2017 岩手医科(医)

第1問

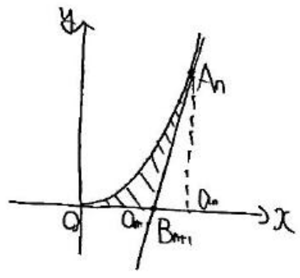
問1.

$L_n: y = 3a_n^2(x - a_n) + a_n^3$
$= 3a_n^2 x - 2a_n^3$

↓ $(a_{n+1}, 0)$ 通る

$0 = 3a_n^2 a_{n+1} - 2a_n^3$

$\therefore a_{n+1} = \frac{2}{3}a_n \quad a_n = \left(\frac{2}{3}\right)^{n-1}$



S_n
$= \int_0^{a_n} x^3 dx - \frac{1}{3}a_n \times a_n^3 \times \frac{1}{2}$
$= \frac{1}{4}a_n^4 - \frac{1}{6}a_n^4 = \frac{1}{12}a_n^4$
$= \frac{1}{12}\left(\frac{16}{81}\right)^{n-1}$

$\therefore S_1 = \frac{1}{12}$

$\sum_{n=1}^{\infty} S_n = \frac{\frac{1}{12}}{1 - \frac{16}{81}} = \frac{27}{260}$

問2.

$2wz + w = 3z + 1$

$\Leftrightarrow (2w - 3)z = -w + 1$

$\therefore z = \frac{-w+1}{2w-3} \quad (w \neq \frac{3}{2})$

↓ zが純虚数 $(w \neq 1)$

$\frac{-w+1}{2w-3} = \frac{-\bar{w}+1}{2\bar{w}-3}(-1)$

$\Leftrightarrow -2w\bar{w} + 2\bar{w} + 3w - 3$
$= 2w\bar{w} - 2w - 3\bar{w} + 3$

$\Leftrightarrow 0 = 4w\bar{w} - 5w - 5\bar{w} + 6$

$\Leftrightarrow 0 = w\bar{w} - \frac{5}{4}w - \frac{5}{4}\bar{w} + \frac{3}{2}$
$= \left(w - \frac{5}{4}\right)\left(\bar{w} - \frac{5}{4}\right) - \frac{1}{16}$

$\therefore \left|w - \frac{5}{4}\right| = \frac{1}{4}$

wは中心が$\frac{5}{4}$, 半径が$\frac{1}{2}$から2点$\frac{3}{2}$, 1を除く.

問3.

$P(x_1x_2x_3$が4の倍数$)$
$= 1 - P($すべて奇数$)$
$\quad - P(2か6が1つ出る)$
$= 1 - \left(\frac{1}{2}\right)^3 - {}_3C_1 \frac{1}{3} \cdot \left(\frac{1}{2}\right)^2$
$= 1 - \frac{1}{8} - \frac{1}{4} = \frac{5}{8}$

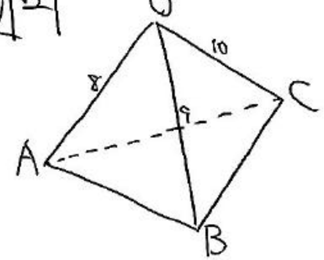
さいころ	1	2	3	4	5	6
x_1	1	2	3	0	1	2
x_2^2	1	0	1	0	1	0
x_3^3	1	0	3	0	1	0

上の表はさいころの目とx_1, x_2^2, x_3^3を4で割ったあまり.

$n(x_2^2 \equiv 1, x_3^3 \equiv 1, x_1 \equiv 2)$
$+ n(x_2^2 \equiv 1\ x_3^3 \equiv 0, x_1 \equiv 3)$
$+ n(x_2^2 \equiv 1, x_3^3 \equiv 3, x_1 \equiv 0)$
$+ n(x_2^2 \equiv 0, x_3^3 \equiv 1, x_1 \equiv 3)$
$+ n(x_2^2 \equiv 0, x_3^3 \equiv 0\ x_1 \equiv 0)$
$+ n(x_2^2 \equiv 0, x_3^3 \equiv 3, x_1 \equiv 1)$
$= 3 \cdot 2 \cdot 2 + 3 \cdot 3 \cdot 1 + 3 \cdot 1 \cdot 1$
$\quad + 3 \cdot 2 \cdot 1 + 3 \cdot 3 \cdot 1 + 3 \cdot 1 \cdot 2$
$= 12 + 9 + 3 + 6 + 9 + 6 = 45$

$P(x_1 + x_2^2 + x_3^3$が4の倍数$)$
$= \frac{45}{216} = \frac{15}{72}$

第2問



問1

$|\overrightarrow{AB}|^2 = |\vec{b} - \vec{a}|^2$
$= |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{a}|^2$ (81, 128, 64)
$= 17$

$\therefore AB = \sqrt{17}$

$|\overrightarrow{AC}|^2 = |\vec{c} - \vec{a}|^2$
$= 100 - 128 + 64 = 36$

$AC = 6 \rightarrow \triangle OAC$は直角三角形

BからΔOACにおろした垂線の足をHとおくと

$\overrightarrow{OH} = s\vec{a} + t\vec{c}$
$\overrightarrow{BH} = s\vec{a} + t\vec{c} - \vec{b}$

↓ $\overrightarrow{BH} \perp \vec{a}$, $\overrightarrow{BH} \perp \vec{c}$

$64s + 64t - 64 = 0$
$-)\ 64s + 100t - 82 = 0$
$-36t + 18 = 0$

$\therefore t = \frac{1}{2} \quad s = \frac{1}{2}$

$|\overrightarrow{BH}|^2 = \left|\frac{1}{2}\vec{a} - \vec{b} + \frac{1}{2}\vec{c}\right|^2$
$= \frac{1}{4}64 + 81 + \frac{1}{4}100$
$\quad -64 - 82 + 16$
$= 8$

四面体OABC

$= 24 \times |BH| \times \frac{1}{3}$ （$|BH| = 2\sqrt{2}$）

$= 16\sqrt{2}$ //

問2

$\vec{x} = p\vec{a} + q\vec{b} + r\vec{c}$

↓

$\begin{cases} \vec{x}\cdot\vec{a} = 64p + 64q + 64r = 1\cdot 8\cdot\cos\theta \\ \vec{x}\cdot\vec{b} = 64p + 81q + 82r = 1\cdot 9\cdot\cos\theta \\ \vec{x}\cdot\vec{c} = 64p + 82q + 100r = 1\cdot 10\cos\theta \end{cases}$

↓

$\begin{cases} -17q - 18r = -\cos\theta \\ -18q - 36r = -2\cos\theta \end{cases}$

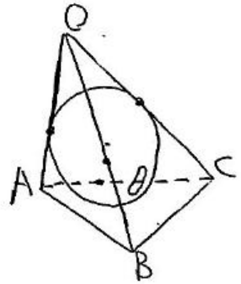
$\therefore q = 0 \quad r = \frac{1}{18}\cos\theta$

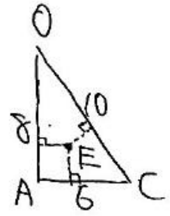
$p + q + r = \frac{1}{8}\cos\theta$

$\therefore p = \left(\frac{1}{8} - \frac{1}{18}\right)\cos\theta$

$\therefore \frac{p}{\cos\theta} = \frac{5}{72}$ //

$\frac{p}{r} = \frac{5}{72}\cos\theta \cdot \frac{18}{\cos\theta} = \frac{5}{4}$ //

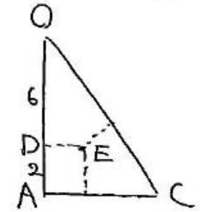




問2より）Eは△ABC内にある．

$\triangle ABC = \frac{r}{2}(8 + 6 + 10)$ （$\triangle ABC = 24$）

$\therefore r = 2$ //（Sの半径）



$\vec{OE} = \vec{OD} + \vec{DE}$

$= \frac{3}{4}\vec{a} + \frac{1}{3}\vec{AC}$

$= \frac{5}{12}\vec{a} + \frac{1}{3}\vec{c}$ //

問4

$OQ = 6$ より

$\vec{OQ} = \frac{6}{9}\vec{b} = \frac{2}{3}\vec{b}$ //

$\frac{V_2}{V_1} = \frac{6\cdot 6\cdot 6}{8\cdot 9\cdot 10} = \frac{9}{2\cdot 15} = \frac{3}{10}$ //

第3問

$x = \sin 2\theta \qquad y = \sin 3\theta$

$\frac{dx}{d\theta} = 2\cos 2\theta \qquad \frac{dy}{dx} = 3\cos 3\theta$

問1

xが最大 $\Leftrightarrow \theta = \frac{\pi}{4}$

$\therefore y = \sin\frac{3}{4}\pi = \frac{\sqrt{2}}{2}$ //

$y = \sin 3\theta = 0$

$\Leftrightarrow \theta = 0, \frac{\pi}{3}$

θは $\frac{1}{3}\pi$ に対応．ここでの接線

$y = \frac{dy}{dx}\Big|_{\theta=\frac{\pi}{3}}\left(x - \frac{\sqrt{3}}{2}\right)$

$= \frac{-3}{-1}\left(x - \frac{\sqrt{3}}{2}\right)$

$= 3x - \frac{3\sqrt{3}}{2}$ //

問2.

$y^2 = \sin^2 3\theta$

$= \frac{1 - \cos 6\theta}{2}$

$y^2\frac{dx}{d\theta}$

$= \frac{1 - \cos 6\theta}{2} 2\cos 2\theta$

$= \cos 2\theta - \cos 6\theta\cos 2\theta$

$= \cos 2\theta - \frac{1}{2}(\cos 4\theta + \cos 8\theta)$

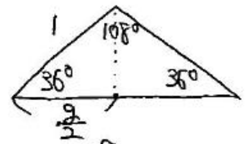
問3.

$\sin 3\theta = \sin 2\theta$

$= \sin(\pi - 2\theta)$

$\therefore 3\theta = \pi - 2\theta$

$\therefore \theta = \frac{1}{5}\pi = \theta_0$ //



$\cos\theta_0 = \frac{\frac{a}{2}}{1}$ （aは黄金数）

$= \frac{1 + \sqrt{5}}{4}$ //

$\alpha^2 = (\sin 2\theta_0)^2$

$= \sin^2 72^\circ$

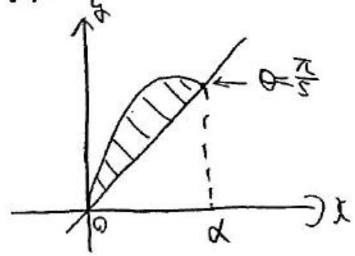
$= \frac{1 - \cos 144^\circ}{2}$

$= \frac{1 + \cos 36^\circ}{2}$

$= \frac{4 + 1 + \sqrt{5}}{8}$

$= \frac{5 + \sqrt{5}}{8}$ //

問14.



V

$= \int_0^{\alpha} \pi y^2 dx - \alpha^2 \pi \cdot \alpha \cdot \frac{1}{3}$

$= \pi \int_0^{\theta_0} y^2 \frac{dx}{d\theta} d\theta - \frac{\pi}{3}\alpha^3$

$= \pi \int_0^{\theta_0} \left\{ \cos 2\theta - \frac{1}{2}\cos 4\theta - \frac{1}{2}\cos 8\theta \right\} d\theta - \frac{\pi}{3}\alpha^3$

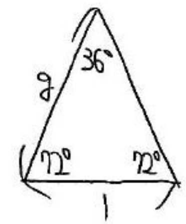
$= \pi \left[\frac{1}{2}\sin 2\theta - \frac{1}{8}\sin 4\theta - \frac{1}{16}\sin 8\theta \right]_0^{\theta_0} - \frac{\pi}{3}\alpha^3$

$= \pi \left(\frac{1}{2}\sin 2\theta_0 - \frac{1}{8}\sin 4\theta_0 - \frac{1}{16}\sin 8\theta_0 \right) - \frac{\pi}{3}\alpha^3$

$\sin 8\theta_0 = \sin 288^\circ = \sin(360^\circ - 72^\circ)$
$= -\sin 2\theta_0$

$= \pi \left(\frac{9}{16}\sin 2\theta_0 - \frac{1}{4}\sin 2\theta_0 \cos 2\theta_0 \right) - \frac{\pi}{3}\alpha^3$

$= \frac{\pi \sin 2\theta_0}{16}(9 - 4\cos 2\theta_0) - \frac{\pi}{3}\alpha^3$ ($2\theta_0 = 72^\circ$)



$\cos 72^\circ = \frac{\frac{1}{2}}{a} = \frac{\sqrt{5}-1}{4}$

$= \frac{\pi\alpha}{16}\left\{ 9 - (\sqrt{5}-1) \right\} - \frac{\pi}{3}\alpha^3$

$= \frac{\pi\alpha}{16}(10 - \sqrt{5}) - \frac{\pi}{3}\alpha^3$

$\therefore \frac{V}{\alpha}$

$= \frac{\pi}{16}(10 - \sqrt{5}) - \frac{\pi}{3}\alpha^2$ ($\alpha^2 = \frac{5+\sqrt{5}}{8}$)

$= \frac{\pi}{48}\left\{ 30 - 3\sqrt{5} - (10 + 2\sqrt{5}) \right\}$

$= \frac{20 - 5\sqrt{5}}{48}\pi$ //