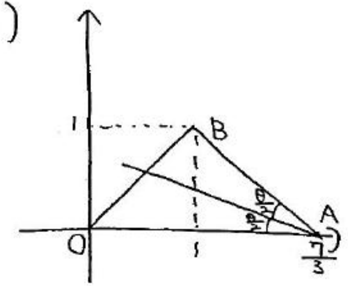
2017 東京医科大

$\boxed{1}$

(1)



$\angle OAB=\theta$ とおく. $\tan\theta=\frac{3}{4}$

$$\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\frac{3}{4}$$

$$\Leftrightarrow 8\tan\frac{\theta}{2}=3-3\tan^2\frac{\theta}{2}$$

$$\Leftrightarrow 3\tan^2\frac{\theta}{2}+8\tan\frac{\theta}{2}-3=0$$

$$\Leftrightarrow \left(3\tan\frac{\theta}{2}-1\right)\left(\tan\frac{\theta}{2}+3\right)=0$$

$$\therefore \tan\frac{\theta}{2}=\frac{1}{3}$$

$\angle OAB$の二等分線 $y=-\frac{1}{3}\left(x-\frac{7}{3}\right)$ と $y=x^2$ を連立.

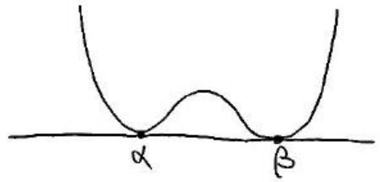
$$x^2=-\frac{1}{3}x+\frac{7}{9}$$

$$\Leftrightarrow 9x^2+3x-7=0$$

$$\therefore P=\frac{-3+\sqrt{9+4\cdot 9\cdot 7}}{18}$$

$$=\underline{\frac{-1+\sqrt{29}}{6}}$$

(2)



$f(x)=(x-\alpha)^2(x-\beta)^2$ とできるので解と係数の関係より

$$\begin{cases} 2\alpha+2\beta=-5 \\ \alpha^2\beta^2=9 \end{cases}$$

$$\therefore \begin{cases} \alpha+\beta=-\frac{5}{2}=\underline{\frac{-5}{2}} \\ \alpha\beta=\pm 3 \end{cases}$$

α, βを解とする2次方程式は

$$t^2+\frac{5}{2}t\pm 3=0$$

$$\Leftrightarrow 2t^2+5t\pm 6=0$$

$$D=25-4\cdot 2(\pm 6)$$

ここで $\alpha\beta=3$ では $D<0$ となるので不適. $\therefore \alpha\beta=\underline{-3}$

再び解と係数の関係より

$$\begin{cases} \alpha^2+\alpha\beta+\alpha\beta+\alpha\beta+\alpha\beta+\beta^2=a \\ \alpha\beta^2+\alpha\beta^2+\alpha^2\beta+\alpha\beta^2=-(-b) \end{cases}$$

$$\Leftrightarrow \begin{cases} (\alpha+\beta)^2+2\alpha\beta=\underline{\frac{1}{4}=a} \\ \alpha\beta(2\alpha+2\beta)=\underline{15=b} \end{cases}$$

$\boxed{2}$

(1)

$$\vec{a}-t\vec{b}=\begin{pmatrix}3-t\\4-2t\end{pmatrix} \quad \vec{a}+t\vec{b}=\begin{pmatrix}3+t\\4+2t\end{pmatrix}$$

$f(t)$

$$=\frac{1}{t}\left(\frac{1}{\sqrt{(3-t)^2+(4-2t)^2}}-\frac{1}{\sqrt{(3+t)^2+(4+2t)^2}}\right)$$

$$=\frac{\frac{1}{\sqrt{5t^2-22t+25}}-\frac{1}{\sqrt{5t^2+22t+25}}}{t}$$

$$=\frac{g(t)-h(t)}{t}$$ とおく.

$$\lim_{t\to 0} f(t)$$

$$=\lim_{t\to 0}\left\{\frac{g(t)-g(0)}{t-0}-\frac{h(t)-h(0)}{t-0}\right\}$$

$$=g'(0)-h'(0)$$

$$\begin{cases} g'(t)=-\frac{1}{2}(5t^2-22t+25)^{-\frac{3}{2}}(10t-22) \\ h'(t)=-\frac{1}{2}(5t^2+22t+25)^{-\frac{3}{2}}(10t+22) \end{cases}$$

$$=11\frac{1}{125}-(-11)\frac{11}{125}$$

$$=\underline{\frac{22}{125}}$$

(2)

$t=ux$ とおくと

$dt=x\,du$

t	$1\to x$
u	$\frac{1}{x}\to 1$

$$f(x)=\int_{\frac{1}{x}}^{1}\frac{x+4ux}{\sqrt{3x^4+u^4x^4}}x\,du$$

$$=\int_{\frac{1}{x}}^{1}\underbrace{\frac{1+4u}{\sqrt{3+u^4}}}_{g(u)}du$$

$$=G(1)-G\left(\frac{1}{x}\right)$$

$f'(x)$

$$=-g\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)=\frac{1}{x^2}g\left(\frac{1}{x}\right)$$

$f'(2)$

$$=\frac{1}{4}\cdot\frac{1+4\frac{1}{2}}{\sqrt{3+\frac{1}{16}}}$$

$$=\frac{1+2}{\sqrt{48+1}}=\underline{\frac{3}{7}}$$

3

$y=6-\frac{9}{2}x^2$

$y'=-9x$

P_tでの接線は

$y=-9t(x-t)+6-\frac{9}{2}t^2$

$=-9tx+\frac{9}{2}t^2+6$

$A_t\left(\frac{\frac{9}{2}t^2+6}{9t},\ 0\right)$

$B_t\left(0,\ \frac{9}{2}t^2+6\right)$

より

$S(t)$

$=\frac{\frac{9}{2}t^2+6}{9t}\cdot\left(\frac{9}{2}t^2+6\right)\frac{1}{2}$

$=\frac{1}{9t}(9t^2+12)^2\frac{1}{8}$

$=\frac{1}{72t}(81t^4+216t^2+144)$

$=\underline{\frac{9}{8}t^3+3t+\frac{2}{t}}$ //

$S'(t)$

$=\frac{27}{8}t^2+3-\frac{2}{t^2}$

$=\frac{27t^4+24t^2-16}{8t^2}$

$=\frac{(9t^2-4)(3t^2+4)}{8t^2}$

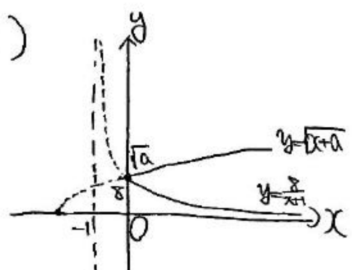
t	$0\cdots\frac{2}{3}\cdots$
$S'(t)$	$-\ \ 0\ \ +$
$S(t)$	$\searrow\ \ \ \nearrow$

$t=\frac{2}{3}$のとき

$m=\frac{1}{3}+2+3=\underline{\frac{16}{3}}$ //

4

(1)



図より $\sqrt{a}\leqq 8$ であればいいので

$\underline{0<a\leqq 64}$ //

(2) $a=1$のとき

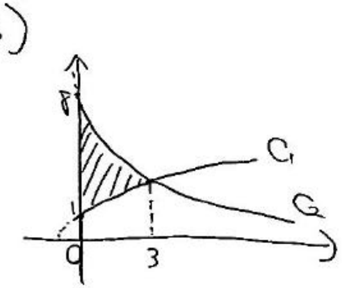
$\sqrt{x+1}=\frac{8}{x+1}$

$\Leftrightarrow (x+1)^{\frac{3}{2}}=8$

$\therefore x=3$

$\therefore \underline{p=3,\ r=2}$ //

(3)



S

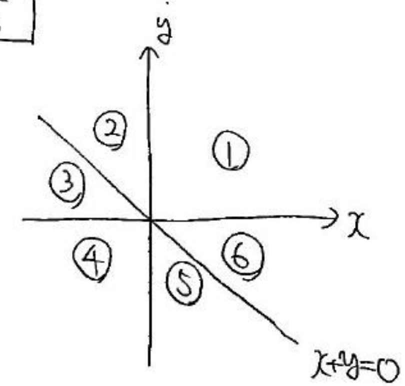
$=\int_0^3\left(\frac{8}{x+1}-\sqrt{x+1}\right)dx$

$=\left[8\log|x+1|-\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_0^3$

$=8\log 4-\frac{2}{3}(8-1)$

$=\underline{16\log 2-\frac{14}{3}}$ //

5



①のとき

$x+y+x+y\leqq 2$

$\Leftrightarrow\ y\leqq -x+1$

②のとき

$-x+y+x+y\leqq 2$

$\Leftrightarrow y\leqq 1$

③のとき

$-x+y-x-y\leqq 2$

$\Leftrightarrow x\geqq -1$

④のとき

$-x-y-x-y\leqq 2$

$\Leftrightarrow y\geqq -x-1$

⑤のとき

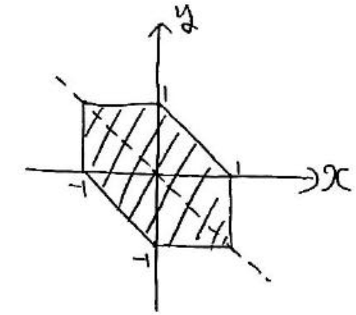
$x-y-x-y\leqq 2$

$\Leftrightarrow y\geqq -1$

⑥のとき

$x-y+x+y\leqq 2$

$\Leftrightarrow x\leqq 1$



領域は斜線部分.

境界線含む.