2017 帝京(医)

本体の数学①

[1]

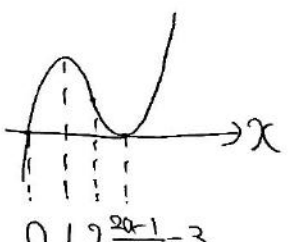
$y=x\{x^2-(a+1)x+2a-1\}$

$y'=3x^2-2(a+1)x+2a-1$

$=\{3x-(2a-1)\}\{x-1\}$

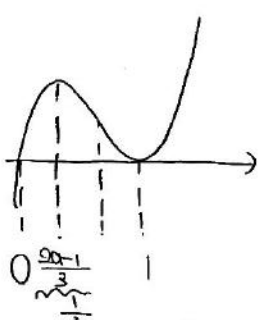
$y'=0 \Leftrightarrow x=\frac{2a-1}{3}, 1$

(i) $a>2$ のとき



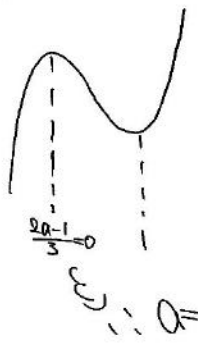
$\therefore a=5$

(ii) $a<2$ のとき



$\therefore a=1$

あるいは

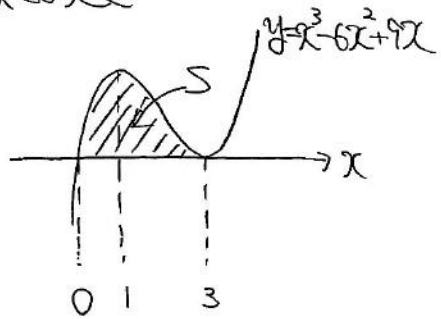


$\therefore a=\frac{1}{2}$

つまり

$t_1=\frac{1}{2}, t_2=1, t_3=5$

$a=5$ のとき



$S=\frac{1}{12}(3-0)^4=\frac{81}{12}=\frac{27}{4}$

(2) $a\neq 2$ かつ 5

$x=1$ で必ず極値をとる.

$x=\frac{2a-1}{3}=7$ で極値をもつ

$a=11$

[2]

(1)

$$\frac{1}{\cos(\alpha-\beta)}+\frac{1}{\cos(\alpha+\beta)}=\frac{2}{\cos\alpha}$$

$$\Leftrightarrow \frac{2\cos\alpha\cos\beta}{\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta}=\frac{2}{\cos\alpha}$$

$$\Leftrightarrow \cos^2\alpha\cos\beta=\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta$$

$$=\cos^2\alpha\cos^2\beta-(1-\cos^2\alpha)(1-\cos^2\beta)$$

$$=\cos^2\alpha+\cos^2\beta-1$$

$$\Leftrightarrow \cos^2\alpha(\cos\beta-1)=\cos^2\beta-1$$

$$\Leftrightarrow \cos^2\alpha=\cos\beta+1=2\cos^2\frac{\beta}{2}$$

$\therefore \cos\alpha=-\sqrt{2}\cos\frac{\beta}{2}$

または

$\cos\alpha=\sqrt{2}\cos\frac{\beta}{2}$

(2)

$0<\cos^2\alpha<1$

$\Leftrightarrow 0<1+\cos\beta<1$

$\Leftrightarrow -1<\cos\beta<0$

$\therefore \frac{1}{2}\pi<\beta<\pi$

(3)

$\cos(\alpha-\beta)+\cos(\alpha+\beta)=-\cos\alpha$

$\Leftrightarrow 2\cos\alpha\cos\beta=-\cos\alpha$

$\Leftrightarrow (1+2\cos\beta)\cos\alpha=0$

$\therefore \cos\beta=-\frac{1}{2}$ ($\because \cos\alpha\neq 0$)

$\therefore \beta=\frac{2}{3}\pi$

$\cos\alpha=\pm\sqrt{2}\cos\frac{\pi}{3}=\pm\frac{1}{\sqrt{2}}$

$\therefore \sin\alpha=\frac{1}{\sqrt{2}}$ ($0<\alpha<\pi$)

$$\alpha=\frac{1}{\cos(\alpha+\beta)}-\frac{1}{\cos\alpha}$$

$$=\frac{1}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}-(\pm\sqrt{2})$$

$$=\frac{-2\sqrt{2}}{\sqrt{3}\pm 1}-(\pm\sqrt{2})$$

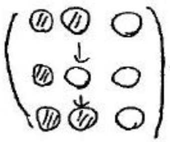
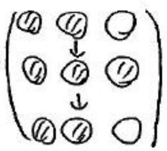
$$=-\sqrt{6}\pm\sqrt{2}-(\pm\sqrt{2})$$

$$=-\sqrt{6}$$

[3]

(1)

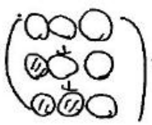
P_2

$= P$  $+ P$ 

$= \frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot 1 = \frac{7}{9}$ //

(2) nが偶数回のときは

●●○ か ○○○ しかない.

P  $= 1 \cdot \frac{2}{3} = \frac{2}{3}$

より

$n-2 \quad\quad n$

●●○ $P_{n-2} \xrightarrow{\frac{7}{9}} P_n$

○○○ $1-P_{n-2} \nearrow \frac{2}{3}$

$P_n = \frac{7}{9}P_{n-2} + \frac{2}{3}(1-P_{n-2})$

$= \frac{2}{3} + \frac{1}{9}P_{n-2}$ //

(特) $\alpha = \frac{2}{3} + \frac{1}{9}\alpha \quad \therefore \alpha = \frac{3}{4}$

$P_n - \frac{3}{4} = \frac{1}{9}\left(P_{n-2} - \frac{3}{4}\right)$

↓

$P_n - \frac{3}{4} = \left(P_0 - \frac{3}{4}\right)\left(\frac{1}{9}\right)^{\frac{n}{2}}$

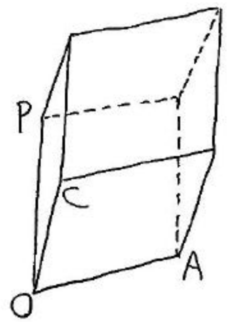
$\Leftrightarrow P_n = \frac{3}{4} + \frac{1}{4} \cdot \frac{1}{9^{\frac{n}{2}}}$

$= \frac{1}{4}\left(3 + \frac{1}{9^{\frac{n}{2}}}\right)$ //

[4]

(1)

(i)



$\square OABC$

$= \frac{1}{2}\sqrt{|\overrightarrow{OA}|^2|\overrightarrow{OC}|^2 - (\overrightarrow{OA}\cdot\overrightarrow{OC})^2} \times 2$

$= \sqrt{9 \cdot 50 - 20^2}$

$= \sqrt{50} = 5\sqrt{2}$ //

(ii)

$\vec{n} \cdot \overrightarrow{OA} = 2a + b - 2 = 0$

$\vec{n} \cdot \overrightarrow{OC} = 3a + 4b - 5 = 0$

$\times 4$: $8a + 4b - 8 = 0$

$-5a + 3 = 0$

$\therefore a = \frac{3}{5}$ //

(iii)

平面OAC : $z = \alpha x + \beta y$ とおく

↓ A.C通る

$\begin{cases} 2 = 2\alpha + \beta \\ 5 = 3\alpha + 4\beta \end{cases}$

$\therefore \alpha = \frac{3}{5}, \ \beta = \frac{4}{5}$

$z = \frac{3}{5}x + \frac{4}{5}y$

$\Leftrightarrow 0 = 3x + 4y - 5z$

これとPとの距離hは

$h = \frac{|-3+12-15|}{\sqrt{9+16+25}} = \frac{6}{\sqrt{50}}$

$\therefore (\text{体積}) = 5\sqrt{2} \times \frac{6}{\sqrt{50}} = 6$ //

(2) $3^x = X, \ 2^y = Y$ とおく.

$3X^2 + 7XY + 4Y^2 - 4X - 4Y = 0$

$\Leftrightarrow (3X+4Y)(X+Y) - 4(X+Y) = 0$

$\Leftrightarrow (3X+4Y-4)(X+Y) = 0$ ($X+Y > 0$)

$\therefore 3 \cdot 3^x + 4 \cdot 2^y = 4$ //

$3^{2x+1} + 2^{2y+2} \quad\quad Y = -\frac{3}{4}X + 1$

$= 3X^2 + 4Y^2$

$= 3X^2 + 4\left(\frac{9}{16}X^2 - \frac{3}{2}X + 1\right)$

$= 3X^2 + \frac{9}{4}X^2 - 6X + 4$

$= \frac{21}{4}\left(X^2 - \frac{24}{21}X\right) + 4$

$= \frac{21}{4}\left(X - \frac{4}{7}\right)^2 \sim$

$X = 3^x = \frac{4}{7}$

$\Leftrightarrow x = \log_3 \frac{4}{7}$ のとき最小 //