2017 順天堂医

[1]

(1)

$c_n = b_n + \alpha$

$b_{n+2} + \alpha = \frac{1}{6}(b_{n+1} + b_n + 2\alpha + 8)$

$\Rightarrow b_{n+2} = \frac{1}{6}(b_{n+1} + b_n) - \frac{2}{3}\alpha + \frac{4}{3}$

∴ $\underline{\alpha = 2}$ //

↓

解

$6\beta^2 = \beta + 1$

$\Leftrightarrow (2\beta - 1)(3\beta + 1) = 0$

$\beta = -\frac{1}{3}, \frac{1}{2}$

$b_{n+2} + \frac{1}{3}b_{n+1} = \frac{1}{2}\left(b_{n+1} + \frac{1}{3}b_n\right)$

$b_{n+2} - \frac{1}{2}b_{n+1} = -\frac{1}{3}\left(b_{n+1} - \frac{1}{2}b_n\right)$

より

$\underline{\beta = \frac{1}{3},\ \gamma = \frac{1}{2}}$ //

$c_n = (\underbrace{b_2}_{\frac{13}{5}} + \underbrace{\tfrac{1}{3}b_1}_{\frac{2}{5}})\left(\frac{1}{2}\right)^{n-1}$

$= \underline{3\gamma^{n-1}}$ //

$d_n = (\underbrace{b_2}_{\frac{13}{5}} - \underbrace{\tfrac{1}{2}b_1}_{\frac{3}{5}})(-\beta)^{n-1}$

$= \underline{2(-\beta)^{n-1}}$ //

↓

$b_{n+1} + \frac{1}{3}b_n = 3\gamma^{n-1}$

$-)\ b_{n+1} - \frac{1}{2}b_n = 2(-\beta)^{n-1}$

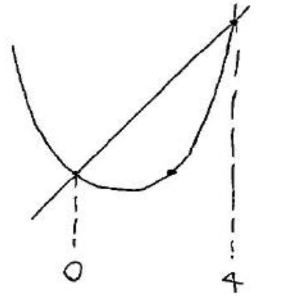
$\frac{5}{6}b_n = 3\gamma^{n-1} - 2(-\beta)^{n-1}$

∴ $b_n = \frac{18}{5}\gamma^{n-1} - \frac{12}{5}(-\beta)^{n-1}$

∴ $\underline{a_n = \gamma^{n-1}\frac{18}{5} - (-\beta)^{n-1}\frac{12}{5} + \alpha}$ //

(2)

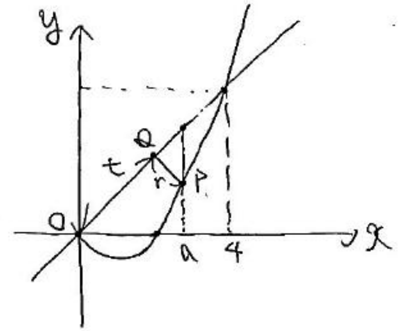
暗算… $\underline{A(4, 4)}$ //



$R = \frac{|a|}{6}(4 - 0)^3$

$= \frac{64}{12}$

$= \underline{\frac{16}{3}}$ //



$\begin{cases} t + r = \sqrt{2}a \\ \sqrt{2}r = a - \left(\frac{a^2}{2} - a\right) \\ \quad = 2a - \frac{a^2}{2} \end{cases}$

↓

$PQ = r = \sqrt{2}a - \frac{a^2}{2\sqrt{2}}$

$= \underline{\sqrt{2}a\left(1 - \frac{1}{4}a\right)}$ //

Qの座標は

$a - \frac{r}{\sqrt{2}}$

$= a - a\left(1 - \frac{1}{4}a\right)$

$= \frac{1}{4}a^2$　∴ $\underline{Q\left(\frac{1}{4}a^2, \frac{1}{4}a^2\right)}$ //

傘型を使う

V

$= \frac{1}{\sqrt{2}}\int_0^4 \left(2a - \frac{a^2}{2}\right)^2 \pi\, dx$

$= \frac{1}{\sqrt{2}}\int_0^4 \left(\frac{a^4}{4} - 2a^3 + 4a^2\right)\pi\, dx$

$= \frac{\pi}{\sqrt{2}}\left[\frac{a^5}{20} - \frac{a^4}{2} + \frac{4}{3}a^3\right]_0^4$

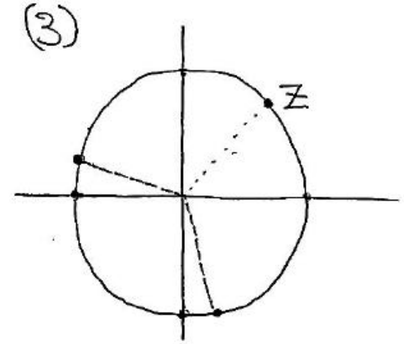
$= \frac{\pi}{\sqrt{2}}\left(\frac{256}{5} - 128 + \frac{256}{3}\right)$

$= \frac{128\pi}{\sqrt{2}}\left(\frac{2}{5} - 1 + \frac{2}{3}\right)$

$= \frac{128\pi}{15\sqrt{2}}$

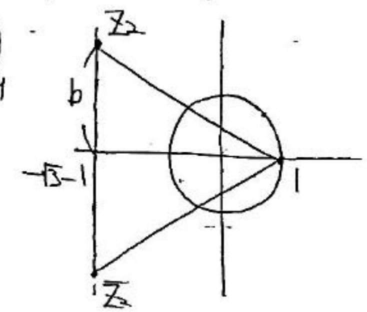
$= \underline{\frac{64\sqrt{2}}{15}\pi}$ //

(3)



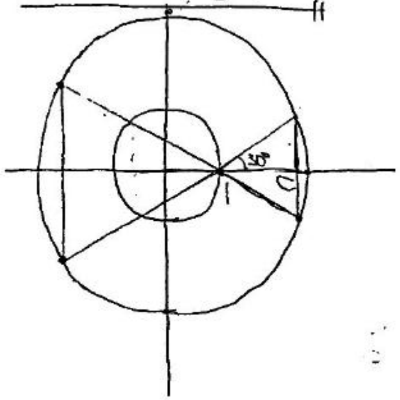
z_1は±120°回転ならばよい。

$\underline{z_1 = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i}$ //



$b = \frac{1}{\sqrt{3}}(\sqrt{3} + 2) = \frac{3 + 2\sqrt{3}}{3}$

$z_2 = -\sqrt{3} - 1 \pm \dfrac{3+2\sqrt{3}}{3} i$ //



$|z_2|^2 = 4 + 2\sqrt{3} + \dfrac{21+12\sqrt{3}}{9}$

$= \dfrac{57+30\sqrt{3}}{9}$

$= \dfrac{19+10\sqrt{3}}{3}$

$c^2 + \left(\dfrac{c-1}{\sqrt{3}}\right)^2 = \dfrac{19+10\sqrt{3}}{3}$

$\Leftrightarrow 4c^2 - 2c + 1 = 19 + 10\sqrt{3}$

$\Leftrightarrow 2c^2 - c - 9 - 5\sqrt{3} = 0$

$c = \dfrac{1 \pm \sqrt{73+40\sqrt{3}}}{4}$

$= \dfrac{1 \pm \sqrt{73+2\sqrt{1200}}}{4}$

$= \dfrac{1 \pm (\sqrt{25}+\sqrt{48})}{4}$

図より

$c = \dfrac{1+5+4\sqrt{3}}{4}$

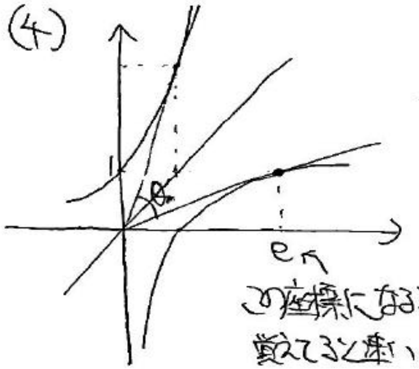
$= \dfrac{3+2\sqrt{3}}{2}$

$\dfrac{c-1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{1+2\sqrt{3}}{2}$

$= \dfrac{6+\sqrt{3}}{6}$

$\therefore z_3 = \dfrac{3+2\sqrt{3}}{2} \pm \dfrac{6+\sqrt{3}}{6} i$ //

(4)



この座標になることは覚えてると楽い

$m = |\vec{OA}||\vec{OB}|\cos\theta_m$

$= \cos\theta_m$

余

$m = \dfrac{2(e^2+1) - \{\sqrt{2}(e-1)\}^2}{2(e^2+1)}$

$= \dfrac{2(e^2+1) - 2(e^2-2e+1)}{2(e^2+1)}$

$= \dfrac{2e}{e^2+1}$

$\log_e m$

$= \log \dfrac{2e}{e^2+1}$

$= 1 + \log_e 2 - \log(e^2+1)$ //

$\boxed{\text{II}}$

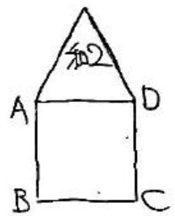
おうぎ形 PRQ'

$= 5 \cdot 5 \pi \times \dfrac{a}{2\pi}$

$= \dfrac{25}{2} a$

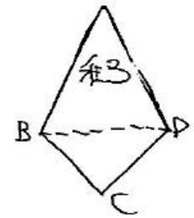
(斜線部分)

$= \dfrac{25}{2} a \times \dfrac{3}{5} = \dfrac{15}{2} a$ //



和 $= 2a = 2 \quad \therefore a = 1$ // $=$ 長半径

図より $b = \dfrac{\sqrt{3}}{2}$ // $=$ 短半径



和 $= 2a = 3$

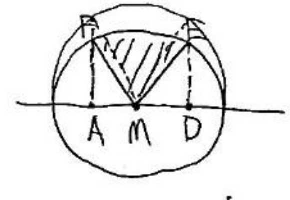
$\therefore a = \dfrac{3}{2}$ // $=$ 長半径

図より

$b^2 = \left(\dfrac{3}{2}\right)^2 - \left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{9}{4} - \dfrac{2}{4} = \dfrac{7}{4}$

$\therefore b = \dfrac{\sqrt{7}}{2}$ // $=$ 短半径



$(\text{斜}) = 1 \cdot 1 \cdot \pi \times \dfrac{1}{6} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{3}}{12} \pi$

$AF = \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{3}{4}$ //

$OF' = \dfrac{3}{2}$

より

$\sin\beta = \dfrac{\frac{1}{\sqrt{2}} AF}{OF'} = \dfrac{2}{3} \cdot \dfrac{3}{4\sqrt{2}}$

$\therefore \sin\beta = \frac{\sqrt{2}}{4}$

扇形FOG $= \frac{9}{4}\pi \times \frac{2\beta}{2\pi}$

$= \frac{9\beta}{4}$

(弧FGとOFとOGで囲まれるところ)

$= \frac{9\beta}{4} \times \frac{\frac{\sqrt{7}}{2}}{\frac{3}{2}}$

$= \frac{3\sqrt{7}}{4}\beta$

(最後)

$= \frac{3\sqrt{7}}{4}\beta \times 4 + \frac{\sqrt{3}}{12}\pi \times 4$

$+ \underbrace{\triangle OME}_{\frac{1}{8}} \times 8$

$= \frac{\sqrt{3}}{3}\pi + 3\sqrt{7}\beta + 1$ //

$\boxed{\text{III}}$

(1) $f\left(\frac{a+b}{2}\right) - \frac{f(a)+f(b)}{2}$

$= -\frac{(a+b)^2}{4} + \frac{a^2+b^2}{2}$

$= \frac{2a^2+2b^2-(a^2+2ab+b^2)}{4}$

$= \frac{(a-b)^2}{4} \geq 0$

$\therefore f\left(\frac{a+b}{2}\right) \geq \frac{f(a)+f(b)}{2}$

(2)

(i) $k=1$のとき

性質(A)に$a=a_1$, $b=a_2$を

代入すれば成立.

(ii) $n=2^k$のときの

成立を仮定する.

$n=2^{k+1}$のとき

$f\left(\frac{a_1+\cdots+a_{2^{k+1}}}{2^{k+1}}\right) - \frac{f(a_1)+\cdots+f(a_{2^{k+1}})}{2^{k+1}}$

$= f\left(\frac{\frac{a_1+\cdots+a_{2^k}}{2^k} + \frac{a_{2^k+1}+\cdots+a_{2^{k+1}}}{2^k}}{2}\right)$

$- \frac{f(a_1)+\cdots+f(a_{2^{k+1}})}{2^{k+1}}$

$= \frac{f\left(\frac{a_1+\cdots+a_{2^k}}{2^k}\right) + f\left(\frac{a_{2^k+1}+\cdots+a_{2^{k+1}}}{2^k}\right)}{2}$

$- \frac{f(a_1)+\cdots+f(a_{2^{k+1}})}{2^{k+1}}$

$\geq \frac{f(a_1)+\cdots+f(a_{2^k})}{2^{k+1}} + \frac{f(a_{2^k+1})+\cdots+f(a_{2^{k+1}})}{2^{k+1}}$

$- \frac{f(a_1)+\cdots+f(a_{2^{k+1}})}{2^{k+1}}$

$= 0$

よって

$f\left(\frac{a_1+\cdots+a_{2^{k+1}}}{2^{k+1}}\right) \geq \frac{f(a_1)+\cdots+f(a_{2^{k+1}})}{2^{k+1}}$

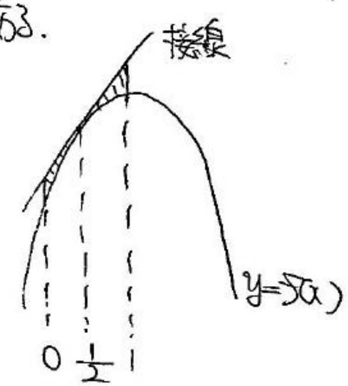
(i)(ii)よりすべての自然数k

において与式は成立.

(3)

性質(A)より$f(x)$は上に凸

である.



$x=\frac{1}{2}$での接線と$y=f(x)$,

直線$x=0$, $x=1$で囲まれた

面積は

$\int_0^1 \left\{f'\left(\frac{1}{2}\right)\left(x-\frac{1}{2}\right)+f\left(\frac{1}{2}\right)-f(x)\right\}dx$

$= \left[f'\left(\frac{1}{2}\right)\left(\frac{x^2}{2}-\frac{x}{2}\right)+f\left(\frac{1}{2}\right)x\right]_0^1$

$-\int_0^1 f(x)dx$

$= f\left(\frac{1}{2}\right) - \int_0^1 f(x)dx \geq 0$

$\therefore \int_0^1 f(x)dx \leq f\left(\frac{1}{2}\right)$