2017 日本大(医)

[I]

(1) $\begin{cases} \alpha+\beta=-\dfrac{5}{3} \\ \alpha\beta=\dfrac{8}{3} \end{cases}$

$\alpha^2+\beta^2=\dfrac{25}{9}-\dfrac{16}{3}=-\dfrac{23}{9}$

$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

$=-\dfrac{5}{3}\left(-\dfrac{23}{9}-\dfrac{8}{3}\right)$

$=\dfrac{235}{27}$

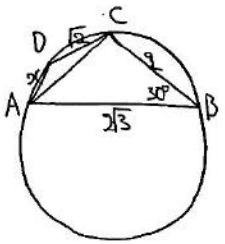
(2)

$\begin{cases} 3x^2+4x-4\leqq 0 \\ -4\leqq 3x+1\leqq 4 \end{cases}$

$\Leftrightarrow \begin{cases} (3x-2)(x+2)\leqq 0 \\ -\dfrac{5}{3}\leqq x\leqq 1 \end{cases}$

$\therefore -\dfrac{5}{3}\leqq x\leqq \dfrac{2}{3}$

(3)



AD=xとおいて△ACDと△ABCに余

$x^2+2-2\sqrt{2}x\cos 150^\circ = CA^2$

$=12+4-2\cdot 2\sqrt{3}\cdot 2\cos 30^\circ$

$\Leftrightarrow x^2+\sqrt{6}x+2=16-12$

$\Leftrightarrow x^2+\sqrt{6}x-2=0$

$\therefore x=\dfrac{-\sqrt{6}+\sqrt{14}}{2}$ $(\because x>0)$

$\therefore AD=\dfrac{\sqrt{14}-\sqrt{6}}{2}$

(4)

$\begin{cases} x-2y-4=0 \\ 4x+2y-6=0 \end{cases} \rightarrow x=2,\ y=-1$

↓3本目に代入

$2m^2-3m-9=0$

$\Leftrightarrow (2m+3)(m-3)=0$

$\therefore m=3,\ -\dfrac{3}{2}$

[2]

(1) n(6桁)

$=n$(1が使われないのが2)

$+n$(　〃　3)

$+n$(　〃　5)

$=\dfrac{6!}{1!3!2!}+\dfrac{6!}{2!2!2!}+\dfrac{6!}{2!3!1!}$

$=60+90+60=210$ 通り

n(6桁∩5の倍数)

$=n$(1が使われないのが2∩1の位が5)

$+n$(　〃　3∩　〃　)

$+n$(　〃　5∩　〃　)

$=\dfrac{5!}{1!3!1!}+\dfrac{5!}{2!2!1!}+\dfrac{5!}{2!3!}$

$=20+30+10=60$ 通り

(2)

$t=\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)$

$\left(\dfrac{\pi}{4}\leqq x+\dfrac{\pi}{4}\leqq\dfrac{5}{4}\pi\right)$

$\therefore -1\leqq t\leqq\sqrt{2}$

$t^2=1+2\sin x\cos x$ より

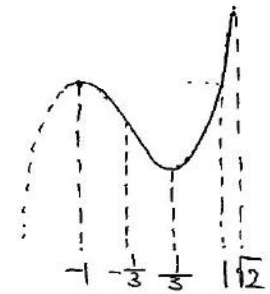
$y=\sin x\cos x(\sin x+\cos x+1)$

$=\dfrac{t^2-1}{2}(t+1)$

$=\dfrac{1}{2}(\underbrace{t^3+t^2-t-1}_{f(t)})$

$f'(t)=3t^2+2t-1$

$=(3t-1)(t+1)$



$\min y=\dfrac{1}{2}f\left(\dfrac{1}{3}\right)=-\dfrac{16}{27}$

(3) $X^2-4\dfrac{1}{X}-6=0$ → 4の約数から解を探す

$\Leftrightarrow X^3-6X-4=0$

$\Leftrightarrow (X+2)(X^2-2X-2)=0$

$\Leftrightarrow X^2-2X-2=0$ $(\because X+2>0)$

$\Leftrightarrow X=1+\sqrt{3}$ $(\because X>0)$

$\Leftrightarrow x=\log_2(1+\sqrt{3})$

(4)

$S_{n+2}-S_{n+1}=4(S_{n+1}-S_n)$

↓一般項

$S_{n+1}-S_n=(S_2-S_1)4^{n-1}$

$=a_2\cdot 4^{n-1}$

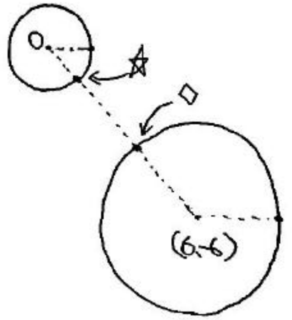
$\Leftrightarrow a_{n+1}=3\cdot 4^{n-1}$

↓$n=5$を代入

$a_6=3\cdot 4^4=768$

[3]

(1)



$P_1\left(\cos\frac{\pi}{2}t, \sin\frac{\pi}{2}t\right)$

$P_2\left(3\cos\left(-\frac{7}{10}\pi\right)t+6, 3\sin\left(-\frac{7}{10}\pi\right)t-6\right)$

P_1が☆の位置に来ればいい。

$$P_1\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) //$$

P_2は◇の位置に来ればいい。
このときP_1P_2は最小。

$$\overline{P_1P_2} = 6\sqrt{2}-1-3 = 6\sqrt{2}-4 //$$

(2)

$$\begin{cases} \frac{\pi}{2}t = \frac{7}{4}\pi + 2m\pi \\ -\frac{7}{10}\pi t = \frac{3}{4}\pi + 2n\pi \end{cases}$$

$$\downarrow$$

$$\frac{7}{2}+4m = -\frac{30}{28} - \frac{20n}{7}$$

$$\Leftrightarrow 49+56m = -15-40n$$

$$\Leftrightarrow 40n+56m = -64$$

$$\Leftrightarrow 5n+7m = -8$$

$$\begin{array}{r} 5n+7m=-8 \\ -)\ 5\cdot4+7(-4)=-8 \\ \hline 5(n-4)+7(m+4)=0 \end{array}$$

$$5(n-4) = 7(-m-4)$$

$$\therefore -m-4 = 5k \quad (k\in\mathbb{Z})$$

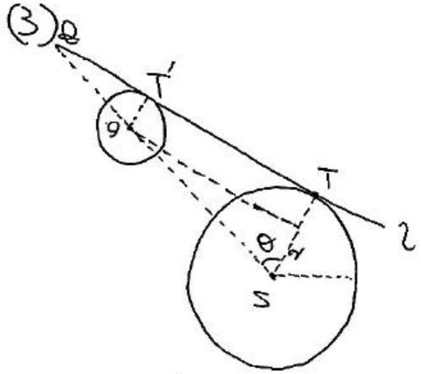
$$\Leftrightarrow m = -5k-4$$

$m>0$より $\min m = 1$ $(k=-1)$

$$\frac{\pi}{2}t = \frac{7}{4}\pi + 2\pi = \frac{15}{4}\pi$$

$$\therefore t = \frac{15}{2} //$$

(3)



$SO = 6\sqrt{2}$より

$$\tan\theta = \frac{\sqrt{72-4}}{2} = \sqrt{17} //$$

(STの傾き)

$$= \tan\left(\frac{3}{4}\pi - \theta\right) = \frac{-1-\sqrt{17}}{1+(-1)\sqrt{17}}$$

$$(l\text{の傾き}) = -\frac{1}{\frac{-1-\sqrt{17}}{1-\sqrt{17}}}$$

$$= \frac{1-\sqrt{17}}{1+\sqrt{17}}$$

$$= \frac{(1-\sqrt{17})^2}{-16}$$

$$= \frac{-9+\sqrt{17}}{8} //$$

$y=-x$とlの交点をQとする

$\triangle QOT' \backsim \triangle QST$が相似比1:3なので$QO = 3\sqrt{2}$

$$\therefore Q(-3, 3) //$$

[4]

(1)

$$\int_0^{\pi} \sin^3 x\, dx$$

$$= \int_0^{\pi} (1-\cos^2 x)\sin x\, dx$$

$$= \int_1^{-1} (1-t^2)(-1)\, dt$$

$$= 2\int_0^1 (1-t^2)\, dt$$

$$= 2\left[t - \frac{t^3}{3}\right]_0^1 = \frac{4}{3} //$$

(2)

$$C_1 = \int_{\pi}^{(1+r)\pi} r\sin^3\left\{\frac{1}{r}(x-\pi)\right\} dx$$

$\frac{1}{r}(x-\pi) = t$ とおく

$$= \int_0^{\pi} r\sin^3 t \cdot r\, dt$$

$$= r^2\int_0^{\pi} \sin^3 t\, dt = \frac{4}{3}r^2 //$$

(3)

C_n

$$= \int_{a_n}^{b_n} r^n \sin^3\left(\frac{1}{r^n}\left\{x - \left(\frac{1-r^n}{1-r}\right)\pi\right\}\right) dx$$

$\frac{1}{r^n}\left(x - \left(\frac{1-r^n}{1-r}\right)\pi\right) = t$ とおく

$$\frac{1}{r^n}dx = dt$$

$$\begin{array}{c|c} x & a_n \to b_n \\ \hline t & 0 \to \frac{b_n - a_n}{r^n} \end{array}$$

$$= \int_0^{\frac{b_n-a_n}{r^n}} r^{2n} \sin^3 t\, dt$$

$$\frac{b_n - a_n}{r^n} = \frac{1}{r^n}\cdot\frac{r^n - r^{n+1}}{1-r}\pi = \pi$$

$$= r^{2n}\int_0^{\pi} \sin^3 t\, dt$$

$$= \frac{4}{3}r^{2n}$$

(4)

$$\lim_{n\to\infty}\sum_{k=1}^{n} C_k = \frac{\frac{4}{3}r^2}{1-r^2}$$

$$= \frac{4r^2}{3(1-r^2)} //$$

[5]

(1) $\frac{dx}{dt} = -2\sin t - 2\sin 2t$

$\frac{dy}{dt} = 2\cos t - 2\cos 2t$

(長さ)

$= \int_0^{\frac{2}{3}\pi} \sqrt{4(\sin t + \sin 2t)^2 + 4(\cos t - \cos 2t)^2}\, dt$

$= \int_0^{\frac{2}{3}\pi} \sqrt{8 - 8(\cos t \cos 2t - \sin t \sin 2t)}\, dt$

$= \int_0^{\frac{2}{3}\pi} \sqrt{8(1 - \underbrace{\cos 3t}_{1-2\sin^2\frac{3}{2}t})}\, dt$

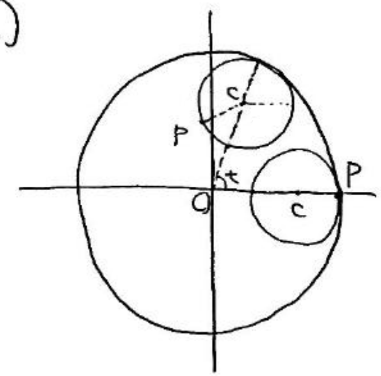
$= \int_0^{\frac{2}{3}\pi} \sqrt{16\sin^2\frac{3}{2}t}\, dt$

$= 4\int_0^{\frac{2}{3}\pi} \sin\frac{3}{2}t\, dt$

$= 4\left[-\frac{2}{3}\cos\frac{3}{2}t\right]_0^{\frac{2}{3}\pi}$

$= 4\left(\frac{2}{3}+\frac{2}{3}\right) = \underline{\frac{16}{3}}$ //

(2)



$\overrightarrow{CP}$と$\begin{pmatrix}1\\0\end{pmatrix}$のなす角は$2t$

$\overrightarrow{CP} = \begin{pmatrix}\cos(-2t)\\ \sin(-2t)\end{pmatrix} = \underline{\begin{pmatrix}\cos 2t\\ -\sin 2t\end{pmatrix}}$ //

$\overrightarrow{OC} = \underline{\begin{pmatrix}2\cos t\\ 2\sin t\end{pmatrix}}$ //

(3)

円Oの半径が3，円Cの半径が1より Pは $t=\frac{2}{3}\pi$ のとき再び円O上に来る．$t=\frac{\pi}{3}$ のとき線分OC上に来る．同様のことを繰り返すと概形は以下の円O内の曲線．