2017 帝京(医)

赤本での数学①

[1]

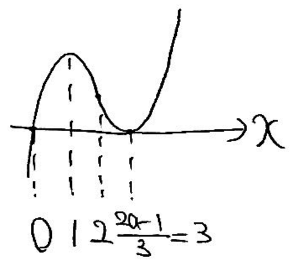
$y=x\{x^2-(a+1)x+2a-1\}$

$y'=3x^2-2(a+1)x+2a-1$

$=\{3x-(2a-1)\}\{x-1\}$

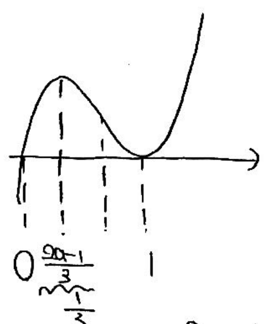
$y'=0 \iff x=\frac{2a-1}{3},\ 1$

(i) $a>2$ のとき



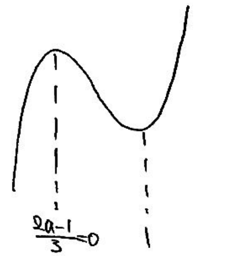
$\therefore a=5$

(ii) $a<2$ のとき



$\therefore a=1$

あるいは

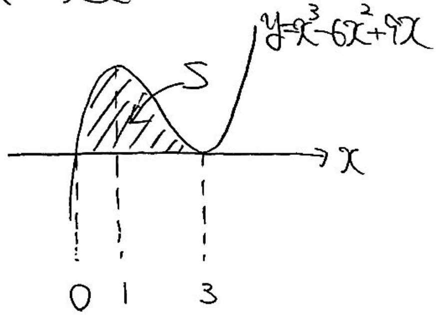


$\therefore a=\frac{1}{2}$

つまり

$\underline{t_1=\frac{1}{2}}$, $\underline{t_2=1}$, $\underline{t_3=5}$ //

$a=5$ のとき



$S=\frac{1}{12}(3-0)^4=\frac{81}{12}=\underline{\frac{27}{4}}$ //

(2) $a \neq 2$ なら

$x=\underline{1}$ で必ず極値をとる.

$x=\frac{2a-1}{3}=7$ で極値をとれば

$\underline{a=11}$ //

[2]

(1)

$\frac{1}{\cos(\alpha-\beta)}+\frac{1}{\cos(\alpha+\beta)}=\frac{2}{\cos\alpha}$

$\iff \frac{2\cos\alpha\cos\beta}{\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta}=\frac{2}{\cos\alpha}$

$\iff \cos^2\alpha\cos\beta=\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta$

$=\cos^2\alpha\cos^2\beta-(1-\cos^2\alpha)(1-\cos^2\beta)$

$=\cos^2\alpha+\cos^2\beta-1$

$\iff \cos^2\alpha(\cos\beta-1)=\cos^2\beta-1$

$\iff \cos^2\alpha=\cos\beta+1=2\cos^2\frac{\beta}{2}$

$\therefore \underline{\cos\alpha=-\sqrt{2}\cos\frac{\beta}{2}}$ //

または

$\underline{\cos\alpha=\sqrt{2}\cos\frac{\beta}{2}}$ //

(2)

$0<\cos^2\alpha<1$

$\iff 0<1+\cos\beta<1$

$\iff -1<\cos\beta<0$

$\therefore \underline{\frac{1}{2}\pi<\beta<\pi}$ //

(3)

$\cos(\alpha-\beta)+\cos(\alpha+\beta)=-\cos\alpha$

$\iff 2\cos\alpha\cos\beta=-\cos\alpha$

$\iff (1+2\cos\beta)\cos\alpha=0$

$\therefore \cos\beta=-\frac{1}{2}$ ($\because \cos\alpha\neq 0$)

$\therefore \underline{\beta=\frac{2}{3}\pi}$ //

$\cos\alpha=\pm\sqrt{2}\cos\frac{\pi}{3}=\pm\frac{1}{\sqrt{2}}$

$\therefore \sin\alpha=\frac{1}{\sqrt{2}}$ $(0<\alpha<\pi)$

d

$=\frac{1}{\cos(\alpha+\beta)}-\frac{1}{\cos\alpha}$

$=\frac{1}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}-(\pm\sqrt{2})$

$=\frac{-2\sqrt{2}}{\sqrt{3}\pm 1}-(\pm\sqrt{2})$

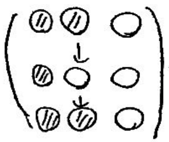
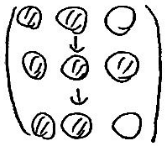
$=-\sqrt{6}\pm\sqrt{2}-(\pm\sqrt{2})$

$=\underline{-\sqrt{6}}$ //

[3]

(1)

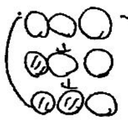
P_2

$= P\Big($  $\Big) + P\Big($  $\Big)$

$= \frac{2}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot 1 = \underline{\frac{7}{9}}$ //

(2) nが偶数回のときは

●●● か ○○○ しかない。

$P\Big($  $\Big) = 1\cdot\frac{2}{3} = \frac{2}{3}$

より

$$\begin{array}{llll} & n-2 & & n \\ ●●● & P_{n-2} & \xrightarrow{\frac{7}{9}} & P_n \\ ○○○ & 1-P_{n-2} & \nearrow_{\frac{2}{3}} & \end{array}$$

$P_n = \frac{7}{9}P_{n-2} + \frac{2}{3}(1-P_{n-2})$

$= \underline{\frac{2}{3} + \frac{1}{9}P_{n-2}}$ //

(特) $\alpha = \frac{2}{3} + \frac{1}{9}\alpha \quad \therefore \alpha = \frac{3}{4}$

$P_n - \frac{3}{4} = \frac{1}{9}\left(P_{n-2} - \frac{3}{4}\right)$

↓

$P_n - \frac{3}{4} = \left(P_0 - \frac{3}{4}\right)\left(\frac{1}{9}\right)^{\frac{n}{2}}$

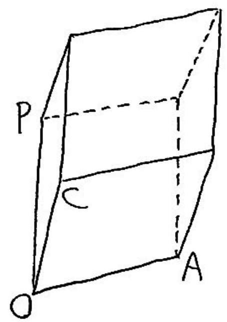
$\Leftrightarrow P_n = \frac{3}{4} + \frac{1}{4}\cdot\frac{1}{9^{\frac{n}{2}}}$

$= \underline{\frac{1}{4}\left(3 + \frac{1}{9^{\frac{n}{2}}}\right)}$ //

[4]

(1)

(i)



$\square OABC$

$= \frac{1}{2}\sqrt{|\vec{OA}|^2|\vec{OC}|^2 - (\vec{OA}\cdot\vec{OC})^2} \times 2$

$= \sqrt{9\cdot 50 - 20^2}$

$= \sqrt{50} = \underline{5\sqrt{2}}$ //

(ii)

$$\left.\begin{array}{l} \vec{n}\cdot\vec{OA} = 2a+b-2=0 \\ \vec{n}\cdot\vec{OC} = 3a+4b-5=0 \end{array}\right\} \times 4$$

$$\begin{array}{r} 8a+4b-8=0 \\ \hline -5a+3=0 \end{array}$$

$\therefore \underline{a = \frac{3}{5}}$ //

(iii)

平面OAC：$z = \alpha x + \beta y$ とおく

↓ A, C通る

$$\begin{cases} 2 = 2\alpha + \beta \\ 5 = 3\alpha + 4\beta \end{cases}$$

$\therefore \alpha = \frac{3}{5},\ \beta = \frac{4}{5}$

$z = \frac{3}{5}x + \frac{4}{5}y$

$\Leftrightarrow 0 = 3x + 4y - 5z$

これとPとの距離hは

$h = \frac{|-3+12-15|}{\sqrt{9+16+25}} = \frac{6}{\sqrt{50}}$

$\therefore$ (体積) $= 5\sqrt{2} \times \frac{6}{\sqrt{50}} = \underline{6}$ //

(2) $3^x = X,\ 2^y = Y$ とおく.

$3X^2 + 7XY + 4Y^2 - 4X - 4Y = 0$

$\Leftrightarrow (3X+4Y)(X+Y) - 4(X+Y) = 0$

$\Leftrightarrow (3X+4Y-4)\underbrace{(X+Y)}_{>0} = 0$

$\therefore \underline{3\cdot 3^x + 4\cdot 2^y = 4}$ //

$3^{2x+1} + 2^{2y+2} \qquad Y = -\frac{3}{4}X + 1$

$= 3X^2 + 4Y^2$

$= 3X^2 + 4\left(\frac{9}{16}X^2 - \frac{3}{2}X + 1\right)$

$= 3X^2 + \frac{9}{4}X^2 - 6X + 4$

$= \frac{21}{4}\left(X^2 - \frac{24}{21}X\right) + 4$

$= \frac{21}{4}\left(X - \frac{4}{7}\right)^2 \sim$

$X = 3^x = \frac{4}{7}$

$\Leftrightarrow \underline{x = \log_3\frac{4}{7}}$ // のとき最小