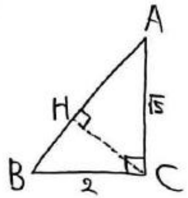
2017 東邦大(医)

1

(1)



$\triangle ACH \backsim \triangle ABC$

$\sqrt{5} : AH = 3 : \sqrt{5}$

$\therefore AH = \dfrac{5}{3}$

$\dfrac{1}{\tan A} + \dfrac{1}{\tan B}$

$= \tan B + \dfrac{1}{\tan B}$

$= \dfrac{\sqrt{5}}{2} + \dfrac{2}{\sqrt{5}}$

$= \dfrac{9}{2\sqrt{5}} = \dfrac{9\sqrt{5}}{10}$

2

両辺 $1+x^3$ 倍する.

$1-x = (a+bx)(1+x) + c(x^2-x+1)$

$= \underbrace{(b+c)}_{0}x^2 + \underbrace{(a+b-c)}_{-1}x + \underbrace{a+c}_{1}$

$a+2b=-1$

$-)\ a-b=1$

$3b=-2$

$\therefore b=-\dfrac{2}{3} \quad c=\dfrac{2}{3} \quad a=\dfrac{1}{3}$

$\displaystyle\int_0^1 \frac{1-x}{1+x^3}\,dx$

$\displaystyle= \int_0^1 \left(\frac{\frac{1}{3}-\frac{2}{3}x}{x^2-x+1} + \frac{\frac{2}{3}}{1+x} \right) dx$

$= \left[-\dfrac{1}{3}\log|x^2-x+1| + \dfrac{2}{3}\log|1+x| \right]_0^1$

$= \dfrac{2}{3}\log 2$

3

$2^x - 2^8 \leqq 4 - 2^{10}\cdot 2^{-x}$

$\Leftrightarrow (2^x)^2 - (2^8+4)2^x + 2^{10} \leqq 0$

$\Leftrightarrow (2^x - 2^8)(2^x - 2^2) \leqq 0$

$\Leftrightarrow 2^2 \leqq 2^x \leqq 2^8$

$\therefore 2 \leqq x \leqq 8$

$f(x) = \dfrac{\log_2 x}{\log_2 4} + \dfrac{2}{\log_2 x}$

$= \dfrac{\log_2 x}{2} + \dfrac{2}{\log_2 x}$

$\dfrac{\log_2 x}{2} > 0,\ \dfrac{2}{\log_2 x} > 0$

(相加平均) $\geqq$ (相乗平均) より

$f(x) \geqq 2\sqrt{\dfrac{\log_2 x}{2}\cdot\dfrac{2}{\log_2 x}}$

$= 2$

等号成立は $\dfrac{\log_2 x}{2} = \dfrac{2}{\log_2 x}$

$\therefore \log_2 x = 2 \quad x=4$ のとき

$x=4$ のとき最小値 2

$x=2$ ： 最大値 $\dfrac{1}{2}+2=\dfrac{5}{2}$

4

$r = -16\sin\left(\theta+\dfrac{\pi}{3}\right)$

$\Leftrightarrow r^2 = -16r\left(\dfrac{1}{2}\sin\theta + \dfrac{\sqrt{3}}{2}\cos\theta\right)$

$\Leftrightarrow x^2+y^2 = -8y - 8\sqrt{3}x$

$\Leftrightarrow (x+4\sqrt{3})^2 + (y+4)^2 = 64$

中心 $(-4\sqrt{3}, -4)$, 半径 8

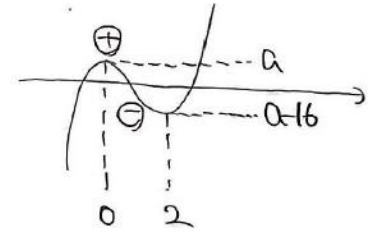
5

$f(x) = x^4 - 4x^3 + ax - 10$

$f'(x) = 4x^3 - 12x^2 + a$

$f''(x) = 12x^2 - 24x = 12x(x-2)$

変曲点の x 座標 $x=0, 2$



極大値をもつには $f'(x)$ の符号が $+$ から $-$ に変化すればよい.

$\therefore a-16 < 0 < a$

$\therefore 0 < a < 16$

6

$\overrightarrow{OA}=\vec{a},\ \overrightarrow{OB}=\vec{b}$ とする.

$\overrightarrow{OG} = \dfrac{1}{3}(\vec{a}+\vec{b})$

$|\overrightarrow{OG}|^2 = \dfrac{1}{9}(25+9+\underbrace{2\vec{a}\cdot\vec{b}}_{20}) = 6$

$\therefore \overrightarrow{OA}\cdot\overrightarrow{OB} = \vec{a}\cdot\vec{b} = 10$

$\triangle OAB$

$= \dfrac{1}{2}\sqrt{|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2}$

$= \dfrac{1}{2}\sqrt{15^2-10^2}$

$= \dfrac{5\sqrt{5}}{2}$

[7]

∠ABC$=\theta$とすると

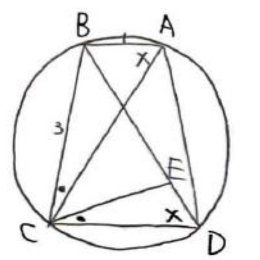
△ABCと△ACDに余弦定理より

$AC^2 = 1^2+3^2-2\cdot1\cdot3\cos\theta$

$-)\ AC^2 = 2^2+3^2-2\cdot2\cdot3\cos(180^\circ-\theta)$

$0 = -3-18\cos\theta$

$\therefore \cos\theta = \cos\angle ABC = \frac{-1}{6}$ //



$AC^2=11 \quad \therefore AC=\sqrt{11}$

△ABC∽△DECより

$\sqrt{11}:3=2:CE$

$\Leftrightarrow CE=\frac{6}{\sqrt{11}}=\frac{6\sqrt{11}}{11}$ //

[8]

$\overline{x}$

$=\frac{1}{25}$(Aの合計+Bの合計)

$=\frac{1}{25}(1050+800)=74$ //

S_x^2

$=\overline{x^2}-(\overline{x})^2$

$=\frac{1}{25}(15\overline{x_A^2}+10\overline{x_B^2})-74^2$

$=\frac{1}{25}\{15(S_{x_A}^2+(\overline{x_A})^2)$

$+10(S_{x_B}^2+(\overline{x_B})^2)\}-74^2$

$=\frac{1}{25}\{15\cdot4910+10\cdot6415\}-74^2$

$=\frac{1}{25}\cdot137800-74^2$

$=5512-5476=36$ //

[9]

$2a_{n+1}=\frac{2a_n-2b_n}{3}-1$

$+)\ b_{n+1}=\frac{2a_n+4b_n}{3}+1$

$2a_{n+1}+b_{n+1}=\frac{2}{3}(2a_n+b_n)$

公比

$2a_n+b_n=(2a_1+b_1)\left(\frac{2}{3}\right)^{n-1}$

$=\frac{19}{12}\left(\frac{2}{3}\right)^{n-1}$

また

$a_{n+1}+b_{n+1}=a_n+b_n+\frac{1}{2}$

$\therefore a_n+b_n=a_1+b_1+(n-1)\frac{1}{2}$

$=\frac{n}{2}+\frac{5}{12}$

$2a_n+2b_n=n+\frac{5}{6}$

$-)\ 2a_n+b_n=\frac{19}{12}\left(\frac{2}{3}\right)^{n-1}$

$b_n=n+\frac{5}{6}-\frac{19}{12}\left(\frac{2}{3}\right)^{n-1}$

$\lim_{n\to\infty}(b_n-n)=\frac{5}{6}$ //

[10] $104=2^3\cdot13$

a+b+c	d+e
26	4
13	8
8	13

$n(a+b+c=26,\ d+e=4)$

$=1\cdot2$

$=2$

abc	de
899	13
	22

$n(a+b+c=13,\ d+e=8)$

$=12\cdot4$

$=48$

abc	de
139	17
22	26
148	35
23	44
157	
24	
33	
166	
25	
34	
355	
44	

$n(a+b+c=8,\ d+e=13)$

$=5\cdot3$

$=15$

abc	de
116	49
125	58
134	67
22	
233	

以上より $2+48+15=65$ //

この中で$a\leqq b\leqq c\leqq d\leqq e$

なのは

abc	de
134	49
22	
233	
125	58
134	
22	
233	
116	67
125	
134	
22	
233	

全部で12通り //