2017 金沢医科大

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(1) P(PとQが同じ)

$= \frac{2}{36}$ 　(太,中)=(5,3),(6,2)

$= \frac{1}{18}$ //

(2) $P(PQ=1)$

$= \frac{6}{36}$ 　PQが AJ, JI, JE, EJ, ED, EF のとき

$= \frac{1}{6}$ //

(3)

max PQは例えば $BG=\sqrt{13}$ //

$P(PQ=\sqrt{13})$

$= \frac{2}{36}$ 　PQが BG, CHのとき

$= \frac{1}{18}$ //

(4) P(△PQRが存在しない)

$= P$(PとQが同じ)

$+ P$(PとQとRが一直線上)

$= \frac{1}{18} + \frac{1}{72}\cdot 12$

$= \frac{2}{9}$ //

P	Q	R	P	Q	R
A	J	O_2	E	H	O_2
	G	O_2		F	O_1
B	E	O_1	J	G	O_2
	F	O_1		I	O_2
C	J	O_1			
	I	O_1			
D	E	O_2			
	H	O_2			

2

(1)

$(n+1)a_{n+1} = \frac{2na_n}{k} + \frac{1}{k}$

$\downarrow$ $b_n = na_n$ 　$b_1 = a_1 = 1$ //

$b_{n+1} = \underline{\frac{2}{k}}b_n + \underline{\frac{1}{k}}$ //

$k=2$のとき

$b_{n+1} = b_n + \frac{1}{2}$

$\downarrow$ 一般項

$b_n = b_1 + (n-1)\frac{1}{2}$

$= \frac{1}{2}n + \frac{1}{2}$

$\therefore a_n = \frac{1}{2} + \frac{1}{2n}$ //

(2) $k \neq 2$のとき

特

$\alpha = \frac{2}{k}\alpha + \frac{1}{k}$

$\Leftrightarrow \frac{k-2}{k}\alpha = \frac{1}{k}$

$\Leftrightarrow \alpha = \frac{1}{k-2}$

$b_n - \frac{1}{k-2} = \left(b_1 - \frac{1}{k-2}\right)\left(\frac{2}{k}\right)^{n-1}$

$\Leftrightarrow b_n = \frac{k-3}{k-2}\left(\frac{2}{k}\right)^{n-1} + \frac{1}{k-2}$ //

(3)

$k=\underline{3}$のとき b_nは一定

(4)

$k=1$のとき

$b_n = 2\cdot 2^{n-1} - 1$

$= 2^n - 1$

$a_n = \frac{1}{n}(2^n - 1)$

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{n}{n+1}\cdot\frac{2^{n+1}-1}{2^n-1} = 2$ //

3

$\begin{cases} \sqrt{a^2-b^2} = 2 \\ 2a = 2\sqrt{5} \end{cases}$

$\therefore a=\sqrt{5},\ b=1$ //

$y = -\frac{1}{2}x + n$ と $\frac{x^2}{5} + y^2 = 1$

を連立

$\frac{x^2}{5} + \left(-\frac{1}{2}x+n\right)^2 = 1$

$\Leftrightarrow \frac{x^2}{5} + \frac{1}{4}x^2 - nx + n^2 - 1 = 0$

$\Leftrightarrow 9x^2 - 20nx + 20(n^2-1) = 0$ 　…①

$\frac{D}{4} = 100n^2 - 180(n^2-1)$

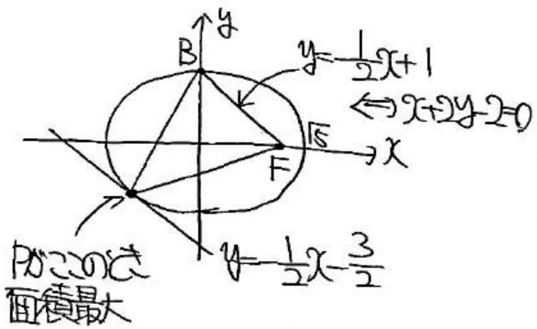
$= -80n^2 + 180 = 0$

$\Leftrightarrow n^2 = \frac{9}{4} \Leftrightarrow n = \pm\frac{3}{2}$

つまり

$y = -\frac{1}{2}x \pm \frac{3}{2}$

$\therefore x + \underline{2}y \pm \underline{3} = 0$ //



$n = -\frac{3}{2}$ を①に代入

$9x^2 + 30x + 25 = 0$

$\therefore x = -\frac{5}{3}$

$P\left(-\frac{5}{3}, -\frac{2}{3}\right)$ //

$x+2y-2=0$ と $\left(-\frac{5}{3}, -\frac{2}{3}\right)$ との

距りhは

$$h=\frac{\left|-\frac{5}{3}-\frac{4}{3}-2\right|}{\sqrt{1+4}}$$

$$=\sqrt{5}$$

$\therefore \max\triangle FBP = \underbrace{FB}_{\sqrt{5}} \times \underbrace{h}_{\sqrt{5}} \times \frac{1}{2} = \frac{5}{2}$ //

$S_1 : S_2 : S_3$

$\frac{2}{3}$ 　　 1 ← 高さでれを

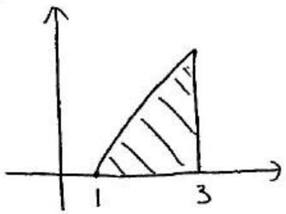
　　 $\frac{5}{3}$ 　 2 ← 出す

$\frac{4}{3} : \frac{5}{3} : 2$

$\therefore 4:5:6$ //

$\boxed{4}$

(1)



$$\int_1^3 x\sqrt{x^2-1}\,dx \qquad \begin{array}{l} x^2=t \\ \downarrow 2x\,dx=dt \end{array}$$

$$=\int_1^9 \frac{1}{2}\sqrt{t-1}\,dt$$

$$=\left[\frac{1}{2}\cdot\frac{2}{3}(t-1)^{\frac{3}{2}}\right]_1^9$$

$$=\frac{16\sqrt{2}}{3}$$ //

(2)

$$y'=\sqrt{x^2-1}+x\cdot\frac{1}{2}(x^2-1)^{-\frac{1}{2}}2x$$

$$=\frac{x^2-1}{\sqrt{x^2-1}}+\frac{x^2}{\sqrt{x^2-1}}$$

$$=(2x^2-1)(x^2-1)^{-\frac{1}{2}}$$

$$y''=4x(x^2-1)^{-\frac{1}{2}}+(2x^2-1)\left(-\frac{1}{2}\right)(x^2-1)^{-\frac{3}{2}}2x$$

$$=4x(x^2-1)^{-\frac{1}{2}}-(2x^3-x)(x^2-1)^{-\frac{3}{2}}$$

$$=\{4x(x^2-1)-(2x^3-x)\}(x^2-1)^{-\frac{3}{2}}$$

$$=x(2x^2-3)(x^2-1)^{-\frac{3}{2}}$$

$x^2=\frac{3}{2}$ $\therefore x=\frac{\sqrt{6}}{2}$ のとき

変曲点 $\left(\frac{\sqrt{6}}{2}, \frac{\sqrt{3}}{2}\right)$ //

(3)

連立

$$x\sqrt{x^2-1}=x$$

$$\Leftrightarrow \sqrt{x^2-1}=1 \quad (x\geqq1)$$

$$\therefore x=\sqrt{2}$$

$A(\sqrt{2},\sqrt{2})$ での接線は

$y=3(x-\sqrt{2})+\sqrt{2}=3x-2\sqrt{2}$ //

(4) 連立

$$x\sqrt{x^2-1}=3x-2\sqrt{2}$$

↓ 2乗

$$x^4-x^2=9x^2-12\sqrt{2}x+8$$

$$\Leftrightarrow x^4-10x^2+12\sqrt{2}x-8=0$$

$$\Leftrightarrow (x-\sqrt{2})(x^3+\sqrt{2}x^2-8x+4\sqrt{2})=0$$

$$\Leftrightarrow (x-\sqrt{2})^2(x^2+2\sqrt{2}x-4)=0$$ //

$\therefore x=\sqrt{2}, -\sqrt{2}\pm\sqrt{6}$

Aと異なるのは

$x=\sqrt{6}-\sqrt{2}$ // $(\because x\geqq1)$