2016.東京医科大

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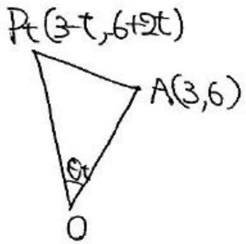
(1)



$$\cos\theta_t=\frac{\overrightarrow{OP_t}\cdot\overrightarrow{OA}}{|\overrightarrow{OP_t}||\overrightarrow{OA}|}$$

$$=\frac{9-3t+36+12t}{\sqrt{(3-t)^2+(6+2t)^2}\sqrt{45}}$$

$$=\frac{3t+15}{\sqrt{5t^2+18t+39}\cdot\sqrt{15}}$$

$$\lim_{t\to\infty}\cos\theta_t=\frac{3}{5}$$ //

(2) 解をα, 4αとおくと

解と係数の関係より

$$\begin{cases}\alpha+4\alpha=-a\\ \alpha\cdot4\alpha=4a \iff \alpha^2=a\end{cases}$$

↓

$$5\alpha=-\alpha^2$$

$\therefore \alpha=-5\ (\because \alpha\neq0)$, $\underline{a=25}$ //

2

(1)

$$|\vec{c}|^2=|(1-t)\vec{a}+t\vec{b}|^2$$

$$=(1-t)^2+2(1-t)t\,\vec{a}\cdot\vec{b}+t^2$$

$$|\vec{a}-\vec{b}|^2=|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$$

$$=2-2\vec{a}\cdot\vec{b}=\frac{25}{44}$$

$$\therefore 2\vec{a}\cdot\vec{b}=\frac{63}{44}$$

$$=2t^2-2t+1+(1-t)t\frac{63}{44}$$

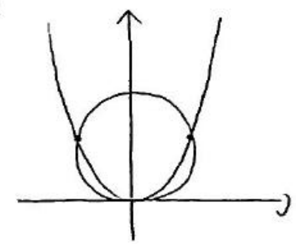
$$=\frac{25}{44}t^2-\frac{25}{44}t+1=5$$

$$\iff 25t^2-25t-176=0$$

$$\iff (5t-16)(5t+11)=0$$

$\therefore \underline{t=\frac{16}{5}}$ $(\because t>0)$ //

(2)



連立

$$\frac{5}{4}y+y^2-2ay+a^2=a^2$$

$$\iff y\left\{y+\left(\frac{5}{4}-2a\right)\right\}=0$$

$$\iff y\left\{y-\left(2a-\frac{5}{4}\right)\right\}=0$$

原点以外の交点のy座標が

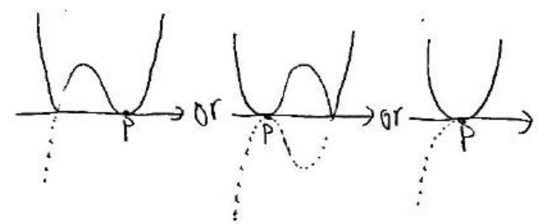
図から正でなければならないので

$$2a-\frac{5}{4}>0$$

$\therefore \underline{a>\frac{5}{8}}$ //

3

問題文から



が考えられるので $f(x)=|g(x)|$

としたとき

$$\begin{cases}g(p)=2p^3-p^2-ap-36=0\\ g'(p)=6p^2-2p-a=0\end{cases}$$

が必要. aを消すと

$$2p^3-p^2-(6p^2-2p)p-36=0$$

$$\iff 4p^3-p^2+36=0$$

$$\iff (p+2)(\underbrace{4p^2-9p+18}_{D<0より正})=0$$

$$\therefore p=-2$$

$$\therefore a=6p^2-2p=28$$

このとき

$$g'(x)=6x^2-2x-28=0$$

$$\iff 3x^2-x-14=0$$

$$\iff (3x-7)(x+2)=0$$

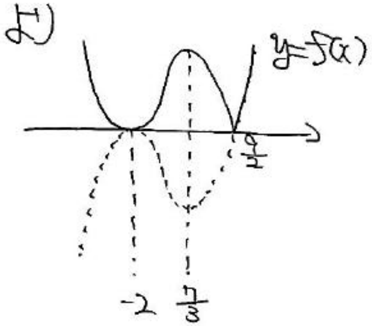
$$g(x)=2x^3-x^2-28x-36$$

$$=(x+2)(2x^2-5x-18)$$

$$=(x+2)(2x-9)(x+2)$$

$$=(x+2)^2(2x-9)$$

より



図から明らかに $\underline{x=\frac{9}{2}}$ で

微分不可能 //

4

(1)

$y = \dfrac{1}{(x-\frac{1}{2})^2+\frac{3}{4}}$

$0 \leqq x \leqq 1$ で $y>0$ より

S

$= \displaystyle\int_0^1 \frac{1}{(x-\frac{1}{2})^2+\frac{3}{4}}dx$

$x-\dfrac{1}{2} = \dfrac{\sqrt{3}}{2}\tan\theta$ とおく

$dx = \dfrac{\sqrt{3}}{2}\cdot\dfrac{1}{\cos^2\theta}d\theta$

$= \displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{\frac{3}{4}(1+\tan^2\theta)}\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{\cos^2\theta}d\theta$

$= \displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{2}{\sqrt{3}}d\theta$

$= 2\displaystyle\int_0^{\frac{\pi}{6}} \frac{2}{\sqrt{3}}d\theta$

$= 2\cdot\dfrac{2}{\sqrt{3}}\cdot\dfrac{\pi}{6}$

$= \dfrac{2\sqrt{3}}{9}\pi$ //

(2)

V

$= \pi\displaystyle\int_0^1 \left\{\frac{1}{(x-\frac{1}{2})^2+\frac{3}{4}}\right\}^2 dx$

$= \pi\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{\frac{9}{16}(1+\tan^2\theta)^2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{\cos^2\theta}d\theta$

$= \pi\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{8\sqrt{3}}{9}\cos^2\theta\, d\theta$

$= \dfrac{16\sqrt{3}\pi}{9}\displaystyle\int_0^{\frac{\pi}{6}} \frac{1+\cos 2\theta}{2}d\theta$

$= \dfrac{16\sqrt{3}\pi}{9}\left[\dfrac{\theta}{2}+\dfrac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{6}}$

$= \dfrac{4\sqrt{3}\pi^2}{27} + \dfrac{2}{3}\pi$ //