2018 福岡大(医)

[I]

(i) 234234

$= 234\cdot 1001$

$= 2\cdot 3^2\cdot 13\cdot 1001$

$= 2\cdot 3^2\cdot 13\cdot 7\cdot 143$

$= 2\cdot 3^2\cdot 7\cdot 11\cdot 13^2$

$9\,)\,234 = 26$

$7\,)\,1001 = 143$ (7, 30, 28, 21)

正の約数… $2\cdot 3\cdot 2\cdot 2\cdot 3$

$= \underline{72\text{コ}}$

$n = 7\cdot 11\cdot 13$

$= 1001$

$8\,)\,1001$

$8\,)\,125 \cdots 1$

$8\,)\,15 \cdots 5$

$1 \cdots 7$

$n = \underline{1751_{(8)}}$

(ii)

A E D B C

97 96 86 86 66

$\quad\quad\;\;x$

$\quad\quad\;\;\wr$

A B E D C

97 86 86 76 66

$\quad\quad\quad\;\;x$

$\underline{76 \leqq x \leqq 86}$

A B C D E

97 86 66 x $x+10$

A.B.Cの平均が83なので D、Eの平均も83になれば分散が最小、そのときのxは78.

s^2

$= \frac{1}{5}\left(14^2+3^2+17^2+5^2+5^2\right)$

$= \frac{1}{5}\left(196+9+289+50\right)$

$= \underline{\frac{544}{5}}$

別

$(\text{平均}) = \frac{1}{5}(97+86+66+x+x+10)$

$= \frac{2x+259}{5}$

s^2

$= \frac{1}{5}\left(97^2+86^2+66^2+x^2+(x+10)^2\right) - \left(\frac{2x+259}{5}\right)^2$

$= \frac{6}{25}x^2 - \frac{936}{25}x + C$ ← 残り

$= \frac{6}{25}(x^2-156x) + C$

平方完成すれば $x=78$ で分散最小.

後は同じ.

(iii) [1] $b \neq 0$ のとき

$f(x) = bx^2-2ax-b-4$

$= b\left(x-\frac{a}{b}\right)^2 - \frac{a^2}{b} - b - 4$

(i) $b>0$ のとき

$\begin{cases} f(-1-\sqrt{2}) \leqq 0 \\ f(1+\sqrt{2}) \leqq 0 \end{cases}$

$\Longleftrightarrow \begin{cases} b \leqq -a+2\sqrt{2}-2 \\ b \leqq a+2\sqrt{2}-2 \end{cases}$

(ii) $b<0$ のとき

① $\frac{a}{b} \leqq -1-\sqrt{2} \Longleftrightarrow b \geqq -(\sqrt{2}-1)a$ のとき

$f(-1-\sqrt{2}) \leqq 0$

$\Longleftrightarrow b \leqq -a+2\sqrt{2}-2$

② $-1-\sqrt{2} < \frac{a}{b} < 1+\sqrt{2}$

$\Longleftrightarrow b < -(\sqrt{2}-1)a$ かつ $b < (\sqrt{2}-1)a$ のとき

$f\left(\frac{a}{b}\right) = -\frac{a^2}{b} - b - 4 \leqq 0$

$\Longleftrightarrow a^2+(b+2)^2 \leqq 4$

③ $\frac{a}{b} \geqq 1+\sqrt{2} \Longleftrightarrow b \geqq (\sqrt{2}-1)a$ のとき

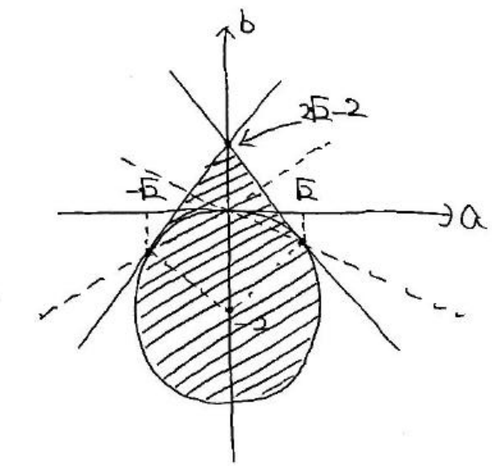
$f(1+\sqrt{2}) \leqq 0$

$\Longleftrightarrow b \leqq a+2\sqrt{2}-2$

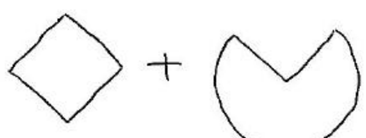
[2] $b=0$ のとき

$\begin{cases} f(-1-\sqrt{2}) \leqq 0 \\ f(1+\sqrt{2}) \leqq 0 \end{cases}$

$\Longleftrightarrow \begin{cases} b \leqq -a+2\sqrt{2}-2 \\ b \leqq a+2\sqrt{2}-2 \end{cases}$



求める面積は

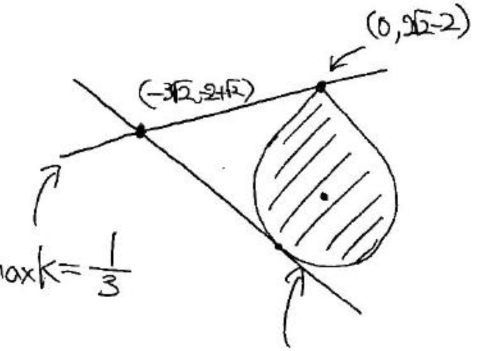


$= 2\sqrt{2}\cdot 2\sqrt{2}\cdot\frac{1}{2} + 2\cdot 2\cdot\pi\cdot\frac{3}{4}$

$= \underline{3\pi + 4}$ //

$k = \frac{b+2-\sqrt{2}}{a+3\sqrt{2}}$

$\iff b = k(a+3\sqrt{2}) - (2-\sqrt{2})$



これと(0, −2)の距離 = 2

$\frac{|3\sqrt{2}k+\sqrt{2}|}{\sqrt{k^2+1}} = 2$

解くと $k = -1, \frac{1}{7}$

$\therefore \min k = -1$

以上より

$\underline{-1 \leqq k \leqq \frac{1}{3}}$ //

[II]

(1)

$f(x)$

$= 4\cos^3 x - 3\cos x + 2\cos^2 x - 1 + \cos x$

$= 4\cos^3 x + 2\cos^2 x - 2\cos x - 1$

$\downarrow \cos x = t$

$= 4t^3 + 2t^2 - 2t - 1$

$= (2t+1)(2t^2 \quad -1)$

$= 4\left(t+\frac{1}{2}\right)\left(t-\frac{1}{\sqrt{2}}\right)\left(t+\frac{1}{\sqrt{2}}\right)$

$f(x) = 0$

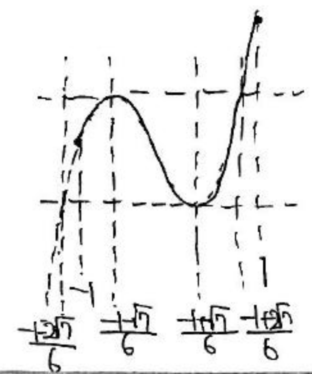
$\iff t = -\frac{1}{2}, \pm\frac{1}{\sqrt{2}}$

$\therefore \underline{x = \frac{\pi}{4}, \frac{2}{3}\pi, \frac{3}{4}\pi}$ //

$\frac{df(x)}{dt} = 12t^2 + 4t - 2$

$= 2(6t^2 + 2t - 1)$

$\frac{df(x)}{dt} = 0 \iff t = \frac{-1\pm\sqrt{7}}{6}$ ← 約2.64



$t = \frac{-1+\sqrt{7}}{6}$ のとき $f(x)$ が最小.

$$
\begin{array}{rrrrr}
 & & \frac{2}{3} & \frac{1}{9} & \\
\hline
6\ 2\ -1 \,) & 4 & 2 & -2 & -1 \\
 & 4 & \frac{4}{3} & -\frac{2}{3} & \\
\hline
 & & \frac{2}{3} & -\frac{4}{3} & -1 \\
 & & \frac{2}{3} & \frac{2}{9} & -\frac{1}{9} \\
\hline
 & & & -\frac{14}{9} & -\frac{8}{9}
\end{array}
$$

つまり

$4t^3 + 2t^2 - 2t - 1$

$= (6t^2+2t-1)\left(\frac{2}{3}t+\frac{1}{9}\right) - \frac{14}{9}t - \frac{8}{9}$

最小値は $\downarrow t = \frac{-1+\sqrt{7}}{6}$

$-\frac{14}{9}\cdot\frac{-1+\sqrt{7}}{6} - \frac{8}{9} = \underline{\frac{-17-7\sqrt{7}}{27}}$ //

(ii)

$\lim_{x\to+\infty} \sqrt{x}\left(\sqrt{1+x}-\sqrt{x}\right)$

$= \lim_{x\to+\infty} \sqrt{x}\cdot\frac{1}{\sqrt{1+x}+\sqrt{x}} = \underline{\frac{1}{2}}$ //

$f(x) = \cos^2 x$ とおくと

平均値の定理より

$$
\begin{cases}
\dfrac{f\left(\frac{k}{n}\pi\right) - f\left(\frac{k-1}{n}\pi\right)}{\frac{k}{n}\pi - \frac{k-1}{n}\pi} = f'(c) \\
\dfrac{k-1}{n}\pi < c < \dfrac{k}{n}\pi
\end{cases}
$$

なる c が存在

$\left|\cos^2\frac{k}{n}\pi - \cos^2\frac{k-1}{n}\pi\right|$

$= |-2\sin c\cos c|\frac{\pi}{n}$

$= \frac{\pi}{n}|\sin 2c|$

(与式)

$= \lim_{n\to\infty}\sum_{k=1}^{2n}\frac{\pi}{n}|\sin 2c|$

$= \pi\int_0^2 |\sin 2\pi x|$

$= \pi\cdot 4\int_0^{\frac{1}{2}}\sin 2\pi x\,dx$

$= 4\pi\left[-\frac{1}{2\pi}\cos 2\pi x\right]_0^{\frac{1}{2}}$

$= 4\pi\left(\frac{1}{2\pi}+\frac{1}{2\pi}\right) = \underline{4}$ //

[III]

(i)

$$(e^{2x}-1)^2 = 1+\frac{2}{e^{2x}-e^{-2x}}$$

$$\Leftrightarrow e^{4x}-2e^{2x} = \frac{2}{e^{2x}-e^{-2x}}$$

$$\Leftrightarrow e^{2x}-2 = \frac{2}{e^{4x}-1}$$

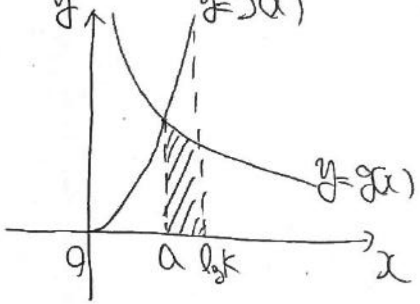
$$\Leftrightarrow e^{6x}-2e^{4x}-e^{2x}=0$$

$$\Leftrightarrow e^{4x}-2e^{2x}-1=0$$

$$\Leftrightarrow e^{2x}=1+\sqrt{2} \quad (\because e^{2x}>0)$$

$$\Leftrightarrow 2x=\log(1+\sqrt{2})$$

$$\therefore x=a=\frac{1}{2}\log(1+\sqrt{2})$$



$$\int_a^{\log k}\left(1+\frac{2}{e^{2x}-e^{-2x}}\right)dx$$

$$=\int_a^{\log k} 1\cdot dx + \int_a^{\log k}\frac{2e^{2x}}{e^{4x}-1}dx \qquad \left\{\begin{array}{l} e^{2x}=t \\ 2e^{2x}dx=dt\end{array}\right.$$

$$=\log k - a + \int_{e^{2a}}^{k^2}\frac{1}{t^2-1}dt$$

$$=\log k - a + \int_{e^{2a}}^{k^2}\left(\frac{1}{t-1}-\frac{1}{t+1}\right)\frac{1}{2}dt$$

$$=\log k - a + \frac{1}{2}\Big[\log|t-1|-\log|t+1|\Big]_{e^{2a}}^{k^2}$$

$$=\log k - a + \frac{1}{2}\log\frac{k^2-1}{k^2+1} - \frac{1}{2}\log\frac{\sqrt{2}}{2+\sqrt{2}}$$

$$=\frac{1}{2}\log k^2 - a + \frac{1}{2}\log\frac{k^2-1}{k^2+1} + a$$

$$=\frac{1}{2}\log\frac{k^2(k^2-1)}{k^2+1} = \log 6 - \frac{1}{2}\log 5$$

$$\Leftrightarrow \log\frac{k^2(k^2-1)}{k^2+1} = \log\frac{36}{5}$$

$$\Downarrow$$

$$5k^2(k^2-1)=36k^2+36$$

$$\Leftrightarrow 5k^4-41k^2-36=0$$

$$\Leftrightarrow (5k^2+4)(k^2-9)=0$$

$$\therefore k^2=9$$

$$\therefore \underline{k=3} \quad (\because \log k \geq a)$$