{-2\leqq x\leqq 2}$

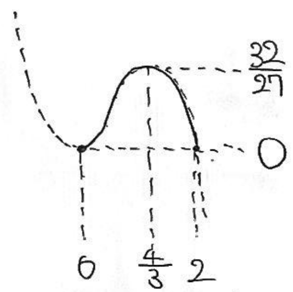
Cはx軸, y軸に関して対称である.

第1象限を調べる.

$y^2=x^2(2-x)=-x^3+2x^2$ …①

$\dfrac{d(y^2)}{dx}=-3x^2+4x$

$=-3x\left(x-\dfrac{4}{3}\right)$



y^2の最大値 $\dfrac{32}{27}$.

yの最大値 $\sqrt{\dfrac{32}{27}}=\dfrac{4\sqrt{2}}{3\sqrt{3}}=\underline{\dfrac{4\sqrt{6}}{9}}$

問2.

①をxで微分.

$2yy'=-3x^2+4x$

$\Leftrightarrow y'=\dfrac{-3x^2+4x}{2y}$

(1,1)での接線

$y=\dfrac{-3+4}{2}(x-1)+1$

$=\underline{\dfrac{1}{2}x+\dfrac{1}{2}}$

(1,1)での法線

$y=-2(x-1)+1$

$=\underline{-2x+3}$

問3

(体積)

$=\displaystyle\int_0^2 y^2\pi\,dx$

$=\pi\displaystyle\int_0^2(-x^3+2x^2)\,dx$

$=\pi\left[-\dfrac{1}{4}x^4+\dfrac{2}{3}x^3\right]_0^2$

$=\pi\left(-4+\dfrac{16}{3}\right)$

$=\underline{\dfrac{4}{3}\pi}$

四(4).

Cはx軸, y軸に関して対称、なので第1象限の面積を4倍する.

(総面積)

$$=4\int_0^2 x\sqrt{2-x}\,dx$$

$\downarrow$ $2-x=t$, $dx=-dt$

$$=4\int_2^0 (2-t)\sqrt{t}\,(-dt)$$

$$=4\int_0^2 \left(2t^{\frac{1}{2}}-t^{\frac{3}{2}}\right)dt$$

$$=4\left[\frac{4}{3}t^{\frac{3}{2}}-\frac{2}{5}t^{\frac{5}{2}}\right]_0^2$$

$$=4\left(\frac{4}{3}2\sqrt{2}-\frac{2}{5}4\sqrt{2}\right)$$

$$=4\left(\frac{1}{3}-\frac{1}{5}\right)8\sqrt{2}$$

$$=\frac{64\sqrt{2}}{15}$$

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